This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

SCHOOL ARITHMETICS

BOOK THREE

WENTWORTH-SMITH

Educ.T 119.22.890 (III)

HARVARD
COLLEGE
LIBRARY

The School Committee of the City of Boston
ADMINISTRATION LIBRARY
15 Beacon Street

School Committee of the City of Boston
ADMINISTRATION LIBRARY

WENTWORTH-SMITH MATHEMATICAL SERIES

SCHOOL ARITHMETICS

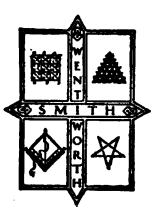
BOOK THREE

BY

GEORGE WENTWORTH

AND

DAVID EUGENE SMITH



GINN AND COMPANY

BOSTON · NEW YORK · CHICAGO · LONDON
ATLANTA · DALLAS · COLUMBUS · SAN FRANCISCO

Educ T 119.20.875 (III)
Educ T 119.22.890 (III)
HARVARD COLLEGE LIBRARY
GIFT OF
BOSTON SCHOOL COMMITTEE LIBRARY
NOV 9 1936

COPYRIGHT, 1919, 1920, BY GEORGE WENTWORTH
AND DAVID EUGENE SMITH
ENTERED AT STATIONERS' HALL
ALL RIGHTS RESERVED

522.6

The Athenæum Press
GINN AND COMPANY · PROPRIETORS · BOSTON · U.S.A.

PREFACE

This book is intended for pupils in Grades VII and VIII. It covers the work as generally taught in a majority of the schools in this country. Most of our elementary schools extend through eight school years, and the topics studied are generally rather uniform. For systems in which the junior high school begins in the seventh grade the Junior High School Mathematics, Books I, II, and III, published in this series, will be found well arranged, but for the more common type of school the work of Grades VII and VIII will for a long time be of the nature herein set forth.

For the convenience of teachers the book has been divided into four chapters, each chapter containing sufficient work for a half year. Teachers may omit as many exercises as necessary to allow for carrying out this plan. Indeed, one of the greatest failures on the part of teachers using any textbook is due to the fact that they feel that all exercises must be solved. A textbook should be the servant of the teacher, not the master, and under each topic the teacher should choose only such exercises and as many of them as are necessary to secure good work on the part of the pupils.

The arrangement of the book is topical, so that a pupil stays long enough with a subject at one time to acquire that feeling of mastery which is his right and privilege. Along with this sequence of topics, however, there are several features which are noteworthy. One is the Little Examinations, a brief series of tests covering each chapter in turn; a second feature is the Review and Drill section, also placed near the end of each chapter, and furnishing a cumulative review of all preceding work; a third is the sets of Problems without Numbers and Problems for Completion, each of which requires new lines

of independent thought on the part of the pupils; and a fourth is the Material for Frequent Drill found on page 315. By the aid of these features a teacher may be assured that the pupil is instructed in and kept refreshed upon those essentials of computation without which he cannot hope to succeed. There have also been inserted several elementary psychological tests relating to arithmetic, which may be used or not in the discretion of the teacher.

In the theory of the work the authors have no more sympathy with the idea that a pupil should be told to do a thing in a certain way, with no knowledge of why this way is the right one, than they have with the notion that he must explain every operation with all the care that a textbook writer would show. They believe that every process should be learned with an appeal to the pupil's understanding, and that thereafter it should become entirely mechanical; and in this way each operation has been presented in this book.

A feature of the book is the large number of modern applications to the actual needs of our American people, but these applications are not permitted to exclude the abstract drill work without which no pupil has ever become a good computer. To balance adequately the abstract and the concrete, the drill work and the applied problem, the review work and the new material, has been one of the earnest endeavors of the authors in the preparation of this series.

The authors hope that their effort to prepare a perfectly usable textbook, free from those eccentricities which, while attracting momentary attention, fail to give the pupils the power they need, will prove helpful to the schools of the country.

GEORGE WENTWORTH
DAVID EUGENE SMITH

CONTENTS

CHAPTER I

PAGE

I. Review and Efficiency Tests 1
II. Percentage 25
III. Percentage Applied 51
IV. Little Examinations 78
V. Review and Drill 79
Minimum Essentials 79
Problems without Numbers 80
Problems for Completion 81
Alertness Test 82
Industrial Problems 83

CHAPTER II

I. Practical Measurements 85
II. Problems of Daily Life 123
III. Little Examinations 140
IV. Review and Drill 141
Minimum Essentials 141
Problems without Numbers 142
Problems for Completion 143
Alertness Test 144
Industrial Problems 145

CHAPTER III

I. Banks and Banking 149
II. Our Duty to the Government 191
III. Little Examinations 210
IV. Review and Drill 211
Minimum Essentials 211
Problems without Numbers 212
Problems for Completion 213
Alertness Test 214
Industrial Problems 215

CHAPTER IV

PAGE

I. Business Relations 219
II. Graphs 247
III. Advanced Mensuration 261
IV. Little Examinations 280
V. Review and Drill 281
Minimum Essentials 281
Problems without Numbers 282
Problems for Completion 283
Alertness Test 284
Industrial Problems 285

GENERAL REVIEW 293
MATERIAL FOR FREQUENT DRILL 315
SUPPLEMENT 321
TABLES FOR REFERENCE 335
SUGGESTIONS TO TEACHERS 339
INDEX 341

SCHOOL ARITHMETICS

BOOK THREE

CHAPTER I

I. REVIEW AND EFFICIENCY TESTS

Why we Review Arithmetic. Why do people fail so often in the ordinary arithmetic of daily life? *Because they cannot add, subtract, multiply, and divide correctly.*

Why are so many people inaccurate in their work? Chiefly *because they do not know, with certainty, such simple facts as these:*

$6 + 8 = 14$	$2\frac{1}{2} + 3\frac{3}{4} = 6\frac{1}{4}$	$7 \times \$1.50 = \$10.50$
$17 - 9 = 8$	$\frac{3}{4}$ of $24 = 18$	$7 \times 8 + 3 = 59$
$6 \times 9 = 54$	$2\frac{1}{2} \times 16 = 40$	$68 \div 7 = 9$, and 5 over
$72 \div 8 = 9$	$1\frac{1}{2} \times 2\frac{1}{4} = 3\frac{3}{8}$	4% of $\$15 = \$0.60$

Why do so many people fail on simple facts which everyone needs in ordinary life? Chiefly *because they did not learn them thoroughly in school.* They studied about arithmetic, but they failed to learn the most important facts about numbers.

Why do you need to review your studies? *So that you will not be counted among the failures* when you begin your life work.

The review consists in drill upon the fundamentals. More work is provided than most teachers will need to use. Teachers should take only what is necessary and reach the new work as quickly as possible.

READING AND WRITING NUMBERS

Numbers 1 to 4, oral

1. Large business houses have machines for filling out their checks. These machines print in words the number $142.75 like this: "One hundred forty-two dollars and seventy-five cents," and then print the figures. Why do they print both the words and the figures? Why do they print "One hundred forty-two" instead of the old-fashioned "One hundred and forty-two"?

Efficiency avoids waste of time and of words. Efficiency requires accuracy. If you write "one hundred *and* forty-two thousandths" for 0.142, you will have no good way in which to write 100.042. *Omit the "and" except between a whole number and a fraction,* as in $2\frac{1}{2}$, 3.42, 100.023, and the like.

2. In the Fourth Liberty Loan the amount issued was $6,989,047,000. Read this number.

This is to show the use and the proper reading of large numbers.

Read the following statements:

3. The amount of the First Liberty Loan issued during the World War was $2,000,000,000.

4. The amount of the Second Liberty Loan issued was $3,808,766,150, and the amount of the Third Liberty Loan issued was $4,176,516,850.

Write the following numbers in words:

5. $2,718,314.
6. $14,800,621.
7. 3,000,003.
8. 15,015,015.
9. $5,207,836,298.37.
10. $8,008,008,008.08.

Write the following numbers in figures:

11. Four million, four.
12. Seven billion, twenty-one.
13. Sixty-three million, two hundred dollars and eight cents.

MEANING OF COMMON WORDS

All work oral

1. What do you mean by *units' place*? by *hundreds' place*?

2. What does the word *unit* mean? Give an example of the kind of unit that you mean when you speak of *units' place*; when you speak of a *unit of length*; of a *unit of area*.

3. What do you mean by a *Roman numeral*? Where are such numerals commonly used? Give an example of the two Roman numerals made up by combining I and V.

4. What do you mean by the word *addend*? *subtrahend*? *minuend*? *multiplier*? *multiplicand*? *divisor*? *dividend*?

5. In what operation is the result called the *quotient*? the *sum*? the *difference*? the *product*?

6. What do you mean by the word *fraction*? *numerator*? *denominator*? by the expression *mixed number*?

7. What do you mean by the expression *per cent*? Illustrate by the case of 6%.

8. What do you mean by an *aliquot part* of a number? Illustrate by giving four aliquot parts of \$1, expressing each result both as a fraction of \$1 and also as cents.

Strictly, an aliquot part is a part like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, but not $\frac{2}{3}$, $\frac{3}{4}$, and the like.

9. What do you mean by a *denominate number*? Give three illustrations of such a number.

10. What does the word *decimal* mean? Give two cases in which it is used in arithmetic.

The purpose of the above questions is not to secure memorized definitions, but to see if the pupils can use the terms properly.

It will be interesting to many pupils to consult a large dictionary and see the origin of such words as are mentioned on this page.

ADDING ONE-FIGURE NUMBERS

All work oral

Add the following rows from left to right, checking by adding from right to left:

1.	3	8	4	9	5	6	3	6	8	7	3	7	5	8	4	7	6	4
2.	7	5	3	4	7	8	6	5	3	9	2	9	7	7	4	6	7	8
3.	5	6	4	5	7	8	2	6	9	7	6	5	6	4	5	8	7	5
4.	6	5	6	7	4	5	8	7	5	8	4	2	6	6	5	9	6	7
5.	4	6	5	6	5	6	9	6	6	7	5	1	7	5	6	8	7	5
6.	2	7	5	8	6	5	4	5	9	6	4	6	6	7	5	6	7	4
7.	1	4	4	3	8	7	7	4	4	8	3	8	8	6	5	5	8	8
8.	0	3	5	2	9	6	8	3	5	7	4	7	9	5	6	4	9	8
9.	8	2	6	1	9	6	9	2	6	6	5	8	8	4	7	5	7	9
10.	9	5	6	5	6	5	9	6	7	6	6	0	8	4	7	7	8	5

In Exs. 1–10 the pupil should add and check each row in 30 sec.

Add the following columns upward, checking by adding downward:

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
2	7	5	3	9	8	4	6	1	3
6	2	6	8	7	8	7	6	7	8
8	8	4	5	8	9	3	2	6	2
7	3	7	6	6	5	8	7	8	9
3	7	3	4	3	4	9	8	4	6
4	2	8	9	4	3	6	7	3	4
9	6	4	3	2	3	8	7	4	5
5	9	6	8	6	5	2	1	9	8
8	8	9	7	4	5	8	9	9	7

In Exs. 11–20 the pupil should add and check each column in 15 sec.

SELF-CHECKING ADDITION

1. Copy the following; add each line horizontally and write the sum in the right-hand column; add each column and write the sum below; then add the sums at the right and add the sums underneath. The results should be the same and the number should be written in the space marked ***.

4	3	4	2	8	6	9	4	8	3	8	5	
3	9	7	3	2	1	7	2	6	5	2	4	
7	2	2	9	7	4	5	9	4	7	9	3	
6	8	8	6	5	2	6	8	2	9	7	2	
4	6	3	4	3	5	3	1	7	6	4	6	
												***

2. A traveling man's expense account for a week is given in the statement below. Perform the additions as in Ex. 1.

DATE	FARES		BOARD		TELE-PHONE		POSTAGE		MISCEL-LANEOUS		TOTAL	
1	2	20	3	90		20				55		
2	1	40	3	85		30		12		60		
3		60	4	20		75		08	1	10		
4	1	95	4	10	1	20		16		75		
5	2	60	3	95		45		24		45		
6	1	46	4	25		25		12	1	00		
7	2	32	4	35		20		18		75		
TOTAL												

ADDING TWO-FIGURE NUMBERS

All work oral

1. Add 39 and 26 as a business man would. Think of $39 + 26$ as $39 + 20 + 6$ and then as $59 + 6$.

Add the following as in Ex. 1:

2.	22	22	22	22	22	42	65
	10	16	18	28	29	39	38
3.	27	34	45	55	78	58	47
	36	27	35	26	12	14	27
4.	38	68	27	47	57	68	78
	18	18	19	19	29	12	13
5.	90	95	95	96	98	57	67
	10	10	15	15	64	13	13
6.	85	85	86	88	64	89	99
	20	25	25	37	36	42	56
7.	58	58	58	58	65	65	67
	50	52	56	57	40	45	48
8.	77	77	78	78	78	78	79
	20	23	30	32	35	56	68

This is precisely the kind of calculation we use in making purchases. If the pupil gets into the habit of adding from the left, in cases like those given above, he will soon come to do it easily, rapidly, and accurately.

DRILL WORK IN ADDITION

Copy the following, add, and check each result:

1.	2.	3.	4.	5.	6.
$3.27	$5.75	$5.80	$7.28	$5.96	$6.89
.42	2.86	2.60	2.96	6.82	9.72
1.27	.92	3.07	.39	3.48	8.34
2.68	3.	4.26	8.72	7.56	4.08
.39	4.05	.39	.48	8.29	6.29
4.	.75	.57	6.75	3.07	.70
7.68	2.10	2.61	3.29	6.40	8.32
1.29	3.76	1.75	4.36	3.36	1.49

The work given above is precisely the kind that everyone meets in adding grocery bills and in keeping accounts. The teacher should first have the pupils copy Exs. 1–6; then, at a given signal, all should begin, and the teacher should time each paper. The teacher's marks should show (1) the number of correct results, (2) the number of incorrect ones, (3) the time. Since each result is checked, an incorrect result should be rare. If the work is reviewed occasionally, the record should show a decrease in time and an increase in accuracy for most of the pupils.

Copy the following, add, and check each result:

7.	8.	9.	10.	11.
$2864.75	$5873.96	473.82	5738.2	5.428
3904.08	2981.44	698.87	698.3	3.206
628.37	3837.62	438.37	4000.	4.783
5072.99	897.68	507.96	783.6	.896
832.87	3278.48	296.81	3408.8	4.008
6271.03	687.49	300.09	629.3	.009
876.53	853.61	41.76	4426.3	6.71
273.06	3400.07	391.63	3900.	3.409
4269.34	289.63	512.19	862.3	2.672

SUBTRACTING TWO-FIGURE NUMBERS

All work oral

1. How do you subtract 28¢ from 54¢? Do you have to take pencil and paper to do what every business man does in his head? If it were 54¢ − 30¢ you would give the result at once as 24¢; but 28¢ lacks only 2¢ of being 30¢, so the result is 24¢ + 2¢. So think like this: "28¢ + **2**¢ = 30¢ and 30¢ + **24**¢ = 54¢," and then just add the 2¢ and 24¢.

2. Think of 61 − 34 by thinking that 34 + **6** = 40 and then that 40 + **21** = 61. What is the result?

This is simply the familiar making-change method of subtraction.

Subtract in each of the following cases:

3.	47	47	47	57	56	54	92
	32	37	38	38	38	37	49
4.	60	61	61	71	82	83	85
	32	32	35	37	36	45	58
5.	72	71	70	73	73	84	92
	42	42	42	47	48	58	76
6.	98	92	90	90	92	92	93
	52	52	52	61	63	68	79
7.	87	87	86	84	84	92	93
	30	37	37	37	39	39	65

If the teacher finds the class better able to take the following short method, it will be found about equally good. Think of 54−28 as 54−20−8, or 34 − 8, or 26. In any case, pencil and paper should not be necessary.

DRILL WORK IN SUBTRACTION

Numbers 1 to 8, oral

1. If you make a purchase amounting to \$2.46 and give the dealer a \$5 bill, how much change is due?

The pupil should think, as the dealer does, "\$2.46 + 4¢ = \$2.50, \$2.50 + 50¢ = \$3, \$3 + \$2 = \$5, and so the change is \$2.54."

Make change for \$5 on purchases as follows:

2. \$2.57. 3. \$1.72. 4. \$2.61. 5. \$0.36. 6. \$3.52.

7. If you have to subtract 4726 from 9000, it is easier to begin at the left, thinking of 9000 as 8990 + 10, written as here shown. Thinking of it in this way, how much is 9000 − 3268?

8	9	9	10
4	7	2	6
4	2	7	4

8. How do you check your work in subtraction? Illustrate by the case of 42 − 17 = 25; by the case of 100 − 72.

Subtract in each of the following cases:

9. 3000	12. 5726	15. 52,123	18. 128,300
1786	3987	18,068	69,426
10. 4000	13. 8209	16. 61,023	19. 301,220
2938	6138	52,836	49,726
11. 5100	14. 7012	17. 72,133	20. 802,308
2673	3648	29,239	476,829

The pupils should copy each column, subtract, and check in 1 min. Since four methods of subtraction are in common use in this country, and the pupil has by this time acquired mechanical efficiency in one of them, it is not desirable hereafter to insist upon any particular one of the methods.

DRILL WORK IN SUBTRACTION

Numbers 1 to 5, oral

In each of the following cases begin at the left and subtract from 50 until 0 remains:

1.	5	2	3	4	6	4	2	4	5	7	6	2

The pupil should say: "45, 43, 40," and so on to 0.

2.	5	7	3	5	4	6	5	2	3	4	2	4
3.	3	7	2	8	5	6	5	4	3	2	2	3
4.	7	5	2	3	8	1	8	6	5	4	0	1
5.	5	8	7	6	5	1	2	3	4	2	4	3

Subtract in each of the following cases, and check:

6. \$1768	**11.** \$2089.70	**16.** \$8273.86	**21.** \$12,782.68
492	379.69	4969.98	698.98
7. \$4107	**12.** \$8231.21	**17.** \$7234.32	**22.** \$23,072.08
298	4681.96	2989.75	16,789.79
8. \$5123	**13.** \$9210.82	**18.** \$6000.	**23.** \$30,292.72
1768	7876.98	4783.81	17,693.49
9. \$8234	**14.** \$8023.81	**19.** \$7000.	**24.** \$52,293.37
3968	6789.96	4383.69	16,820.98
10. \$9230	**15.** \$7333.03	**20.** \$8000.	**25.** \$75,000.
4768	5768.79	3270.08	968.39

26. From the sum of \$432.86, \$981.08, and \$75.82 take the sum of \$42.38, \$681.96, \$29.07, and \$57.36.

Multiplication. You know how to multiply one number by another, and we shall now briefly review the operation.

4674	$45.87	3.24
49	2007	0.21
42066	321 09	324
18696	9174	648
229026	$92061.09	0.6804

1. Multiply 4674 by 49.

Here we multiply 4674 first by 9 and then by 40 and add the products.

2. Multiply $45.87 by 2007.

The products corresponding to the zeros in the multiplier are zero, and therefore, as you have already learned, they need not be written.

3. Multiply 3.24 by 0.21.

In this example we multiply in the ordinary way, and notice that 1 (hundredth) × 4 (hundredths) = 4 (ten-thousandths) and that the 4 of the product must therefore go in the ten-thousandths' place.

From Ex. 3 we have the following useful rule:

To multiply by a decimal, multiply as if by an integer and point off from the right as many decimal places in the product as there are decimal places in both factors together, prefixing zeros if necessary.

Cross Multiplication. With reasonably small numbers the method known as *cross multiplication* is commonly used.

For example, multiply $0.48 by 35.

$5 \times 8 = 40$; write 0 and "carry" 4;

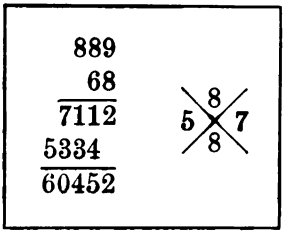
$3 \times 8 + 4 = 28$
$5 \times 4 = 20$ } making 48; write 8 and "carry" 4;

$3 \times 4 + 4 = 16$; write 16.

$0.48
35
$16.80

Checks in Multiplication. We may check our work in multiplication in various ways. If we are multiplying 889 by 68, for example, we may check by multiplying 68 by 889. Instead of this we may multiply 889 by 34, which is half of 68, and then multiply the product by 2. Each of these checks, however, takes too much time, and so we rarely use either of them.

Check of Nines. Another check, which is much simpler than either of the others and rarely fails to discover an error, is known as the *check of casting out nines* or simply the *check of nines.* Briefly described, it is applied as follows:



In the multiplier, $6+8=14=9+5$, we reject, or "cast out," the 9, leaving 5. We write 5 in the left angle of the cross, as shown.

In the multiplicand, $8+8+9=25=2\times 9+7$, we cast out the two 9's, leaving 7. We write 7 in the angle at the right.

The product of this 5 and 7 is 35, or $3\times 9+8$. We cast out the three 9's, leaving 8. We write 8 in the angle at the top.

In the product, $6+0+4+5+2=17=9+8$, we cast out the 9, leaving 8. We write 8 in the angle at the bottom.

Since the figure at the top (8) agrees with the figure at the bottom (8) the work is probably correct.

This check is so interesting and valuable that its use is recommended, but this is optional with the teacher. No further explanation should be attempted in an elementary class. The check is widely used by computers and will save the pupils from many mistakes in their work in arithmetic.

The work of casting out nines can often be greatly shortened by not including in the sum any 9 or any combination of digits whose sum is 9. Thus, in 60,452, since $4+5=9$, we see that the remainder is $6+2$, or 8.

DRILL WORK IN MULTIPLICATION

Multiply the following:

1. 17,236	3. 15,009	5. 61,432	7. 87,087	9. 50,752
24	37	53	87	86
2. 31,408	4. 28,808	6. 60,098	8. 69,960	10. 37,823
46	49	72	36	58

The above exercises should be solved and the check of nines applied in 6 min. after the numbers are copied. Speed tests should begin after the pupils have all written the numbers on paper. Remember that accuracy is more important than speed and that many good computers are not particularly rapid in their work.

Multiply by cross multiplication:

11. 73	12. 82	13. 78	14. 64	15. 76	16. 87
21	61	32	24	35	56

Exs. 11–16 should be omitted unless cross multiplication has been studied by the class. The method is useful in making out or verifying bills of goods. The check of nines will usually detect all errors.

Multiply the following:

17. 401×3664.	25. $283 \times \$48.25$.	33. 704×6.25.
18. 503×2763.	26. $342 \times \$29.75$.	34. 172×31.7.
19. 630×4876.	27. $276 \times \$49.42$.	35. 776×0.125.
20. 580×6924.	28. $428 \times \$75.19$.	36. 0.175×424.
21. 847×6476.	29. 3574×0.05.	37. 1.75×424.
22. 972×3502.	30. 0.25×536.	38. 175×424.
23. 707×8763.	31. 0.25×5.36.	39. 175×4.24.
24. 5002×7983.	32. 0.25×0.536.	40. 0.425×584.

Aliquot Part. A number that will exactly divide another number is called an *aliquot part* of that number.

Thus, 5, 10, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, and $33\frac{1}{3}$ are aliquot parts of 100; $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10; 125, $333\frac{1}{3}$, and 250 are aliquot parts of 1000.

Aliquot Parts of 100. Memorize these aliquot parts of 100:

$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100	$20 = \frac{1}{5}$ of 100
$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100	$10 = \frac{1}{10}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100	$5 = \frac{1}{20}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100	$4\frac{1}{6} = \frac{1}{24}$ of 100	$2 = \frac{1}{50}$ of 100

Other Important Fractional Parts of 100. Memorize the following important fractional parts of 100:

$75 = \frac{3}{4}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100	$66\frac{2}{3} = \frac{2}{3}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100	$83\frac{1}{3} = \frac{5}{6}$ of 100

Important Fractional Parts of \$1. Memorize the following:

$5¢ = \$\frac{1}{20} = \$0.05$	$40¢ = \$\frac{2}{5} = \$0.40$
$6\frac{1}{4}¢ = \$\frac{1}{16} = \$0.06\frac{1}{4}$	$50¢ = \$\frac{1}{2} = \$0.50$
$8\frac{1}{3}¢ = \$\frac{1}{12} = \$0.08\frac{1}{3}$	$60¢ = \$\frac{3}{5} = \$0.60$
$10¢ = \$\frac{1}{10} = \$0.10$	$62\frac{1}{2}¢ = \$\frac{5}{8} = \$0.625$
$12\frac{1}{2}¢ = \$\frac{1}{8} = \$0.125$	$66\frac{2}{3}¢ = \$\frac{2}{3} = \$0.66\frac{2}{3}$
$16\frac{2}{3}¢ = \$\frac{1}{6} = \$0.16\frac{2}{3}$	$70¢ = \$\frac{7}{10} = \$0.70$
$20¢ = \$\frac{1}{5} = \$0.20$	$75¢ = \$\frac{3}{4} = \$0.75$
$25¢ = \$\frac{1}{4} = \$0.25$	$80¢ = \$\frac{4}{5} = \$0.80$
$30¢ = \$\frac{3}{10} = \$0.30$	$83\frac{1}{3}¢ = \$\frac{5}{6} = \$0.83\frac{1}{3}$
$33\frac{1}{3}¢ = \$\frac{1}{3} = \$0.33\frac{1}{3}$	$87\frac{1}{2}¢ = \$\frac{7}{8} = \$0.875$
$37\frac{1}{2}¢ = \$\frac{3}{8} = \$0.375$	$90¢ = \$\frac{9}{10} = \$0.90$

Use of Fractional Parts in Multiplication. The aliquot parts and other fractional parts studied on page 14 are very useful in multiplication, as you learned in Book Two. For example:

Do not multiply by 25 in full form; just multiply by 100 and then divide by 4.

This is because $25 = \frac{100}{4}$. It is easier to divide by 4 than to multiply by 25. Thus, $25 \times 79 = 7900 \div 4 = 1975$.

Do not multiply by 125 in full form; just multiply by 1000 and then divide by 8.

Do not multiply in full form $12\frac{1}{2}¢$ by any number; it is easier to multiply $\$\frac{1}{8}$ by the number, whatever it is.

Always use fractional parts when it is easier to do so.

USE OF FRACTIONAL PARTS

Multiply each of the following by 25:

1. 740. 2. 964. 3. 1284. 4. 17,225. 5. 14,007.

Multiply each of the following by 125:

6. 648. 7. 544. 8. 1488. 9. 15,230. 10. 27,634.

Find the value of each of the following:

11. 5×848.
12. $12\frac{1}{2} \times 72$.
13. $6\frac{1}{4} \times 160$.
14. $33\frac{1}{3} \times 96$.
15. $16\frac{2}{3} \times 78$.
16. $8\frac{1}{3} \times 144$.
17. $4\frac{1}{6} \times 480$.
18. $37\frac{1}{2} \times 16$.
19. $62\frac{1}{2} \times 72$.
20. $87\frac{1}{2} \times 88$.
21. $66\frac{2}{3} \times 96$.
22. $83\frac{1}{3} \times 78$.
23. $12 \times 8\frac{1}{3}¢$.
24. $16 \times 12\frac{1}{2}¢$.
25. $36 \times 16\frac{2}{3}¢$.
26. $168 \times 25¢$.
27. $9 \times 33\frac{1}{3}¢$.
28. $16 \times 37\frac{1}{2}¢$.
29. $32 \times 62\frac{1}{2}¢$.
30. $48 \times 66\frac{2}{3}¢$.
31. $15 \times 12\frac{1}{2}¢$.
32. $20 \times 16\frac{2}{3}¢$.
33. $25 \times 37\frac{1}{2}¢$.
34. $35 \times 66\frac{2}{3}¢$.

DRILL WORK IN MULTIPLICATION

Multiply the following:

1. $10 \times \$0.25$.
2. $10 \times \$8251$.
3. $10 \times \$6.25$.
4. $10 \times \$5.50$.
5. $100 \times \$0.37$.
6. $100 \times \$2.87\frac{1}{2}$.
7. $1000 \times \$0.25$.
8. $1000 \times \$6.37\frac{1}{2}$.
9. $50 \times \$64{,}224$.
10. $50 \times \$752.20$.
11. $50 \times \$861.55$.
12. $25 \times \$144$.
13. $25 \times \$640$.
14. $25 \times \$325$.
15. $675 \times \$25$.
16. $25 \times \$25.50$.
17. $25 \times \$75.25$.
18. $12\frac{1}{2} \times \$6400$.
19. $12\frac{1}{2} \times \$4880$.
20. $125 \times \$640$.
21. $125 \times \$6400$.
22. $125 \times \$2800$.
23. $33\frac{1}{3} \times 36$.
24. $33\frac{1}{3} \times 48$.
25. $33\frac{1}{3} \times 144$.
26. $33\frac{1}{3} \times \$312.30$.
27. $16\frac{2}{3} \times 12{,}630$.
28. $16\frac{2}{3} \times \$636.30$.
29. $5 \times 47{,}664.80$.
30. $5 \times \$210{,}650$.
31. $125 \times \$84{,}480$.
32. $125 \times \$844.80$.
33. $16\frac{2}{3} \times 4{,}800{,}000$.

Prove the following statements:

34. $\left.\begin{matrix}654\\732\end{matrix}\right\} + \left\{\begin{matrix}654\\267\end{matrix}\right\} = 653{,}346$, or $\left\{\begin{matrix}654\\999\end{matrix}\right.$

This is a short way of stating that 732×654 added to 267×654 is equal to 653,346, or to the product of 999×654.

35. $\left.\begin{matrix}328\\245\end{matrix}\right\} + \left\{\begin{matrix}328\\754\end{matrix}\right\} = 327{,}672$, or $\left\{\begin{matrix}328\\999\end{matrix}\right.$

36. $\left.\begin{matrix}745\\620\end{matrix}\right\} + \left\{\begin{matrix}745\\379\end{matrix}\right\} = 744{,}255$, or $\left\{\begin{matrix}745\\999\end{matrix}\right.$

37. $\left.\begin{matrix}5627\\4832\end{matrix}\right\} + \left\{\begin{matrix}5627\\5167\end{matrix}\right\} = 56{,}264{,}373$, or $\left\{\begin{matrix}5627\\9999\end{matrix}\right.$

Division. You know how to divide one number by another, and we shall now briefly review the operation.

1. Divide \$21.15 by 9.

We see that $21 \div 9 = 2$, with remainder 3, leaving 3.15 to be divided. We see that $3.1 \div 9 = 0.3$, with remainder 0.4, leaving 0.45 to be divided. We see that $0.45 \div 9 = 0.05$. Therefore the quotient is \$2.35.

$$9\overline{)\$21.15}\quad \$2.35$$

Check. $9 \times \$2.35 = \$21.15$.

The above method, known as *short division*, should be used for all divisors 12 or less, and may easily be used for many divisors greater than 12.

172	$17\frac{9}{37}$	\$3.12
$42\overline{)7224}$	$37\overline{)648}$	$18\overline{)\$56.16}$
42	37	54
302	278	21
294	259	18
84	19	36
84		36

2. Divide 7224 by 42.

Since 72 hundreds $\div$ 42 = 1 hundred, with remainder 30 hundreds, we write 1 in hundreds' place in the quotient. We now have 3024 to be divided. Since 302 tens $\div$ 42 = 7 tens, with remainder 8 tens, we write 7 in tens' place in the quotient. We now have 84 to be divided, and we see that $84 \div 42 = 2$. Therefore the quotient is 172.

3. Divide 648 by 37.

Proceeding as before, we have 17 for the quotient, with remainder 19. Since the 19 is still to be divided by 37, we write the full quotient $17\frac{19}{37}$.

4. Divide \$56.16 by 18.

The process is sufficiently clear from the above work.

DRILL WORK IN DIVISION

Divide the following as indicated:

1.	4)3368	5)1725	6)3942	7)5264	8)4456
2.	3)7779	2)5550	9)4446	8)7464	7)6566
3.	4)9232	8)9048	7)4844	9)6669	5)4405

After writing on paper the numbers in Exs. 1–3, the pupil should perform the divisions and then check the quotients by multiplication. The entire computation should be done within 4 min.

Divide the following as indicated:

4.	2)3425	3)4177	4)6283	5)7291	6)3337
5.	7)6441	8)9649	9)3470	7)8507	5)9643
6.	6)4445	8)7327	9)5555	7)1004	7)3005

In the results found in solving Exs. 4–6 the remainders should not be written separately, but a fraction should appear in each quotient. The computation should take less than 3 min.

Find the quotient, and the remainder if any:

7. 50,568 ÷ 602.
8. 13,020 ÷ 620.
9. 19,251 ÷ 629.
10. 76,109 ÷ 629.
11. 33,750 ÷ 125.
12. 26,288 ÷ 106.
13. 153,186 ÷ 422.
14. 237,975 ÷ 501.
15. 972,360 ÷ 222.
16. $48.72 ÷ 16.
17. $39.54 ÷ 12.
18. $57.81 ÷ 15.
19. $34.51 ÷ 17.
20. $208.65 ÷ 65.
21. $282,127 ÷ 75.
22. $359,333 ÷ 78.
23. $463,481 ÷ 55.
24. $582,603 ÷ 82.

Division by a Decimal. Since we may multiply both divisor and dividend by the same number without affecting the quotient, we may, if the divisor contains decimal places, multiply both divisor and dividend by such a power of 10 as makes the divisor an integer. That is,

In the division of decimals multiply both divisor and dividend by such a power of 10 as makes the divisor an integer. Then divide as by an integer, placing the decimal point in the quotient over the decimal point in the dividend.

$$\begin{array}{r} 14.4 \\ 23\overline{)331.2} \\ 23 \\ \hline 101 \\ 92 \\ \hline 92 \\ 92 \\ \hline \end{array}$$

Thus, in dividing 33.12 by 2.3 we multiply both divisor and dividend by 10, and then have 331.2 ÷ 23. The divisor is now an integer, and we proceed as shown above, finding the quotient to be 14.4.

Check. $2.3 \times 14.4 = 331.2$.

Another Method. If we prefer, we may leave the decimal points in the divisor and dividend as they are and determine the position of the decimal point in the quotient as follows:

In the division of decimals draw a vertical line as many places to the right of the decimal point in the dividend as there are decimal places in the divisor. Then divide as by an integer, placing the decimal point in the quotient above this vertical line.

$$\begin{array}{r} 14.4 \\ 2.3\overline{)33.1\,|\,2} \\ 23 \\ \hline 101 \\ 92 \\ \hline 92 \\ 92 \\ \hline \end{array}$$

Thus, in dividing 33.12 by 2.3 we place a vertical line one place to the right of the decimal point in the dividend. Proceeding as in ordinary division, we find that the quotient is 14.4.

Both the above plans are in common use. The teacher should adopt one of them and adhere to it in teaching, but should not require a pupil to change from the plan that he already knows.

DRILL WORK IN DIVISION

Divide as indicated, carrying the quotient to three decimal places if possible:

1. $177\overline{)342781}$
2. $42.3\overline{)2869.4}$
3. $239\overline{)734168}$
4. $63.8\overline{)427.52}$
5. $125\overline{)349250}$
6. $39.7\overline{)79.797}$

The above work should be done within 12 min.

Divide as indicated:

7. $62.9 \div 37.$
8. $6.29 \div 37.$
9. $62.9 \div 3.7.$
10. $6.29 \div 3.7.$
11. $80.5 \div 35.$
12. $80.5 \div 3.5.$
13. $8.05 \div 3.5.$
14. $46.2 \div 22.$
15. $46.2 \div 2.2.$
16. $4.62 \div 2.2.$
17. $600 \div 0.6.$
18. $600 \div 0.06.$
19. $6 \div 0.0006.$
20. $627 \div 0.3.$
21. $6.27 \div 0.3.$
22. $7.14 \div 0.42.$
23. $62.32 \div 7.6.$
24. $6.232 \div 0.76.$
25. $8190 \div 0.9.$
26. $61.20 \div 1.2.$
27. $7.200 \div 1.2.$
28. $78.4 \div 0.08.$
29. $3.12 \div 0.04.$
30. $7.81 \div 1.1.$
31. $8.91 \div 0.11.$
32. $4.55 \div 0.07.$
33. $2.73 \div 0.03.$
34. $8.96 \div 12.8.$
35. $4.41 \div 0.21.$
36. $17.28 \div 14.4.$
37. $1231.2 \div 2.4.$
38. $1.2312 \div 2.4.$
39. $62.75 \div 123.$
40. $6.275 \div 1.23.$
41. $36.08 \div 32.8.$
42. $3.608 \div 0.328.$
43. $1.036 \div 0.560.$
44. $882.32 \div 0.82.$
45. $4.4116 \div 0.41.$
46. $11{,}300 \div 5.65.$
47. $6.51021 \div 3.207.$
48. $651.021 \div 320.7.$
49. $176.4 \div 0.0072.$
50. $5.44 \div 0.0068.$
51. $717.75 \div 4.125.$

Teachers should use only so much of a page of this kind as is necessary to accomplish the purpose. If the pupils can divide accurately and with fair rapidity, they should pass on to the next topic. The large number of examples allows the teacher to select different ones from time to time. Always remember that the textbook should be a servant, not a master.

Short Methods in Division. *To divide by 10, move the decimal point one place to the left, prefixing a zero if necessary.*

To divide by 100 or 1000, move the decimal point to the left, two or three places respectively, prefixing zeros if necessary.

To divide by 5, multiply by 2 and divide by 10.

To divide by 25, multiply by 4 and divide by 100.

To divide by 125, multiply by 8 and divide by 1000.

To divide by $12\frac{1}{2}$, multiply by 8 and divide by 100.

To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.

To divide by $66\frac{2}{3}$, multiply by 3 and divide by 200.

To divide by $16\frac{2}{3}$, multiply by 6 and divide by 100.

Since the pupil is already familiar with these processes, from his work in Book Two, and since the explanation follows easily from pages 14 and 15 of this book we proceed at once to the necessary drill work.

DRILL WORK IN SHORT METHODS

Divide by short methods, writing only the answers:

1. $7945 \div 5$.
2. $3875 \div 25$.
3. $7925 \div 12\frac{1}{2}$.
4. $9676 \div 33\frac{1}{3}$.
5. $3375 \div 125$.
6. $9486 \div 66\frac{2}{3}$.
7. $3739 \div 16\frac{2}{3}$.
8. $8347 \div 33\frac{1}{3}$.
9. $7969 \div 12\frac{1}{2}$.
10. $59{,}625 \div 125$.
11. $3978 \div 16\frac{2}{3}$.
12. $7168 \div 66\frac{2}{3}$.
13. $9375 \div 12\frac{1}{2}$.
14. $4873 \div 33\frac{1}{3}$.
15. $67{,}500 \div 125$.

16. Divide by 5: 125; 875; \$375; \$67,295; \$575,265.

17. Divide by 25: 650; 825; \$175; 98,750; \$482,000.

18. Divide by 125: \$250,125; \$375,375; \$276,625; \$452,625.

19. Divide 452,346,125 by 125 and also by 25, using long division, and see how many seconds it takes. Do the same by the short methods.

THE COMMON MEASURES

Fill the following parentheses:

1. 72 in. = ( ) yd.
2. $2\frac{3}{4}$ yd. = ( ) in.
3. $5\frac{1}{2}$ ft. = ( ) in.
4. 48 in. = ( ) ft.
5. 48 ft. = ( ) yd.
6. $\frac{1}{2}$ mi. = ( ) yd. = ( ) ft.
7. 835 ft. = ( ) yd. ( ) ft.
8. 2.5 mi. = ( ) yd. = ( ) ft.
9. 8000 yd. = ( ) mi. ( ) yd.
10. 8000 ft. = ( ) yd. ( ) ft.

If necessary, the pupils should consult the tables on pages 335–338.

Express the following as quarts:

11. 27 gal. 12. $6\frac{3}{4}$ gal. 13. 16 pt. 14. $\frac{1}{2}$ bu. 15. 9 pk.

Express the following as pounds:

16. $2\frac{1}{2}$ T. 17. $\frac{3}{4}$ T. 18. 48 oz. 19. 64 oz. 20. 8 oz.

Express the following as square feet:

21. 2 sq. yd. 22. 1 sq. rd. 23. 1 A. 24. 432 sq. in.

25. Some boys are laying out a quarter-mile track. How many feet long is it?

26. Suppose the boys in Ex. 25 have only a yardstick with which to measure, how many yards must they allow?

27. Land surveyors often use a chain which is 66 ft. long. How many chains must the boys in Ex. 25 allow?

28. There are 10 square chains in an acre, the chain being the length given in Ex. 27. If a man has a rectangular garden 90 ft. by 121 ft., what part of an acre does it contain?

29. If a 10-acre field is 400 ft. wide, how long is it?

In solving Ex. 29 the pupils may use the information given in Ex. 28.

ADDING AND SUBTRACTING FRACTIONS

Find the sums of the following:

1. $\frac{1}{2}, \frac{3}{8}$.
2. $\frac{1}{4}, \frac{3}{16}$.
3. $\frac{1}{4}, \frac{5}{16}$.
4. $\frac{1}{4}, \frac{9}{16}$.
5. $\frac{3}{4}, \frac{13}{16}$.
6. $\frac{1}{8}, \frac{3}{16}$.
7. $\frac{3}{8}, \frac{7}{16}$.
8. $\frac{3}{8}, \frac{15}{16}$.
9. $\frac{5}{8}, \frac{15}{16}$.
10. $\frac{7}{8}, \frac{13}{16}$.
11. $\frac{1}{2}, \frac{7}{32}$.
12. $\frac{1}{2}, \frac{15}{32}$.
13. $\frac{1}{4}, \frac{25}{32}$.
14. $\frac{3}{4}, \frac{31}{32}$.
15. $\frac{3}{8}, \frac{5}{32}$.
16. $\frac{3}{4}, \frac{7}{24}$.
17. $\frac{3}{4}, \frac{17}{24}$.
18. $\frac{5}{8}, \frac{5}{24}$.
19. $\frac{7}{8}, \frac{23}{24}$.
20. $\frac{5}{16}, \frac{1}{64}$.
21. To the sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$ add the sum of $\frac{3}{5}$, $\frac{7}{15}$, and $\frac{2}{3}$.
22. To the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{3}{16}$ add the sum of $\frac{1}{2}$, $\frac{7}{8}$, and $\frac{9}{16}$.

Find the sums of the following:

23. $8\frac{3}{4}, 5\frac{1}{3}$.
24. $6\frac{3}{4}, 8\frac{2}{3}$.
25. $2\frac{1}{3}, 7\frac{1}{8}$.
26. $6\frac{1}{3}, 5\frac{3}{8}$.
27. $8\frac{2}{3}, 9\frac{5}{8}$.
28. $7\frac{2}{3}, 8\frac{7}{8}$.
29. $3\frac{3}{4}, 5\frac{5}{12}$.
30. $4\frac{3}{8}, 6\frac{5}{16}$.
31. $62\frac{5}{6}, 29\frac{3}{5}$.
32. $75\frac{7}{8}, 62\frac{15}{16}$.
33. $92\frac{1}{3}, 75\frac{4}{15}$.
34. $73\frac{2}{3}, 68\frac{7}{15}$.
35. $52\frac{4}{5}, 75\frac{13}{15}$.
36. $48\frac{9}{10}, 73\frac{11}{12}$.
37. $52\frac{1}{5}, 61\frac{7}{20}$.
38. $37\frac{4}{15}, 32\frac{11}{20}$.
39. $5\frac{4}{5}, 6\frac{2}{3}, 4\frac{2}{15}$.
40. $6\frac{7}{8}, 3\frac{1}{2}, 5\frac{15}{16}$.
41. $4\frac{1}{3}, 3\frac{3}{5}, 5\frac{1}{30}$.
42. $2\frac{7}{8}, 5\frac{13}{16}, 6\frac{5}{32}$.
43. $3\frac{5}{6}, 2\frac{5}{12}, 6\frac{7}{24}$.
44. $4\frac{7}{8}, 3\frac{2}{3}, 5\frac{11}{24}$.
45. $3\frac{1}{16}, 5\frac{3}{32}, 7\frac{11}{64}$.
46. $9\frac{2}{15}, 13\frac{13}{30}, 15\frac{17}{20}$.

Subtract the following as indicated:

47. $\frac{3}{4} - \frac{1}{8}$.
48. $\frac{5}{8} - \frac{1}{2}$.
49. $\frac{5}{8} - \frac{3}{16}$.
50. $\frac{7}{8} - \frac{5}{16}$.
51. $\frac{1}{2} - \frac{5}{32}$.
52. $\frac{7}{8} - \frac{5}{32}$.
53. $\frac{15}{32} - \frac{3}{64}$.
54. $32 - 4\frac{7}{8}$.
55. $63 - 5\frac{3}{4}$.
56. $72 - 8\frac{9}{10}$.
57. $48 - 3\frac{7}{32}$.
58. $67 - 5\frac{3}{16}$.
59. $71 - 6\frac{5}{24}$.
60. $80 - 9\frac{5}{48}$.
61. $9\frac{7}{16} - 7\frac{3}{8}$.
62. $8\frac{9}{16} - 3\frac{3}{8}$.
63. $6\frac{11}{16} - 4\frac{5}{8}$.
64. $7\frac{15}{16} - 3\frac{7}{8}$.
65. $9\frac{31}{32} - 6\frac{3}{4}$.
66. $8\frac{29}{32} - 5\frac{3}{8}$.
67. $23\frac{1}{8} - 8\frac{3}{16}$.

FURTHER DRILL IN FRACTIONS

Add the following:

1. $\frac{1}{2}+\frac{1}{4}$. 3. $\frac{1}{2}+\frac{1}{8}$. 5. $\frac{1}{2}+\frac{5}{8}$. 7. $\frac{1}{4}+\frac{1}{8}$. 9. $\frac{3}{4}+\frac{3}{8}$.
2. $\frac{1}{2}+\frac{3}{4}$. 4. $\frac{1}{2}+\frac{3}{8}$. 6. $\frac{1}{2}+\frac{7}{8}$. 8. $\frac{1}{4}+\frac{7}{8}$. 10. $\frac{3}{4}+\frac{5}{8}$.

Subtract the following:

11. $\frac{1}{2}-\frac{1}{4}$. 13. $\frac{1}{2}-\frac{1}{8}$. 15. $\frac{7}{8}-\frac{1}{2}$. 17. $\frac{1}{4}-\frac{1}{8}$. 19. $\frac{3}{4}-\frac{3}{8}$.
12. $\frac{3}{4}-\frac{1}{2}$. 14. $\frac{1}{2}-\frac{3}{8}$. 16. $\frac{5}{8}-\frac{1}{2}$. 18. $\frac{7}{8}-\frac{1}{4}$. 20. $\frac{3}{4}-\frac{5}{8}$.

Multiply the following:

21. $\frac{1}{2}\times\frac{1}{4}$. 23. $\frac{1}{2}\times\frac{1}{8}$. 25. $\frac{5}{8}\times\frac{1}{2}$. 27. $\frac{1}{4}\times\frac{1}{8}$. 29. $\frac{3}{4}\times\frac{3}{8}$.
22. $\frac{1}{2}\times\frac{3}{4}$. 24. $\frac{1}{2}\times\frac{3}{8}$. 26. $\frac{7}{8}\times\frac{1}{2}$. 28. $\frac{7}{8}\times\frac{3}{4}$. 30. $\frac{3}{4}\times\frac{5}{8}$.

Divide the following:

31. $\frac{3}{4}\div\frac{3}{8}$. 33. $\frac{3}{4}\div\frac{5}{8}$. 35. $\frac{3}{4}\div\frac{7}{8}$. 37. $\frac{2}{3}\div\frac{3}{4}$. 39. $\frac{2}{3}\div\frac{7}{8}$.
32. $\frac{3}{8}\div\frac{3}{4}$. 34. $\frac{5}{8}\div\frac{3}{4}$. 36. $\frac{7}{8}\div\frac{3}{4}$. 38. $\frac{3}{4}\div\frac{2}{3}$. 40. $\frac{3}{5}\div\frac{2}{3}$.

The 40 examples above should be copied and solved in 5 min.

Multiply the following:

41. $34 \times 3\frac{1}{2}$ 44. $96 \times 15\frac{7}{8}$ 47. $29\frac{1}{2} \times 36$ 50. $5\frac{1}{2} \times 2\frac{1}{2}$ 53. $8\frac{1}{4} \times 2\frac{3}{8}$
42. $48 \times 23\frac{1}{4}$ 45. $64 \times 35\frac{5}{8}$ 48. $38\frac{2}{3} \times 45$ 51. $7\frac{3}{4} \times 3\frac{1}{2}$ 54. $6\frac{3}{4} \times 5\frac{5}{8}$
43. $56 \times 38\frac{3}{4}$ 46. $84 \times 22\frac{2}{3}$ 49. $52\frac{7}{8} \times 25$ 52. $9\frac{7}{8} \times 6\frac{1}{2}$ 55. $7\frac{2}{3} \times 3\frac{3}{8}$

Divide the following:

56. $7\frac{3}{4}\div 2\frac{1}{2}$. 57. $9\frac{7}{8}\div 3\frac{3}{4}$. 58. $8\frac{2}{3}\div 3\frac{1}{4}$. 59. $3\frac{1}{4}\div 8\frac{2}{3}$.

II. PERCENTAGE

Per Cent. As you learned in Book Two, another name for hundredths is *per cent.* We shall now study more about per cents and their uses in practical life.

Thus 0.01, or $\frac{1}{100}$, is the same as 1 per cent; 0.06 is the same as 6 per cent; 100 per cent is the same as $\frac{100}{100}$.

That part of arithmetic which treats of per cent is called *percentage.* The word "percentage" is often used for "per cent," as in speaking of the percentage of base hits in a baseball game.

Symbol for Per Cent. The symbol for per cent is written thus: %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% either as "6 per cent" or as "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and is equal to the whole number 8; 225% means the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}$% means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either "$\frac{1}{2}$ per cent" or, quite commonly, "$\frac{1}{2}$ of 1%."

Per Cents as Common Fractions. As you learned in Book Two, $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$. Therefore,

To express per cent as a common fraction, write the number indicating the per cent as the numerator of the fraction and 100 as the denominator, and reduce to lowest terms.

For example:

$3\% = \frac{3}{100}$ $\qquad$ $10\% = \frac{10}{100} = \frac{1}{10}$

$4\% = \frac{4}{100} = \frac{1}{25}$ $\qquad$ $25\% = \frac{25}{100} = \frac{1}{4}$

$5\% = \frac{5}{100} = \frac{1}{20}$ $\qquad$ $50\% = \frac{50}{100} = \frac{1}{2}$

$6\% = \frac{6}{100} = \frac{3}{50}$ $\qquad$ $275\% = \frac{275}{100} = 2\frac{75}{100} = 2\frac{3}{4}$

PER CENTS AS COMMON FRACTIONS

All work oral

1. By using these columns show that $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, and $80\% = \frac{4}{5}$.

				20%
			20%	20%
		20%	20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
20%	40%	60%	80%	100%

2. Show that 20% is $\frac{1}{2}$ of 40%, $\frac{1}{3}$ of 60%, $\frac{1}{4}$ of 80%, and $\frac{1}{5}$ of 100%.

3. Show that 40% is $\frac{2}{3}$ of 60%, $\frac{1}{2}$ of 80%, and $\frac{2}{5}$ of 100%.

4. Show that 60% is $1\frac{1}{2}$ times 40%, 3 times 20%, $\frac{3}{4}$ of 80%, and $\frac{3}{5}$ of 100%.

These columns should be written on the board. Columns of other per cents, such as $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, 25%, and $33\frac{1}{3}\%$, are also useful for drill work.

5. A foot is what fraction of a yard? A foot is what per cent of a yard?

6. A pint is what fraction of a gallon? A pint is what per cent of a gallon?

7. Five cents is what fraction of a dollar? Five cents is what per cent of a dollar?

8. If you wish to find $33\frac{1}{3}\%$ of a number, what fractional part of the number do you wish to find?

9. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

10. What per cent of the perimeter of a square is one side of the square? What per cent are two sides? three sides?

11. If there are 30 pupils in a class and 50% are boys, what fraction are boys? How many boys are there?

PER CENTS AS COMMON FRACTIONS

Numbers 1 to 16, oral

1. A man said that he gained 100% on some property that he sold. What does this mean?

2. A dealer said that he would sell some goods at 40% less than the marked price. Express this as a common fraction.

3. A boy beginning to play tennis won only 25% of the games played the first week. Express this as a common fraction.

Express as common fractions, whole numbers, or mixed numbers:

4. 20%.	7. 60%.	10. 25%.	13. 200%.
5. 30%.	8. 70%.	11. 125%.	14. 300%.
6. 50%.	9. 90%.	12. 225%.	15. 325%.

16. A man said that he marked some goods $33\frac{1}{3}\%$ more than they cost him, and another said that he marked his goods $\frac{1}{3}$ more than they cost him. Was there any difference?

17. How much is $\frac{1}{2}\frac{1}{5}$ of 50? 4% of 50? 4% of 60?

18. How much is $\frac{1}{4}$ of 440? 25% of 440? 25% of 3724?

19. How much is $\frac{1}{2}$ of 3628? 50% of 3628? 50% of 9376?

20. How much is $\frac{3}{4}$ of 6480? 75% of 6480? 75% of 3872?

21. How much is $1\frac{1}{4} \times 964$? 125% of 964? 125% of 896?

22. Instead of multiplying 72 by $12\frac{1}{2}\%$, what fractional part of 72 may we take? Do this, and thus find $12\frac{1}{2}\%$ of 72.

23. Instead of multiplying 640 by $37\frac{1}{2}\%$, what fractional part of 640 may we take? Do this, and thus find $37\frac{1}{2}\%$ of 640.

24. Find $87\frac{1}{2}\%$ of 160 and of 3200 in the shortest way.

25. Find $33\frac{1}{3}\%$ of 96 and of 9672 in the shortest way.

26. Find $3\frac{1}{4} \times 488$; 325% of 488; 325% of 872.

Per Cents as Decimals. If a baseball team has lost 32% of its games, how many hundredths of its games has it lost? How do you write 32% as hundredths?

Since 32%, 0.32, and $\frac{32}{100}$ all mean the same thing, we see that we can easily write per cents as decimals.

To express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.

When we omit the per cent sign we must indicate the hundredths in some other way, as by moving the decimal point two places to the left.

Thus, $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015; $125\% = 1.25$; $0.6\% = 0.006$.

PER CENTS AS DECIMALS

Numbers 1 to 9, oral

1. If 2% of the people in this town are in the seventh grade of school, how many out of every 100 are in the seventh grade? How many hundredths of the people are in this grade?

2. If 2% of the pupils of this school are absent today, how many pupils are absent out of every 100? out of every 50?

3. How much is $\frac{1}{100}$ of \$200? 1% of \$200? 1% of \$1700?

4. How much is 0.06 of \$100? 6% of \$100? 6% of \$300?

Read the following as decimal fractions:

5. 35%. 6. 19%. 7. 75%. 8. 120%. 9. 325%.

10. Express $\frac{1}{10}$ as a decimal; as per cent.

11. Express $\frac{1}{10}\%$ as a decimal; as a common fraction.

Express as decimal fractions, whole numbers, or mixed decimals:

12. 800%. 13. 0.5%. 14. 150%. 15. 0.005%.

Decimals as Per Cents. If a basketball team has won 0.6 of the games it has played, how many hundredths of the games has it won? What per cent of the games has it won?

Since per cent means hundredths, to express a decimal as per cent we ask how many hundredths the decimal represents.

1. Express 0.3 as per cent.

Since we know that $0.3 = 0.30$, we see that 0.3 is the same as 30 hundredths, or 30%.

2. Express 0.375 as per cent.

Here $0.375 = 37.5$ hundredths, or $37\frac{1}{2}$ hundredths. Therefore 0.375 is the same as 37.5%, or $37\frac{1}{2}$%.

3. Express 0.00125 as per cent.

Here $0.00125 = 0.00\frac{125}{1000} = 0.00\frac{1}{8}$, and so $0.00125 = \frac{1}{8}$%.

4. Express $6.5\frac{1}{2}$ as per cent.

Here $6.5\frac{1}{2} = 6.55 = \frac{655}{100}$, and therefore $6.5\frac{1}{2} = 655$%.

To express a decimal as per cent, move the decimal point two places to the right and place the per cent sign after the number.

DECIMALS AS PER CENTS

Numbers 1 to 5, oral

1. A pint, being 0.5 qt., is what per cent of a quart?
2. A peck, being 0.25 bu., is what per cent of a bushel?
3. A quart, being 0.125 pk., is what per cent of a peck?
4. What per cent of a dollar is 49¢? is 49 dimes? is $1?
5. Express 1 qt. as per cent of a pint; of 2 bu.; of 4 bu.

Express the following as per cents:

6. 0.9.	8. 0.59.	10. 0.325.	12. $6.66\frac{2}{3}$.
7. 6.2.	9. 0.72.	11. 0.650.	13. 0.00375.

Common Fractions as Per Cents. If you are finding a discount of 20% on a bill of goods bought for the Boy Scouts, which is easier, to take $\frac{1}{5}$ or to take 20%? Is it easier for you to divide by 5 or to multiply by 2? We see that sometimes it is easier to use per cents than to use common fractions.

Since per cent means hundredths, to express a common fraction as per cent we have only to reduce it to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, and this is the same as 0.80, or 80%. Likewise $\frac{2}{25} = \frac{8}{100} = 8\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\%$.

To express a common fraction as per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.

COMMON FRACTIONS AS PER CENTS

Numbers 1 to 8, oral

1. 25¢ is what fraction of 50¢? what per cent?
2. $33\frac{1}{3}$¢ is what fraction of $1? what per cent?

Express the following as per cents:

3. $\frac{1}{4}$. 4. $\frac{1}{5}$. 5. $\frac{1}{10}$. 6. $\frac{3}{4}$. 7. $\frac{2}{5}$. 8. $\frac{7}{10}$.

9. $\frac{5}{18}$ of some games played is what per cent of the games?
10. An inch is what per cent of a foot? of a yard?
11. A pint is what per cent of a peck? of a bushel?
12. What per cent of a gallon is 2 qt.? $2\frac{1}{2}$ qt.? 4 qt.?
13. What per cent of a pound is 1 oz.? $3\frac{1}{5}$ oz.? 16 oz.?
14. What per cent of a rod is 1 ft.? 1 yd.? $5\frac{1}{2}$ yd.? 11 yd.?
15. If you have increased $\frac{1}{9}$ in weight during the past year, what per cent have you increased in weight?

Reading Per Cents. We have found that 2.75 may be read "two and seventy-five hundredths" or "two hundred seventy-five per cent." In business it is more common to read 2.75 "two point seven five."

READING PER CENTS

Numbers 1 to 3, oral

1. If our school has twice as many pupils as it had 10 yr. ago, it has now what per cent as many as then?

2. Read the following numbers and tell which of them are equal: 2; $\frac{200}{100}$; 200%; $\frac{100}{50}$; 7; 700%.

3. Because $3.75 = \frac{375}{100}$, then 3.75 is what per cent?

Write the following in words:

4. 475%. 5. 4.75%. 6. 3.9%. 7. 39%. 8. 0.39%. 9. 390%. 10. 5.25. 11. 7.35. 12. 0.6%. 13. 0.06%.

Express the following as decimals:

14. 225%. 15. $\frac{1}{4}$%. 16. 300%. 17. $\frac{3}{8}$%. 18. 0.4%.

Express as common fractions or mixed numbers:

19. 78%. 20. $6\frac{1}{4}$%. 21. 325%. 22. 475%. 23. $33.3\frac{1}{3}$%.

Express the following as per cents:

24. $\frac{3}{5}$. 25. $\frac{1}{3}$. 26. $\frac{3}{4}$. 27. $\frac{4}{5}$. 28. $2\frac{1}{2}$. 29. $3\frac{3}{4}$.

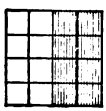
Express as decimals, whole numbers, or mixed decimals:

30. 28%. 31. 34%. 32. 0.6%. 33. $66\frac{2}{3}$%. 34. 600%. 35. 355%. 36. 250%. 37. 275%. 38. 325%. 39. 5000%. 40. 0.006%. 41. 4550%.

PER CENTS AND FRACTIONS

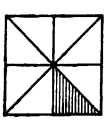
1. Read 0.50, using the words "per cent"; using the word "hundredths." How many halves are there in 1? Then how many times is 50% contained in 1? in 100%?

2. If this square is called 16, how much is the shaded part? How many sixteenths of the square are shaded? How many hundredths of it are shaded? What per cent of it is shaded?



3. Read 0.25, using the words "per cent"; using the word "hundredths." How many fourths are there in 1? Then how many times is 25% contained in 1? in 100%?

4. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? What per cent of the square is shaded?



5. How many eighths are there in 1? Then how many times is $12\frac{1}{2}\%$ contained in 1?

6. Draw a square on the blackboard and shade 10% of it. Draw another square and shade 75% of it. How many tenths of the square is 10% of it? How many fourths of the square is 75% of it?

7. How many times is $16\frac{2}{3}\%$ contained in 1? in 2?

8. How many times is $6\frac{1}{4}\%$ contained in 1? in 3?

9. How many times is 20% contained in 1? in 5?

10. How many times is 50% contained in 7? in 12?

11. How many times is 25% contained in 9? in 20?

12. Express $\frac{15}{100}$, 15%, and 0.15 as twentieths.

13. Express $\frac{3}{4}\%$ as a decimal; as a common fraction.

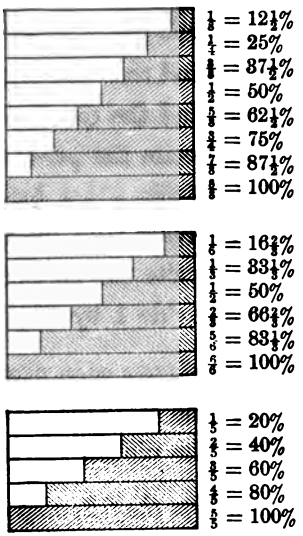
14. Express $666\frac{2}{3}\%$ as a decimal; as an improper fraction.

Important Per Cents. Which do you find the easier, to find $\frac{3}{8}$ of 72 games played or to find $37\frac{1}{2}\%$ of 72 games played?

Sometimes it is easier to use the common fraction and sometimes it is easier to use the per cent. It is easier in this case, of course, to use $\frac{3}{8}$ than to use $37\frac{1}{2}\%$.

You have already learned the common fractions that are equal to some of the most important per cents. The most important ones are given in the following table:

$50\% = \frac{1}{2}$	$33\frac{1}{3}\% = \frac{1}{3}$
$25\% = \frac{1}{4}$	$66\frac{2}{3}\% = \frac{2}{3}$
$12\frac{1}{2}\% = \frac{1}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$37\frac{1}{2}\% = \frac{3}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$
$62\frac{1}{2}\% = \frac{5}{8}$	$20\% = \frac{1}{5}$
$87\frac{1}{2}\% = \frac{7}{8}$	$40\% = \frac{2}{5}$
$6\frac{1}{4}\% = \frac{1}{16}$	$60\% = \frac{3}{5}$
$3\frac{1}{8}\% = \frac{1}{32}$	$80\% = \frac{4}{5}$



The pupils should become so familiar with these equivalents that they are able to change at once from one to the other. No pupil should ever multiply by 25%, $12\frac{1}{2}\%$, $33\frac{1}{3}\%$, or any other per cent when the common fraction is easier for the purpose.

Illustrative Problems. 1. Find $62\frac{1}{2}\%$ of \$1640.

$62\frac{1}{2}\%$ of \$1640 = $\frac{5}{8}$ of \$1640 = 5 × \$205 = \$1025.

2. Find $83\frac{1}{3}\%$ of \$720.

$83\frac{1}{3}\%$ of \$720 = $\frac{5}{6}$ of \$720 = 5 × \$120 = \$600.

3. Find 80% of \$725.

Here there is nothing gained by taking $\frac{4}{5}$ instead of 80%.

IMPORTANT PER CENTS

Numbers 1 to 9, oral

1. Recite the per-cent equivalents of the eighths.
2. $87\frac{1}{2}\%$ is how many times $12\frac{1}{2}\%$?

The pupil should use the equivalent common fractions.

3. Find the quotient of 75% divided by $62\frac{1}{2}\%$.
4. Find the quotient of $62\frac{1}{2}\%$ divided by 50%.
5. Recite the per-cent equivalents of the sixths.
6. Divide $83\frac{1}{3}\%$ by $33\frac{1}{3}\%$; $33\frac{1}{3}\%$ by $66\frac{2}{3}\%$.
7. Divide 50% by $16\frac{2}{3}\%$; 50% by 25%; 25% by 50%.
8. Recite the per-cent equivalents of the fifths.
9. Divide 1 by 80%; 80% by 1; 80% by 100%.

Draw figures as on page 33 and learn the per-cent equivalents of:

10. The sevenths.
11. The ninths.
12. The twelfths.
13. The sixteenths.
14. The tenths.
15. The twentieths.

Perform the following operations:

16. $50\% \div 40\%$.
17. $80\% \div 50\%$.
18. $25\% \div 60\%$.
19. $12\frac{1}{2}\%$ of 80%.
20. $83\frac{1}{3}\%$ of 60%.
21. 60% of $33\frac{1}{3}\%$.
22. $50\% \div 12\frac{1}{2}\%$.
23. $33\frac{1}{3}\%$ of 75%.
24. $66\frac{2}{3}\% \div 40\%$.

Find the values of the following:

25. 50% of \$1273.80.
26. 25% of \$3724.64.
27. $12\frac{1}{2}\%$ of \$5128.72.
28. $6\frac{1}{4}\%$ of \$23,376.16.
29. $37\frac{1}{2}\%$ of \$17,768.32.
30. $33\frac{1}{3}\%$ of \$77,163.69.
31. $66\frac{2}{3}\%$ of \$80,070.12.
32. $83\frac{1}{3}\%$ of \$71,108.10.
33. $112\frac{1}{2}\%$ of 87,264.48.
34. $137\frac{1}{2}\%$ of 96,328.32.

Terms used in Percentage. The number of which a required per cent is to be taken is often called the *base.*

The number of hundreths of the base is called the *rate.*

In 25% of \$300, \$300 is the base and 25% is the rate.

Sometimes 25 is called the *rate per cent,* 25% being called the *rate,* but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage.* Therefore

The percentage is the product of the base and the rate.

The terms "base" and "percentage" are introduced because they are occasionally used. They are not needed in this book and are not often used in business in the way that they were formerly used in school. The terms "interest" "profit," "loss," "commission," "premium," "taxes," and so on are used instead of "percentage." The term "percentage" is often used to designate that part of arithmetic in which per cents are used. The terms "principal," "selling price," "cost," "capital," "amount invested," "amount insured," and "face of note" are used instead of "base."

Three Important Problems of Percentage. There are, as we shall soon see, three important kinds of problems of percentage.

The first problem is that of finding a required per cent of a number.

That is, given the base and rate, find the percentage. This problem is considered on page 36.

The second problem is that of finding what per cent one number is of another.

That is, given the base and percentage, find the rate. This problem is considered on page 39.

The third problem is that of finding the number of which a given number is a given per cent.

That is, given the percentage and rate, find the base. This problem is considered on page 43.

First Important Problem. The most important kind of problem in percentage is that of finding some per cent of a number.

For example, a man who has a contract to remove 9600 cu. yd. of earth in building a canal reports that $23\frac{1}{4}\%$ of the earth has been removed. How many cubic yards does this represent?

Since $23\frac{1}{4}\% = 0.23\frac{1}{4}$, multiply 9600 by $0.23\frac{1}{4}$.

$$\begin{array}{r} 9600 \\ 0.23\frac{1}{4} \\ \hline 2400 \\ 288 \\ 192 \\ \hline 2232. \end{array}$$

To find a required per cent of a number, multiply the number by the given rate.

A still easier way would be simply to multiply 96 by $23\frac{1}{4}$. The student should always look for such short cuts.

FIRST IMPORTANT PROBLEM

All work oral

1. In a school of 200 pupils 9% were in the seventh grade. How many pupils were in that grade?

2. In a school of 200 pupils 11% were in the third grade. How many pupils were in that grade?

3. Of 300 pupils 5% have been tardy or absent this month. How many pupils have not been tardy or absent?

4. In a school of 300 pupils, if there should be an increase of $3\frac{1}{3}\%$ next year, how many pupils would there be then?

5. In a school of 250 pupils there were 10% fewer pupils four years ago. How many pupils were there then?

6. If $16\frac{2}{3}\%$ of the pupils in a school of 240 pupils graduate this year, how many will graduate?

7. A man borrowed \$200 and paid 6% of that sum for the use of it for a year. How much was this?

FIRST IMPORTANT PROBLEM

1. A suit of clothes was marked \$45, but after the holidays the price was reduced 15%. How much was it reduced, and what was the price after the reduction?

2. Mollie had a test in spelling. There were 50 words in the test and she missed 6% of them. How many words did she miss? How many did she spell correctly?

3. Mr. Russell received \$125 for some work which he did on a house. He paid 20% of this money for his board and with the rest he bought a Treasury Savings Certificate for \$84.60. How much money had he left?

4. Mr. Baker saved coal during the winter by using a little less each day and making up for it by taking better care of his furnace. If he used $14\frac{1}{2}$ T. last year and saved 9% this winter, how many tons did he save?

Pupils should be required to use judgment in deciding how far to carry decimals in an answer. It rarely happens, in a practical problem in this part of arithmetic, that a result is needed beyond hundredths. Thus, 1.31 is a better answer to Ex. 4 than 1.305.

5. If a manufacturer sells shoes at a profit of 15% on the total cost, and it costs him \$3.70 a pair to make and sell them, how much is his profit on 1000 pairs?

6. If a shop manufactures 276 locomotives and sells 75% of them for \$14,125 each and the rest for \$12,825 each, how much is received for all?

7. A man can have a wall built for 80¢ a foot, but he can have a better job done for 15% more. How much will it cost to have built 54 ft. of the better kind?

8. A farmer owned 1424 sheep and sold $12\frac{1}{2}$% of them. How many sheep did he sell? How many were left?

FIRST IMPORTANT PROBLEM

Numbers 1 to 6, oral

1. If 20% of the pupils in a school of 250 pupils are in the seventh and eighth grades, how many are in those grades?

2. If 25% of the pupils of a school of 300 pupils are in the first and second grades, how many are in those grades?

3. If 55% of the pupils in a school of 300 pupils are girls, how many girls are there in the school?

4. How much must be added to 250 ft. to increase it 50%? to increase it 100%?

5. How much must be added to 240 lb. to make it 125% of the present weight?

6. A column of mercury in a thermometer is 6 in. high. If it increases $16\frac{2}{3}\%$ in height, how high is it then?

7. How much butter fat is there in 425 lb. of milk brought to a creamery, the creamery test showing that 3.9% of the weight of the milk is butter fat?

8. A farmer takes 420 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of the milk. Find the weight of the butter fat and the amount of money received.

9. A girl who works in a store is paid $25 a week; another girl is paid $24 a week. The first girl saves 6.2% of her salary and the second girl saves 6.5%. Which girl saves more in a year, and how much more does she save?

10. A farmer owns 90 hogs which average 225 lb. each in weight, and he decides to sell 60% of them. If he sells the hogs at the rate of $12 per hundredweight, how much money does he receive for them?

Second Important Problem. The second important kind of problem in percentage mentioned on page 35 is that of finding what per cent one number is of another.

The solution of problems of this kind depends upon this principle:

Given the product of two factors and one of the factors, the other factor may be found by dividing the product by the given factor.

That is, if we have given the product 10 and one factor 5, we can find the other factor 2 by dividing 10 by 5; or, if the given factor is 2, we can find the other factor 5 by dividing 10 by 2.

This is a very important principle in mathematics, and in order that the principle may be clearly understood by the pupils, there are given below in Exs. 1–3 a few simple number puzzles that will be found interesting and that will, at the same time, lead up to the work in percentage.

PRODUCT AND ONE FACTOR

All work oral

1. I am thinking of the number which multiplied by 9 gives the product 63. What is the number?

2. There has been blotted out here ■ a number which multiplied by 7 gives the product 56. What is the number?

3. Mary weighs four times as much as her baby sister. Mary weighs 72 lb. How much does the baby weigh?

The first number in each of the following is the product of two factors, and the second is one of the factors. Find the other factor:

4. 36, 4.	8. 80, 8.	12. 6, 3.	16. 5, $\frac{1}{4}$.
5. 36, 12.	9. 80, 10.	13. $\frac{6}{10}$, 3.	17. 7, $\frac{1}{2}$.
6. 72, 8.	10. 200, 10.	14. 0.6, 3.	18. 7, 0.5.
7. 70, 10.	11. 200, 20.	15. 0.60, 3.	19. 8, 50%.

Finding What Per Cent One Number is of Another. The second problem of importance in percentage is to find what per cent one number is of another.

For example, if Fred was at bat 20 times in the baseball games of last June and made 6 base hits, what per cent of base hits did he make?

We have a certain per cent $\times$ 20 = 6; that is, we have given the product, 6, of two factors and one factor, 20, to find the other factor, a certain per cent. Therefore

$6 \div 20 = 0.30 = 30\%$, the per cent of base hits.

RELATIONS OF NUMBERS AS PER CENTS

All work oral

1. If Jack was at bat 24 times and made 12 base hits, what per cent of base hits did he make?

2. If a baseball team wins 16 games out of 20, what per cent of games does it win?

3. If there are 30 pupils in a class and 3 of them are absent today, what per cent are absent?

4. On an automobile trip of 90 mi. what per cent has a man traveled when he has traveled 30 mi.?

Find what per cent the second number is of the first:

5. 2, 1.	10. 40, 20.	15. 44, 22.	20. 25, 5.
6. 3, 1.	11. 40, 30.	16. 44, 44.	21. 25, 2.5.
7. 4, 1.	12. 90, 45.	17. 60, 40.	22. 25, $12\frac{1}{2}$.
8. 4, 2.	13. 60, 15.	18. 72, 72.	23. 25, 25.
9. 4, 3.	14. 32, 16.	19. 36, 27.	24. 25, 50.

Illustrative Problem. An agent bought an automobile for $600 and sold it at a profit of $120. What per cent of the cost did he gain?

Here we have $120 equal to some per cent of $600.

That is, we have given the product, 120, of two factors, and one of the factors, 600, to find the other factor. Therefore

$120 ÷ $600 = 0.20 = 20%, the per cent of gain.

DRILL WORK IN PER CENTS

1. $130 is what per cent of $2600? of $3900?
2. $11.25 is what per cent of $375? of $56.25?
3. $10.12 is what per cent of $253? of $202.40?
4. $29.10 is what per cent of $465.60? of $174.60?
5. A quart is what per cent of a gallon? of 25 gal.?
6. A rod is what per cent of a foot? of a yard?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. $33\frac{1}{3}$% is what per cent of $66\frac{2}{3}$%? of 1? of $133\frac{1}{3}$%?
10. $16\frac{2}{3}$% is what per cent of $33\frac{1}{3}$%? of $66\frac{2}{3}$%? of 1?

Find the per cent that the first number is of the second:

11. 387; 2709. 12. 52.8; 31.68. 13. 56.7; 453.6.

14. If a house that costs $4000 rents for $360 a year, what per cent of the cost is the rent?

15. If $12\frac{1}{2}$ tons of iron are obtained from 235 tons of ore, what per cent of the ore is iron?

16. In 10 yr. the population of a certain city increased from 26,275 to 31,530. What was the per cent of increase?

SECOND IMPORTANT PROBLEM

1. If the seventh grade devotes 45 min. a day to arithmetic and the sixth grade 42 min., what per cent more time does the seventh grade devote to arithmetic than does the sixth grade?

2. If a class had 16 examples to solve on Monday and 20 on Tuesday, what was the per cent of increase?

3. If you have read 69 pages in a book of 300 pages, what per cent of the pages have you read?

4. If the last chapter of a book is Chapter XXXII and you have finished reading Chapter XXIV, what per cent of the chapters have you read?

5. A man's income is $2300 a year and he spends $1386. What per cent of his income does he save?

6. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys?

7. A man invests $13,500 and gains $1620. His gain is what per cent of the amount invested?

8. The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring? in a 14-carat ring?

9. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine?

10. If 2 members of a class of 32 have not been either tardy or absent during the year, and 4 members have not been absent, what per cent have not been either tardy or absent? What per cent have not been absent?

11. A man's income is $4350 a year, and he saves $1957.50. What per cent of his income does he save? What per cent does he spend? What is the sum of these two per cents?

Third Important Problem. The third important problem in percentage mentioned on page 35 is that of finding the number of which a given number is a given per cent.

For example, if a baseball team lost 18 games, which was 45% of the number of games played, how many games were played?

Here we have given that 45% of the number of games is 18.

That is, we have given the product, 18, of two factors and one of the factors, 0.45, to find the other factor. Therefore

$18 \div 0.45 = 40$, the number of games played.

Check. 45% of $40 = 18$.

THIRD IMPORTANT PROBLEM

1. If 20% of the inhabitants of a certain city are school children and there are 18,275 school children in the city, what is the total population of the city?

2. If 36% of the inhabitants of a certain city voted at a certain election and there were 7848 ballots cast, what was the total population of the city?

3. A player reached first base 29 times, or $33\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

4. A player reached home base 24 times, or $8\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

5. If you pay $37\frac{1}{2}\%$ of the expenses of a camping trip, your contribution being \$6.24, what are the expenses of the trip?

6. In a certain school there are 117 boys, which is 45% of the total number of pupils. How many pupils are there in the school? How many of the pupils are girls?

7. If a school is in session 50% of the days of a certain year and is in session 183 da., is that year a leap year or not?

THIRD IMPORTANT PROBLEM

Find the numbers of which the following are the given per cents:

1. 88, 22%.
2. 75, 5%.
3. 24, 6%.
4. 98, 7%.
5. 44, 4%.
6. 37, $12\frac{1}{2}$%.
7. 49, $3\frac{1}{8}$%.
8. 53, $33\frac{1}{3}$%.
9. 186, $37\frac{1}{2}$%.
10. 325, $62\frac{1}{2}$%.
11. 32.45, $16\frac{2}{3}$%.
12. 57.8, 17%.
13. 22.22, 110%.
14. 65.72, 106%.
15. 622.6, 5.5%.

16. Of what number is 385 equal to 5%?

17. \$4.65 is 15% of what amount? 20% of what amount?

18. \$7.84 is 16% of what amount? 10% of what amount?

19. 12 ft. 5.6 in. is 11% of what length? 5% of what length?

20. 126 lb. 14 oz. is 35% of what weight? 2% of what weight?

21. If a farmer sold a horse for 80% of what it cost him and received \$160, how much did the horse cost him?

22. A poultry raiser set some eggs, and 15% failed to hatch. If the number that failed to hatch was 66, how many eggs did the poultry raiser set?

23. A man sells some lumber for \$360, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

24. A man saves \$1350.40 a year, which is 32% of his income. How much is his income?

25. A school has 20% of its pupils in the sixth grade, which numbers 24. How many pupils are there in the school? How many are not in the sixth grade?

From the examples on pages 43 and 44 it may be seen that the third important problem of percentage is not nearly so practical as the first and second problems mentioned on page 35.

Per Cents Greater or Less. Mr. Lane had been earning \$30 and his wages were raised to \$36 a week. His wages were then what per cent greater than before?

The increase is \$36 − \$30, or \$6.

The question reduces to this: \$6 is what per cent of \$30?

This is like the work on page 40, and we have

$$\$6 \div \$30 = 0.20 = 20\%.$$

That is, his wages were raised 20%.

No new principle is involved here. This kind of problem comes under the second important kind of problem in percentage.

PER CENTS GREATER OR LESS

All work oral

1. \$101 is what per cent greater than \$100?
2. \$99 is what per cent less than \$100?
3. \$106 is what per cent greater than \$100?
4. \$95 is what per cent less than \$100?
5. A man who is now earning \$6 a day formerly earned \$5 a day. What is the per cent of increase?
6. A man who earned \$150 in a certain month worked only part time the next month and earned \$100. His wages were then what per cent less than before?
7. \$125 is what per cent greater than \$100?
8. \$250 is what per cent greater than \$200?
9. \$750 is what per cent less than \$1000?
10. \$1500 is what per cent less than \$2000?
11. 150 ft. is what per cent greater than 100 ft. and what per cent less than 200 ft.?

PER CENTS GREATER OR LESS

1. \$302.50 is what per cent greater than \$275?

2. A mile is what per cent greater than 2640 ft.?

3. A half mile is what per cent less than $\frac{3}{4}$ mi.?

4. \$4080 is what per cent greater than \$3400?

5. \$21,000 is what per cent greater than \$17,500?

6. \$3500 is what per cent greater than \$3000 and what per cent less than \$4000?

7. \$251.45 is what per cent greater than \$235?

8. \$218.55 is what per cent less than \$235?

9. 40 in. is what per cent longer than 1 yd.?

10. A yard is what per cent longer than 1 ft.?

11. A foot is what per cent shorter than 1 yd.?

12. A boiler that supplies steam for an engine has a steam gauge attached to it that shows how many pounds the steam presses against every square inch of the boiler surface. If the steam pressure increases from 120 lb. to 150 lb. per square inch, it is then what per cent greater than before?

13. A man bought a horse for \$150 and sold it so as to gain \$30. Find the per cent of gain.

Instead of saying that he bought it for \$150 and sold it for \$180, as in the above exercises, the gain is here given directly.

Find the per cent of gain or loss in each of these cases:

	Cost	Gain		Cost	Loss
14.	\$425	\$29.75	18.	\$575	\$74.75
15.	\$235	\$16.45	19.	\$725	\$166.75
16.	\$320	\$44.80	20.	\$2750	\$247.50
17.	\$225	\$11.25	21.	\$12,375	\$1361.25

Using What You Have Learned. We shall now take some miscellaneous problems, and here you will find it necessary to decide upon the method of solution. You will find it best to ask yourself exactly what is given and then to consider carefully whether you have two factors given, to find a required per cent of a number, or whether you have a product and one factor given, to find the other factor.

USING WHAT YOU HAVE LEARNED

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that weighs 125 lb. What per cent of the water is drawn off?

2. To a class of 32 pupils 8 more are admitted. What per cent of the original number are admitted? What per cent of the present number are admitted? If 10 pupils now leave the class, what per cent leave the class?

3. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 problems have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

4. If 45% of the 240 pupils that attend a certain school are boys and the rest are girls, how many are girls?

5. A man purchased 530 head of cattle and sold 30% of them. How many head of cattle had he left?

6. If an automobile going at the rate of 14 mi. an hour increases its speed 37%, at what rate will it then be going?

7. If a man earns $2400 a year and has his salary increased 5%, how much does he then receive?

8. If a farmer sold 15 cows, and this is $33\frac{1}{3}$% of the number he had, how many cows did he have at first?

USING WHAT YOU HAVE LEARNED

1. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than pressed gold?

2. A clerk's salary is $24 a week. If he spends 60% of it for board and room, 10% for clothes, and 15% for other expenses, and saves the rest, how much does he save in 50 wk.?

3. If a baker uses 1278 lb. of flour in making bread and adds to the flour 426 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture?

4. If 3 teaspoons of sugar make a tablespoon, a teaspoon of sugar is what per cent of a tablespoon?

5. If there are 16 tablespoons to a cup, a tablespoon is what per cent of a cup?

6. A teaspoon of butter is $\frac{1}{3}$ of a tablespoon. Express a tablespoon of butter as per cent of a teaspoon.

7. The record of the champion team in baseball in a certain year was 52 games won and 14 lost. What per cent of the games did the champion team win that year?

8. One of the best records in baseball of recent years was when Brooklyn won the pennant with a total of 82 games won and 54 lost. What was the per cent of games won?

9. Two of the best averages for runs in baseball were 597 times at bat, 139 runs, and 543 times at bat, 145 runs. What was the per cent of runs in each case?

10. If the expenses of a certain village were $16,500 last year and $16,005 this year, what is the per cent of decrease?

11. A man bought a horse for $225 and sold it at a loss of 8%. How much did he receive for the horse?

USING WHAT YOU HAVE LEARNED

1. If a salesman receives a commission of $45 on a sale amounting to $2250, what is his rate of commission?

2. If a dairyman keeps 108 cattle this year and kept 72 last year, what is the per cent of increase?

3. If the rainfall in a certain place was 3.2 in. last month and 2.8 in. the month before, what was the per cent of increase?

4. If a dealer sold daily 225 qt. of milk last month and 270 qt. this month, what is the rate of increase?

5. If you have read 48 pages in a book of 320 pages, what per cent have you read? What per cent have you still to read?

6. If you have read 73 pages of a book, and this is 25% of the book, how many pages are there in all? How many pages have you still to read?

7. If you have read $37\frac{1}{2}\%$ of the pages of a book, and have read 39 pages, how many pages are there in the book?

8. A hardware dealer bought some tools for $67.50 less 20% discount. He sold them for $67.50. What per cent did the dealer gain on the amount he paid for the tools?

9. Allowing 4 oz. of Paris green to 25 gal. of water in preparing a spray to destroy leaf-eating insects, and considering a gallon of water to weigh 8.4 lb., the weight of the Paris green is what per cent of the weight of the water?

10. A cork is pushed below the surface of a glass full of water, and the water that runs over is weighed. The cork is found to weigh 0.6 oz. and the water to weigh 2.5 oz. Cork is what per cent as heavy as an equal volume of water?

11. A dressmaker bought some silk for $45 and sold it for $60. What per cent did she gain on the cost?

USING WHAT YOU HAVE LEARNED

1. A certain number when increased by 30% of itself becomes 650. What is the number?

Explain to the class that once the number (100% of the number) and 30% of the number are 130% of the number, and so 130% of the number is 650. Then ask how the number may be found.

This kind of problem has various applications, but such problems are not so important as the others considered in the book. The teacher may therefore treat this page as optional, to be omitted if desired.

2. A certain number when decreased by 20% of itself becomes 400. What is the number?

Explain to the class that once the number (100% of the number) less 20% of the number is 80% of the number, and so 80% of the number is 400. Then ask how the number may be found.

3. A man sold some goods for $577.50, making a profit of 10% of the cost. How much did the goods cost him?

4. A dealer sold a span of horses for $594, making a profit of 10% of the cost. How much did the horses cost him?

5. If you weigh 78.1 lb. and have increased 42% in weight in the last five years, how much did you weigh five years ago?

6. A manufacturer sells a suit of clothes to a merchant at a profit of $12\frac{1}{2}\%$. The merchant sells it to you for $36 and makes a profit of $33\frac{1}{3}\%$. How much does the suit cost the merchant? How much does the suit cost the manufacturer?

It should be noticed that $133\frac{1}{3}\%$ of the cost to the merchant is $36.

7. An agent bought 100 doz. eggs, which he sold to a grocer at a profit of 10%. The grocer sold the eggs at retail for $66 and made a profit of 20%. How much per dozen did the eggs cost the grocer and how much profit did the agent make?

It should be noticed that 120% of the cost to the grocer is $66.

III. PERCENTAGE APPLIED

Discount. In Book Two we learned something about discount. We shall now review the subject and learn more about the way in which business men use discounts.

After the baseball season you may be glad to sell your catcher's mask for less than it cost you so that you can use the money toward buying a football. If you sell it for 25% less than it cost you, you sell it at a *discount* of 25%. You might speak of this as "a quarter off," or as "75¢ on a dollar," but the selling price is found by deducting 25%, or $\frac{1}{4}$, of the cost.

The price of goods in a printed list or catalogue such as manufacturers or big dealers usually have is called the *list price.*

In a store the price you must pay is usually marked on the goods, and this price is called the *marked price.*

A reduction from the list price or from the amount of a bill of goods is called *commercial discount.*

A reduction from the list price such as is regularly given by a wholesaler to a dealer is called *trade discount.*

A reduction for payment within a fixed time is called a *time discount.*

A reduction for payment either when goods are delivered or promptly upon receipt of the bill is called a *cash discount.*

It is not necessary that the pupil should distinguish between these different discounts, and the definitions need not be learned.

The amount of a bill after the discount has been taken off is called the *net amount.* The amount of a list price or marked price after the discount has been taken off is called the *net price* or the *selling price.*

Thus, the net amount of a bill of \$100 with a discount of 20% is \$100 less \$20, or \$80.

FINDING DISCOUNTS

Numbers 1 to 15, oral

1. If a certain article is marked \$30, and 10% discount is allowed, what is the discount? What is the selling price?

2. If a dealer offers me a \$2 book at a discount of 25%, this is how many cents below the regular price?

3. If a merchant buys \$400 worth of goods and is allowed 10% discount in case he pays for them at once, how much does he gain by prompt payment? How much does he pay?

Find the discounts on the following amounts at the rates specified:

4. \$90, 10%.	8. \$80, 25%.	12. \$100, 10%.
5. \$35, 10%.	9. \$88, 25%.	13. \$200, 20%.
6. \$60, 20%.	10. \$40, 50%.	14. \$240, 25%.
7. \$35, 20%.	11. \$60, 50%.	15. \$360, 50%.

16. If goods marked \$7.35 are sold to a dealer at a discount of 20%, how much do they cost him?

17. If a merchant marks a lot of suits at \$34.50 each and sells them at $\frac{1}{3}$ off, what is the net price of each suit?

Pupils should be asked to watch advertisements in the newspapers to see what discounts are offered. They should state the probable reasons for these discounts and should bring to class the problems which they may make.

Given the marked prices and rates of discount, find the net prices:

18. \$37.50, 10%.	23. \$36.60, 25%.	28. \$81.60, $12\frac{1}{2}$%.
19. \$26.50, 20%.	24. \$75.30, $33\frac{1}{3}$%.	29. \$37.70, 50%.
20. \$37.75, 20%.	25. \$41.70, $33\frac{1}{3}$%.	30. \$55.00, 10%.
21. \$44.40, 25%.	26. \$49.50, $16\frac{2}{3}$%.	31. \$74.75, 20%.
22. \$46.60, 25%.	27. \$92.46, $16\frac{2}{3}$%.	32. \$95.25, 20%.

Debtor and Creditor. A man who owes money to another is called a *debtor* (Dr.). The man to whom he owes money is called the *creditor* (Cr.). So if Rob has bought a baseball from Fred and owes him 50¢, Rob is the debtor in the transaction and Fred is the creditor.

Debit and Credit. The amount owed on a bill is called the *debit*, and any payments that have been made form the *credit*.

Bill. The following is an example of a receipted bill of more elaborate form than those which were given in Book Two:

LORD & TAYLOR

Importers and Dealers in
Foreign and Domestic Dry Goods

Jacksonville, Fla., May 1, 1924

Mrs. John P. Sinclair
Hotel Plaza, City

Apr.	4	2 hatpins	.50	1	00	1	00
	13	4 pr. gloves	2.25	9	00		
		2 yd. lace	2.75	5	50	14	50
	20	2 yd. ribbon	.31		62		
		1 pr. slippers	5.00	5	00	5	62
						21	12

RECEIVED PAYMENT
MAY 3, 1924
LORD & TAYLOR
Per E. F. S.

To fill out the bill by inserting the amounts of the various purchases in the columns at the right is called "making the extensions on the bill."

Discounted Bill. A bill sent by a wholesale dealer usually has some discount allowed. A sample receipted bill follows:

Chicago, Ill., Dec. 17, 1924

Mr. B. S. Cole, Oklahoma City, Okla.

Bought of HILL, SMITH & CO.
Manufacturing Jewelers
8378 Burlington Ave.

Dec.	2	3 doz. silver forks	@ $32	96		
		1/6 doz. salad forks	@ $33	5	50	
				101	50	
			Less 20%	20	30	
						81 20

RECEIVED PAYMENT
DEC. 20, 1924
HILL, SMITH & CO.
Per C.T.M.

Receipt. A receipt may also be written separately, thus:

$81.20 *Chicago, Ill., Dec. 20, 1924*

Received from B. S. Cole

Eighty-one 20/100 *Dollars*

to balance account

Hill, Smith & Co.

Invoice. A bill giving in detail a list of items and prices of goods sold is called an *invoice*. A sample invoice follows:

St. Paul, Minn., Dec. 12, 1925

Mr. James S. Dunbar, Fargo, N.D.

Bought of RANDALL BROS.

Importers of Dry Goods

63 East Fourth Street

Terms: *2/10 net 30*

236	*14*	*pc. blue flannel*				
		40, 41, 41^1, 41^3, 40^2, 40,				
		41, 42, 41^3, 42^1, 41, 42^2,				
		41, 40 576 @ $33\frac{1}{3}$¢	*192*			
427	*8*	*pc. silk*				
		40, 39, 41^2, 40^3, 39^1, 41,				
		42^2, 40 324 @ $1.25	*405*		*597*	

In the above invoice 41^1 means $41\frac{1}{4}$; 41^2 means $41\frac{2}{4}$, or $41\frac{1}{2}$; and 41^3 means $41\frac{3}{4}$. This is the customary way of indicating the number of yards in pieces of goods. The numbers 236 and 427 refer to the price list in which the goods are described. The numbers 14 and 8 indicate the number of pieces (pc.). The number of yards of the first is 576 and of the second 324.

The expression "Terms: 2/10 net 30" means that Mr. Dunbar may take off a 2% discount if he pays the bill within 10 da. or that he may have 30 da. in which to settle the account.

There is no essential difference between a bill and an invoice. The latter term is commonly used in wholesale transactions.

BILLS AND DISCOUNTS

Make out bills, or invoices, for the following, take off the discount given, and write the receipt on each bill:

1. 14 pc. ribbon @ \$1.65; 16 pc. @ \$1.33⅓; 48 pc. @ \$1.66⅔; 36 pc. @ \$1.37½; 24 pc. @ \$1.82½. Terms: 2/10 net 30.

2. 2 carloads coal, 23,800 lb., 25,200 lb., @ \$9.80 per ton; 3 carloads coal, 24,800 lb., 23,700 lb., 22,600 lb., @ \$9.95 per ton. Terms: 3/10 net 90.

3. 1 gro. cans sardines @ \$3.66⅔ per doz.; 9 doz. cans shrimps @ \$2.25; 8 doz. tins herrings @ \$2.62½; 8¼ doz. cans lobster @ \$3.16; 48 cans mackerel @ 23½¢. Terms: 2/10 net 60.

4. 9 doz. boxes codfish @ \$2.25; 8 doz. cans salmon @ \$2.66⅔; 16 doz. cans caviar @ \$3.87½; 14 doz. cans Yarmouth bloaters @ \$2.66⅔; 36 doz. cans tongue @ \$11.25. Terms: 3/10 net 60.

5. 9 doz. jars meat extract @ \$3.87½; 18 doz. cans chicken @ \$3.56; 36 doz. cans corned beef @ \$2.87½; 16 doz. cans soup @ \$3.75; 8 doz. cans clam chowder @ \$3.90; 18 doz. cans clam juice @ \$1.46½. Terms: 3/10 net 30.

6. 7 armchairs @ \$8.30; 36 kitchen chairs @ \$1.37½; 9 kitchen tables @ \$2.75; 3 bedroom sets @ \$54.33⅓; 18 dining tables @ \$16.50; 9 sideboards @ \$24.66⅔. Terms: 2/10 net 30.

7. 20 pc. linen containing 39, 40^2, 38^3, 42, 41^1, 40^2, 40, 39, 40^1, 41^2, 43, 39^3, 40^1, 42^1, 40^2, 40^1, 41^3, 42, 39^1, 40 yd., @ 66⅔¢; 16 pc. silk containing 41^1, 42^1, 40, 39^2, 42, 40^3, 38, 41^1, 43, 41^3, 39, 40^2, 40^1, 41^1, 40, 39^2 yd., @ \$1.33⅓. Discount allowed: 20%.

8. 275 bbl. flour @ \$9.75; 324 bbl. flour @ \$10.25; 300 bu. potatoes @ \$1.15; 1578 lb. butter @ 48¢; 2000 bu. apples @ \$1.75; 500 bu. oats @ \$0.80; 625 bu. corn @ \$1.25; 238 bu. wheat @ \$1.87; 123 bu. rye @ \$1.35. Discount allowed: 15%.

Several Discounts. In some kinds of wholesale business two or more discounts are sometimes allowed on the same bill.

For example, a dealer may buy $400 worth of hardware with discounts of 20% and 10%, often written "20%, 10%" or simply "20, 10." This means that 20% is first deducted from the amount of the bill and then 10% is deducted from the remainder.

The amount of the bill is $400.
The amount less 20% is $320.
The $320 less 10% is $288, the *net amount.*

This explains the general nature of the problem. Practically, however, we should take such short cuts as we can.

The solution is much shorter if we remember that $20\% = \frac{1}{5}$, that $10\% = \frac{1}{10}$, that if we take away $\frac{1}{5}$ we have $\frac{4}{5}$ left, and that if we take away $\frac{1}{10}$ from the result we have $\frac{9}{10}$ left.

Therefore the solution becomes simply this:

$$\frac{9}{10} \text{ of } \frac{4}{5} \text{ of } \$400 = \frac{9 \times 4 \times \overset{80}{\$400}}{10 \times 5} = \$288.$$

In the same way, if the discounts were $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, we should have

$$\frac{5}{6} \text{ of } \frac{7}{8} \text{ of } \$400 = \frac{5 \times 7 \times \overset{50}{\$400}}{\underset{3}{6} \times 8} = \frac{\$875}{3} = \$291.66\tfrac{2}{3}.$$

Therefore the net amount would be $291.67.

Although there is no fixed custom, in solving the problems of this book the pupil should neglect fractions of a cent less than $\frac{1}{2}¢$ in a discount, calling $\frac{1}{2}¢$ or more a full cent.

With the extended use of trade acceptances these chain discounts are becoming less common. The subject of acceptances is discussed later.

SEVERAL DISCOUNTS

Numbers 1 to 4, oral

1. From \$100 take 10%, and 10% from the remainder.
2. From \$400 take 20%, and 1% from the remainder.
3. From \$100 take 20%, and 10% from the remainder.
4. From \$400 take 20%, and 10% from the remainder.

Find the net amounts of bills with discounts as follows:

5. \$400, 20%, 20%.
6. \$600, 25%, 5%.
7. \$700, 25%, 10%.
8. \$300, 15%, 2%.

9. What is the difference between a discount of 50% on \$100 and the two discounts of 25%, 25%?

10. Is there any difference between a discount of 10%, 5%, and one of 5%, 10%, on \$100? How is it on \$200? on \$900?

11. What single discount is equal to the discount of 10%, 8%?

From 100% take 92% of 90%.

Find the single discounts that are equal to the following:

12. 10%, 10%.
13. 10%, 12%.
14. 5%, 16%.
15. 8%, 12%.
16. 8%, 20%.
17. 10%, 20%.

18. Find the net price of a tractor listed at \$1220 less 10%, 5%.

19. An article listed in a catalogue at \$278 is sold at 12% trade discount and 2% for cash. Find the total discount that a dealer would be allowed if he bought for cash.

20. A contractor received two bids for trim on a building that he was erecting. One was \$3500 less 65%, 10%, and the other was \$3400 less 50%, 16%. Which is the cheaper bid, and how much cheaper?

Bill with Several Discounts. The following is a common form of a wholesale bill of goods with several discounts:

Dallas, Tex., June 13, 1922

Mr. R. S. Thorndike, Abilene, Tex.

Bought of GREELEY & COMPANY, Jewelers
1072 Commerce St.

Terms: *30 da.*

June	*10*	*8 doz. spoons* @ *$24.75*	*198*	*00*		
		7 doz. plated forks @ *$7.20*	*50*	*40*		
					248	*40*
		Less 20%, 10%			*69*	*55*
					178	*85*

BILLS AND DISCOUNTS

Make out bills for the following:

1. 36 doz. files @ $8.30. Discounts 30%, 20%.

2. 480 yd. silk @ $2.50; 840 yd. velvet @ $2.75; 425 yd. satin @ $3.10; 285 yd. chiffon @ $2.40. Discounts 25%, 10%.

3. 9 doz. pairs hinges @ $6.75; 36 doz. table forks @ $8.40; 48 doz. table knives @ $12.60. Discounts 20%, 10%.

4. 36 doz. locks @ $6.30; 19 doz. mortise locks @ $6.75; 28 doz. pairs hinges @ $8.85. Discounts 25%, 8%.

5. 360 yd. taffeta @ $1.37½; 9 gross pompons @ $185; 6 doz. pieces braid @ $31.80. Discounts 10%, 5%.

6. 780 yd. silk @ $2.65; 440 yd. lawn @ 45¢; 560 yd. taffeta @ $1.42. Discounts 10%, 5%, 5%.

Sample Price List. The following is a price list of certain school supplies, with the discounts allowed to schools and dealers:

Rulers,	\$0.95 per doz., less 5%
Composition books,	11.00 per gross, less 10%
Thumb tacks,	1.20 per 100, less 40%
Drawing pencils,	11.42 per gross, less 20%
Drawing paper, 9×12,	3.80 per package, less $\frac{1}{10}$
Pens,	1.62 per gross, less $\frac{1}{4}$, 10%
Tubes of paste,	10.10 per gross, less 10%, 5%
Penholders,	8.40 per gross, less 12%, 5%
Drawing compasses,	5.40 per doz., less 10%, 5%

DISCOUNTS FROM LIST PRICE

1. A school buys 8 packages of drawing paper and 400 thumb tacks. Using the above list, how much will they cost?

2. How much will 9 gross of pens and $\frac{3}{4}$ gross of composition books cost? 3 gross of pens and 2 doz. rulers?

3. There are 30 pupils in a class, and each needs drawing compasses and a ruler. How much will they cost the school? How much would they cost if there were 40 pupils?

4. If a dealer sells pens at 2¢ apiece, how much does he gain per gross? If he sells penholders at 8¢ each, how much does he gain per gross?

5. A dealer buys a gross of rulers and a gross of drawing pencils. He sells the rulers at 15¢ each and the pencils at 12¢ each. How much does he gain?

6. If a dealer buys a gross of tubes of paste for mounting pictures and sells the tubes at 15¢ each, how much does he gain on the purchase? how much on 3 gross?

DRILL WORK ON DISCOUNTS

Find the discount on each of the following:

1. \$725, 15%.
2. \$630, 12%.
3. \$480, 14%.
4. \$960, 16%.
5. \$1974, $\frac{1}{3}$ off.
6. \$2764, $\frac{1}{4}$ off.
7. \$5735, $\frac{1}{5}$ off.
8. \$7360, $\frac{1}{8}$ off.
9. \$972.85, 6%.
10. \$374.95, 5%.
11. \$673.80, 4%.
12. \$596.83, 8%.

Find the net amounts of bills with discounts as follows:

13. \$380, 4%.
14. \$795, 5%.
15. \$687, 6%.
16. \$985, 7%.
17. \$745, 9%.
18. \$2736, $\frac{1}{3}$ off.
19. \$3896, $\frac{1}{4}$ off.
20. \$7315, $\frac{1}{5}$ off.
21. \$8436, $\frac{1}{6}$ off.
22. \$9656, $\frac{1}{8}$ off.
23. \$342.80, 4%.
24. \$538.90, 5%.
25. \$738.62, 6%.
26. \$896.48, 8%.
27. \$34.76, 12%.

Find the discount on each of the following:

28. \$1275, 10%, 6%.
29. \$1596, 12%, 4%.
30. \$3286.40, 6%, 3%.
31. \$4739.75, 8%, 5%.
32. \$1286.92, 9%, 6%.
33. \$3425, 6%, 4%, 3%.
34. \$4368, 5%, 5%, $2\frac{1}{2}$%.
35. \$7423, 6%, 5%, $1\frac{1}{2}$%.
36. \$9280, 8%, $2\frac{1}{2}$%, $1\frac{1}{2}$%.
37. \$4800, 9%, $5\frac{1}{2}$%, $1\frac{1}{2}$%.

38. Show that the discount 7, 6, 3, that is, 7%, 6%, 3%, on \$1000 is the same as the discount 3, 6, 7, and also the same as the discount 6, 3, 7.

39. A dry-goods dealer bought a shipment of cloth, and after deducting a discount of 25%, 10%, and paying \$8.50 for freight and cartage, the cloth cost him \$400. Find the gross amount of the bill, that is, the amount before the discounts were deducted.

Profit and Loss on Cost. When a dealer buys goods he expects to sell them for more than he paid, thus making a *profit.*

If he sells them for less than he paid he suffers a *loss.*

Merchants usually speak of their profits and losses as a certain per cent. This per cent is reckoned sometimes on the cost of the goods and sometimes on the selling price. We shall first consider the older plan of finding the per cent on the cost.

Overhead. Such items as freight charges, storage, insurance, interest on the money invested, rent, heat, light, and the cost of selling the goods are called *overhead charges* or simply *overhead.*

In order to simplify the work for pupils in the seventh grade we shall include under *overhead* all the *cost of doing business.* In large establishments the *buying expenses* are considered separately, but these details need not be given in elementary arithmetic.

Illustrative Problems. 1. A man buys goods for \$40, including all overhead charges, and sells them at a profit of 20% on the total cost. What is the selling price?

\$40		\$40	
0.20		8	
\$8.	profit	\$48	selling price

2. A man buys goods for \$60, including all overhead charges, and sells the goods for \$40. What is his rate of loss, that is, he loses what per cent of the cost?

\$60		60)20	
40			$0.33\frac{1}{3}$
\$20	loss		

Since \$20 is lost on \$60, we are asked to find what per cent \$20 is of \$60. Then, as on page 40, we have

$\$20 \div \$60 = 0.33\frac{1}{3} = 33\frac{1}{3}\%$, the per cent of loss.

PROFIT AND LOSS ON COST

1. A dealer buys some goods for $750 and sells them so as to gain $225. Find his per cent of profit.

In all problems on this page the cost should be considered as including all overhead and other expenses, and the per cent of profit or loss should be computed on the cost.

2. A merchant bought some goods for $5700, but owing to hard times he had to sell them at a loss of $285. Find his per cent of loss.

3. If a grocer buys some canned goods for $2475 and sells them for $3217.50, what is his per cent of profit?

4. If a dry-goods merchant buys some silk for $4800 and is compelled to sell it for $4464, what is his per cent of loss?

5. Mr. Allen bought a village lot for $1500 and sold it for $2100. Find the rate of profit.

6. If Mr. Allen had sold the lot mentioned in Ex. 5 for $900, what would have been his per cent of loss?

7. Mr. Robinson bought a house for $5600 and sold it at a profit of 15%. Find the selling price.

8. If Mr. Robinson had sold the house mentioned in Ex. 7 at a loss of 15%, what would have been the selling price?

9. A man bought some horses and sold them at a profit of $1960. If he made 20% on the deal, how much did he pay for the horses and for how much did he sell them?

10. A man bought a city lot for $24,000 and sold it at a profit of $16\frac{2}{3}\%$. Find the amount of profit and also the selling price.

11. A man bought an automobile for $2400, got $360 worth of use out of it, and sold it for $12\frac{1}{2}\%$ less than he paid. Find the profit or the loss, if any.

Profit and Loss on Selling Price. Formerly profit and loss were always computed on the mere cost of goods. As the overhead became more of an expense, the profit and loss were computed on the cost plus the overhead, that is, on the total cost.

At present, however, in large business houses it is the custom to figure the cost of the goods, the overhead, and the profit, each as a per cent of the selling price.

By keeping careful records of their business, including all overhead charges, large dealers are able to determine in advance the per cent of the selling price that should be allowed to the overhead in order to allow them to realize the expected profit. *This profit is then computed on the selling price* instead of, as formerly, on the cost.

This method of figuring profit and loss on the selling price is given because it represents a modern business custom which has been adopted almost universally by large business houses. In small transactions the pupil will find in common use the older method of figuring profit on the cost. However, many merchants have failed in business because they computed profits simply on the first cost of goods and neglected the overhead.

Illustrative Problem. A large department store pays \$8 for a certain article. Experience has shown that the overhead is 20% of the selling price. If the desired profit is 15% of the selling price, what should be the selling price?

The cost is 20% + 15%, or 35%, less than the selling price; that is, since $35\% = \frac{7}{20}$, the cost is $\frac{13}{20}$ of the selling price.

Then $\frac{13}{20}$ of the selling price is \$8,

$\frac{1}{20}$ of the selling price is $\$\frac{8}{13}$,

and the selling price is $20 \times \$\frac{8}{13}$.

Therefore the selling price is $\$\frac{160}{13}$, or $\$12.30\frac{10}{13}$, which would be given as \$12.31.

PROFIT AND LOSS ON SELLING PRICE

1. A merchant sells for \$12 an article that cost \$9, including all overhead. Find the per cent of profit on the selling price.

2. At what price should a department store sell an article that cost \$4.50 so as to make a profit of 10% on the selling price?

In the first six problems on this page the cost should be considered as including the overhead and other expenses.

3. A department store made a profit of 10% on the selling price of an article that cost \$2.25. Find the selling price.

4. A large store sells for \$17.50 an article that cost \$14. Find the per cent of profit on the selling price.

5. A merchant sells for \$7.50 an article that cost \$10. Find the per cent of loss on the selling price.

6. A department store gained \$3.96 on an article that is sold for \$26.40. Find the per cent of profit on the selling price.

7. A certain article costs \$12 and the overhead is 10% of the selling price. If the profit is to be 15% of the selling price, what should be the selling price?

8. A certain article costs \$21 and the overhead is 15% of the selling price. If the profit is to be 15% of the selling price, what should be the selling price?

9. A dealer pays \$3.30 a yard for some silk and the overhead is 20% of the selling price. If the profit is to be 14% of the selling price, what should be the selling price per yard?

10. An automobile dealer pays \$2250 for a car and allows 15% of the selling price for overhead and 10% for profit. What is the selling price?

11. A dealer sells an automobile and makes a profit of 15% on the selling price. If the profit is \$351, what is the selling price?

PROFIT AND LOSS

1. A dealer buys 36 stoves at \$28.75 and pays \$33.70 freight. The other overhead is \$2.10 per stove. If he sells the stoves at \$37, what is the per cent of profit on the total cost?

On this page the pupil is definitely told on what the per cent is to be computed. Results should be carried only to the nearest tenth of 1% in problems of this type.

2. In Ex. 1 find the per cent of profit on the selling price.

3. A dry-goods dealer buys 940 yd. of damask linen at \$1.25 a yard, estimates his overhead at $3\frac{1}{2}\phi$ a yard, and sells the linen at \$1.60 a yard. Find the per cent of profit on the selling price.

4. A manufacturer adds 22% of the original cost of an article to cover all expenses. If the original cost is \$9.75, at what price must he sell the article to make a profit of 20% on the total cost?

5. Taking the result of Ex. 4, find the manufacturer's per cent of profit on the selling price of the article.

6. A manufacturer sells at a profit of $17\frac{1}{2}\%$ on the selling price. How much is his profit on a sale of \$875.60?

7. A gain of 20% on the selling price of an article is equivalent to what per cent of gain on the cost?

The pupils should let 100 represent the selling price, then find the profit, then the cost, and finally the per cent gained on the cost.

8. A dealer marks 75 copies of a book \$1.50 each, thereby expecting a net profit of 20% on the cost. He sold 65 copies at \$1.50 and the rest at \$1.10. Find the per cent of gain or loss, reckoned on the cost.

9. A real-estate dealer gained \$600 by selling a house and lot at a profit of 20% of the cost. What was the cost of the property, and for how much did the dealer sell it?

Interest. Mr. Pomeroy has worked in a grocery store for a number of years and decides to open a store of his own. He has saved some money, but since it will take more than his savings to pay for the rent of the store, to pay his help, and to pay for the goods which he must buy, he must borrow some money to start in business. When a man borrows money he pays for the use of it until such time as he pays it back.

The money paid for the use of money is called *interest.*

For example, if Mr. Pomeroy borrows $2000 for a year and pays $120 for the use of it, the $120 is interest.

Principal. The sum of money on which interest is paid is called the *principal.*

If Mr. Pomeroy borrows $2000, the $2000 is the principal.

Amount. The sum of principal and interest is called the *amount.*

Reckoning Interest. Interest is usually reckoned as a certain per cent of the principal. If the interest for 1 yr. is 6% of the principal, and the principal is $2000, the interest for 1 yr. is 6% of $2000, or $120.

If interest is paid for a fraction of a year, the interest is that fraction of the interest for a year. That is, the interest on $2000 for 6 mo. at 6% is $\frac{1}{2}$ of $120, or $60, and the interest for 2 mo. is $\frac{1}{6}$ of $120, or $20.

Rate of Interest. The per cent of the principal paid as interest for 1 yr. is called the *rate of interest.* That is, if Mr. Pomeroy has to pay 6% of the principal for the use of the money for 1 yr., the rate of interest is 6% per year, or per annum.

The rate of interest varies in cities from less than 2% for certain kinds of loans to a high rate for certain other kinds. Ordinary loans commonly pay 5% or 6%, but the war has led to higher rates generally.

FINDING INTEREST

Numbers 1 to 13, oral

1. Do you know of any banks where people put their money and receive interest until they take the money out again? If so, tell the class about them.

2. Do you know about our government's borrowing money and selling bonds to the people? If so, tell the class about it.

3. Have you ever heard of our government's selling Savings Stamps? If so, tell the class about them.

Find the interest for 1 yr. on the following sums:

4. \$100 at 4%.
5. \$100 at 5%.
6. \$100 at 6%.
7. \$200 at 4%.
8. \$200 at 6%.
9. \$300 at 5%.
10. \$300 at 6%.
11. \$3000 at 5%.
12. \$4000 at 6%.

13. If the interest for 1 yr. on a certain sum is \$12, how much is it for 3 mo.? How much is the interest for 6 mo.? How much is the interest for 8 mo.?

Find the interest for 1 yr. on the following sums:

14. \$750 at 3%.
15. \$875 at 4%.
16. \$475 at 5%.
17. \$425 at 6%.
18. \$75 at $4\frac{1}{2}$%.
19. \$85 at $3\frac{1}{2}$%.
20. \$95 at $4\frac{1}{2}$%.
21. \$65 at $2\frac{1}{2}$%.
22. \$375.60 at 5%.
23. \$550.25 at 4%.
24. \$835.40 at 6%.
25. \$925.20 at 6%.

Find the interest for 6 mo. on the following sums:

26. \$100 at 6%.
27. \$275 at 4%.
28. \$350 at 4%.
29. \$750 at 6%.
30. \$475 at 3%.
31. \$250 at $2\frac{1}{2}$%.
32. \$350 at 6%.
33. \$550 at $4\frac{1}{2}$%.
34. \$695.30 at 3%.
35. \$964.80 at 3%.
36. \$575.75 at 6%.
37. \$950.80 at 5%.

Interest for Months and Days. The general plan of finding interest for any length of time is always the same.

Find the interest for one year and then multiply this by the number representing years or the fractional part of a year.

For example, find the interest on \$250 for 10 mo. at 6%.

The interest for 1 yr. is 6% of \$250, and for 10 mo. it is $\frac{10}{12}$ of that sum. We therefore have to find

$\frac{10}{12}$ of 6% of \$250.

We arrange the work as here shown, cancel, and the result is \$12.50.

$$\frac{1\cancel{0} \times \cancel{6} \times \$25\cancel{0}}{\cancel{12} \times 1\cancel{0}\cancel{0}} = \frac{\$25}{2} = \$12.50$$

This example may be solved mentally by noticing that $\frac{10}{12}$, or $\frac{5}{6}$, of 6% is $5 \times 1\%$, or 5%, and so we have 5% of \$250 = $5 \times \$2.50 = \$12.50$.

In finding interest consider 30 da. as a month. After finding the interest for 1 mo., the interest for 1 da. is $\frac{1}{30}$ of this amount; for 5 da. it is $\frac{1}{6}$ of the interest for 1 mo.; and so on.

Neglect a fraction of a cent less than $\frac{1}{2}$ in any interest found; call any fraction of a cent $\frac{1}{2}$ or greater a full cent.

INTEREST FOR MONTHS AND DAYS

1. How much is the interest on \$872, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

2. How much is the interest on \$720, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

3. How much is the interest on \$1080, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 5 da.? for 1 da.?

4. How much is the interest on \$225.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 1 mo.? for 10 da.?

Interest for Years and Months. Mr. Dean needs \$125 in order to pay for some repairs on his house and borrows the money at 5%. If he does not pay it back until 2 yr. 3 mo. later, how much is the interest?

Since 2 yr. 3 mo. is the same as $2\frac{1}{4}$ yr. or $\frac{9}{4}$ yr., we have to find $2\frac{1}{4} \times 5\%$ of \$125.

We arrange the work as here shown and find the result to be \$14.06$\frac{1}{4}$, which, neglecting the fraction, we give as \$14.06.

$$\frac{9 \times 5 \times \$125}{4 \times 100} = \frac{\$56.25}{4} = \$14.06\tfrac{1}{4}$$

It will be noticed that in this case we did not cancel the factor 25 from \$125 and 100, as we might have done. Had we done so, we should have had the divisor 16 in place of the divisor 4. In such cases it is often easier not to cancel any factors from the 100 in the denominator, but to move the decimal point two places to the left in the result.

In the problem above we also might have said

5% of \$125 = \$6.25, interest for 1 yr.

$2\frac{1}{4} \times \$6.25 = \$14.06\frac{1}{4}$. \$14.06. *Ans.*

INTEREST FOR YEARS AND MONTHS

Find the interest on the following:

1. \$350, at 6%, for 6 mo.
2. \$400, at $5\frac{1}{2}$%, for 8 mo.
3. \$500, at 5%, for $2\frac{1}{2}$ yr.
4. \$725, at 4%, for $3\frac{1}{2}$ yr.
5. \$850, at 6%, for $1\frac{1}{2}$ yr.
6. \$9600, at 5%, for $2\frac{1}{2}$ yr.
7. \$825, at 4%, for $3\frac{1}{2}$ yr.
8. \$95, at 6%, for 2 yr. 6 mo.
9. \$375, at 6%, for 6 mo.
10. \$80, at $4\frac{1}{2}$%, for 1 yr. 6 mo.

Find the interest on \$875, for 2 yr. 3 mo., at:

11. 5%.
12. 6%.
13. 3%.
14. 4%.
15. $5\frac{1}{2}$%.

USING WHAT YOU HAVE LEARNED

1. A man borrows \$950, agreeing to pay 5% interest. He pays the principal and interest in 3 yr. 2 mo. How much money does he pay in all, principal and interest?

2. A dealer in automobiles buys 16 cars at \$1150 each and agrees to pay for them in 9 mo., with interest at 6%. How much does he pay in all, principal and interest?

3. Mr. Russell hires Mr. Baker to make some repairs to his house, agreeing to pay him \$85 for the work. Mr. Baker agrees to wait 3 mo. for his money if he receives interest at 6%. How much money is due Mr. Baker at the end of the 3 mo.?

4. Woodhull & Co. need \$3650 to buy holiday goods. They borrow it Oct. 15 and agree to pay it back in 3 mo., with interest at 5%. How much do they owe in 3 mo.?

5. Pierson & Co. buy \$2250 worth of goods, agreeing to pay for the goods in 3 mo., with interest at 5%. How much do they owe at the end of the 3 mo.?

6. Mr. Holbrook buys from Mr. Jackson a secondhand automobile, agreeing to pay \$580 in 4 mo., with interest at 6%. How much interest is due in 4 mo.?

7. A man borrows \$2500, agreeing to pay 6% interest. He pays the principal and interest in 1 yr. $8\frac{1}{2}$ mo. How much money does he pay in all, principal and interest?

8. A man buys 36 head of cattle at \$90 a head and agrees to pay for them in 10 mo., with interest at 6%. How much does he pay in all, principal and interest?

9. The day a certain boy is 16 yr. old his father invests \$500 for him at 5%, the \$500 and interest being due the day he is 21 yr. old. What will be the total amount due at that time?

Interest for Years, Months, and Days. 1. A man borrows $400 from another man on Sept. 10, 1922, and agrees to pay interest at 6% when he repays the principal. If he pays the loan on Aug. 7, 1923, how much interest is due then?

We find the time that the $400 draws interest as follows:

yr.	mo.	da.	
1923	8	7	= second date
1922	9	10	= first date
	10	27	= difference in time

10 mo. 27 da. = 300 da. + 27 da. = 327 da.

According to the rule given on page 69 the interest for 1 yr. is 6% of $400, and for 327 da. it is $\frac{327}{360}$ of that sum. We therefore have to find

$\frac{327}{360}$ of 6% of $400.

We arrange the work as here shown, cancel, and find the interest due Aug. 7, 1923, to be $21.80.

$$\frac{\overset{109}{\cancel{327}} \times \cancel{6} \times \overset{20}{\cancel{\$400}}}{\cancel{360} \times 100} = \$21.80$$

$\cancel{60}$

$\cancel{20}$

We save time and avoid errors by using cancellation, as shown above, whenever it is possible to do so.

It should be remembered that banks compute interest by means of elaborate interest tables. In charging interest on loans the banks commonly use tables based on 360 da. to the year. In paying interest on deposits they frequently use tables based on 365 da. to the year, this interest being a little less. In a problem like the one above, a bank would find, by means of tables, the exact number of days from Sept. 10, 1922, to Aug. 7, 1923. These matters are, however, technicalities of banking which it is not necessary as yet for the pupil to consider. The problem would be an unusual one under modern banking conditions, as explained in the note at the bottom of page 73.

2. Suppose that the man referred to in Ex. 1, page 72, had kept the $400 from Sept. 10, 1922, to Aug. 7, 1925. What would the interest have amounted to then?

We find the time from Sept. 10, 1922, to Aug. 7, 1925, as follows:

yr.	mo.	da.	
1925	8	7	= second date
1922	9	10	= first date
2	10	27	= difference in time

Since in 2 yr. there are 720 da., and in 10 mo. there are 300 da., in 2 yr. 10 mo. 27 da. there are 1047 da. We therefore have to find

$\frac{1047}{360}$ of 6% of $400.

Arranging the work as shown, we cancel and find the interest due Aug. 7, 1925, to be $69.80.

$$\frac{\overset{349}{\cancel{1047}} \times \cancel{6} \times \overset{20}{\cancel{\$400}}}{\underset{60}{\cancel{360}} \times \cancel{100}} = \$69.80$$

This problem could have been solved by finding first the interest for 2 yr., then the interest for 10 mo. 27 da., and adding these two results, but ordinarily the method employing cancellation, which is shown above, is shorter.

As already stated, problems of this kind are becoming more unusual every year. The great improvement in banking facilities has led to the borrowing of money from banks instead of from individuals. In most parts of this country money is usually borrowed from a bank for a short period, the interest being paid in advance. If the borrower needs the money for a longer time, the bank would cancel the old loan, make a new one for another short period, and require the borrower to pay the interest in advance on the new loan. The older method of borrowing for a longer time, as in the above example, is often found, however, and pupils should know how to solve such problems.

COMPUTING INTEREST

Find the interest on $750, for 1 yr. 2 mo. 6 da., at:

1. 4%. 2. 5%. 3. 6%. 4. $4\frac{1}{2}$%. 5. 7%.

Find the interest at $5\frac{1}{2}$%, for 1 yr. 3 mo. 10 da., on:

6. $375. 7. $480. 8. $560. 9. $675. 10. $950.

Find the interest at $4\frac{1}{2}$%, on $2750.75, for:

11. 1 yr. 6 mo. 12. 1 yr. 8 mo. 12 da. 13. 2 yr. 9 mo. 16 da.

14. Find the interest on $1728.45, for 2 yr. 8 mo. 9 da., at 6%.

15. Find the amount of principal and interest on $3468.75, for 9 mo., at $5\frac{1}{2}$%.

Find the interest on each of the following:

16. $725, for 1 yr. 6 mo. 8 da., at 6%.

17. $600, for 1 yr. 3 mo. 15 da., at 4%; at 5%.

18. $625.50, for 2 yr. 11 mo. 15 da., at $5\frac{3}{4}$%.

19. $475.50, for 3 yr. 10 mo. 15 da., at $4\frac{1}{4}$%.

20. $265.50, for 2 yr. 10 mo. 20 da., at $4\frac{3}{4}$%; at 5%.

21. $3562.50, for 3 yr. 2 mo. 5 da., at $5\frac{1}{2}$%; at $4\frac{1}{2}$%.

22. A man borrowed $1275 on Apr. 7, 1923, at 6%, and $1500 on Apr. 10, 1923, at 5%. He paid the principal and interest in both cases on Dec. 30, 1924. How much did he pay?

23. A man borrowed $1500 on May 1, 1924, at 5%, and $2500 on July 5, 1924, at $4\frac{1}{2}$%. He paid the principal and interest in both cases on June 7, 1926. How much did he pay?

24. What is the amount of principal and interest on $3200 borrowed Feb. 10, 1925, at 6%, and $2800 borrowed Apr. 15, 1925, at 5%, both payments being made on Oct. 15, 1927?

Six-Per-Cent, or 60-Day, Method of Finding Interest. Because banks often charge 6% interest, it is frequently convenient to use what is commonly called the Six-Per-Cent Method.

For example, find the interest on \$420 for 60 da. at 6%.

Since 60 da. $= \frac{1}{6}$ yr., we have

$$\frac{1}{6} \text{ of } 6\% \text{ of } \$420 = 1\% \text{ of } \$420 = \$4.20.$$

Hence the interest at 6% for 60 days is 0.01 of the principal, and for 6 days 0.001 of the principal.

To find the interest for 60 da. at 6%, move the decimal point two places to the left; for 6 da., three places to the left.

For other periods the interest can be found from that for 60 da. or 6 da.

For example, find the interest on \$420 for 5 mo. 10 da. at 6%.

The work should be done mentally except that results should, if necessary, be set down as here shown.

2 mo.,	\$4.20
2 mo.,	4.20
1 mo.,	2.10
$\frac{1}{3}$ mo.,	.70
	\$11.20

The interest is found to be \$11.20.

After finding the interest at 6%, the interest for other rates is easily found. Thus, the interest at 5% is $\frac{5}{6}$ of that at 6%.

SIX-PER-CENT METHOD

Numbers 1 to 5, oral

Find the interest for 60 da. and also for 30 da., at 6%, on:

1. \$250. 2. \$370. 3. \$1250. 4. \$1720. 5. \$2800.

Find the interest at 6% and also at 5% on the following:

6. \$750, 3 mo. 8. \$920, 4 mo. 10. \$1500, 5 mo. 15 da.
7. \$1250, 5 mo. 9. \$870, 1 mo. 11. \$2500, 7 mo. 6 da.

Partial Payments. If a dealer holds a note, or a written promise to pay money, against one of his customers, and it bears interest, and partial payments are made, the amount due on the day of settlement is found in the following way:

Add the interest to the principal whenever the payment, or the sum of the payments, equals or exceeds the interest.

Then deduct the payment or payments and continue as before.

This is the United States Rule for partial payments. In states where other rules are legal such problems as may be given should be solved accordingly.

For example, a note for \$1000 at 5% is dated Jan. 1, 1926. The following payments are indorsed (written across the back): July 1, 1926, \$10; Jan. 1, 1927, \$40; July 1, 1927, \$20; Jan. 1, 1928, \$130. How much is due July 1, 1928?

Explain to the class that the interest due July 1, 1926, is \$25, and the payment is \$10. Hence the payment cannot be deducted. The reason is easily seen, for if we should take \$1025 − \$10, or \$1015, as the new principal, we should be drawing interest on more than the \$1000.

First principal	\$1000
Interest for 1 yr.	50
First amount, Jan. 1, 1927	\$1050
1st and 2d payments	50
Second principal, Jan. 1, 1927	\$1000
Interest for 1 yr.	50
Second amount, Jan. 1, 1928	\$1050
3d and 4th payments	150
Third principal	\$900
Interest for 6 mo.	22.50
Amount due July 1, 1928	\$922.50

Therefore the amount due on the day of settlement is \$922.50.

PROBLEMS IN PARTIAL PAYMENTS

1. A note for \$375 at 5% has a payment of \$18.75 indorsed annually at the close of each year from its date, for three years. How much is due at the end of the fourth year?

2. A note for \$500 at 6% is dated Jan. 1, 1926, and has the following payments indorsed: July 2, 1926, \$100; Dec. 1, 1926, \$2; Jan. 1, 1927, \$98. How much is due July 1, 1927?

3. A note for \$300 at 6% is dated April 1, 1926, and has the following payments indorsed: Sept. 1, 1926, \$50; Feb. 16, 1927, \$75; Jan. 1, 1928, \$98. How much is due July 1, 1928?

4. A note for \$1800 at 5% is dated April 16, 1926, and has the following payments indorsed: July 2, 1926, \$500; Oct. 1, 1926, \$250; Feb. 1, 1927, \$100; May 1, 1927, \$375. How much is due Sept. 5, 1927?

5. A note for \$750 at 4% is dated June 15, 1926, and has the following payments indorsed: March 6, 1927, \$200; Sept. 6, 1927, \$150; Oct. 8, 1927, \$50; Jan. 17, 1928, \$250. How much is due May 14, 1928?

6. A note for \$600 at 5% is dated July 2, 1926, and has the following payments indorsed: Jan. 2, 1927, \$50; Feb. 11, 1927, \$75; July 8, 1927, \$200; Sept. 10, 1927, \$80; Nov. 2, 1927, \$50. How much is due Jan. 2, 1928?

Teachers should remember that the subject of partial payments has lost most of its practical value in the last few years. Money is now usually borrowed at banks for short periods, and if a payment is made on a loan on which the interest was paid in advance, the bank simply rebates whatever portion of the interest is due the borrower. The subject is not obsolete, however, and it affords such excellent drill in computing interest that we have included it in this book. In communities where the so-called "Merchants' Rule" is used the teacher may find it desirable to discuss this rule.

IV. LITTLE EXAMINATIONS

i. 1. \$175 is what per cent more than \$100?
2. \$100 is what per cent less than \$150?
3. How much is 38% of \$350? of \$3500? of \$35,000?
4. How much is 175% of \$3500? $\frac{3}{4}$% of \$8000?

ii. 1. Express 0.2% as a decimal fraction.
2. Express $3\frac{1}{2}$ qt. as per cent of a gallon.
3. Express an inch as per cent of a yard.
4. Express $62\frac{1}{2}$%, $87\frac{1}{2}$%, and $83\frac{1}{3}$% as common fractions.

iii. *Find the values of the following:*

1. 50% of 975.
2. 75% of $87\frac{1}{2}$.
3. $37\frac{1}{2}$% of \$9.36.
4. $33\frac{1}{3}$% of \$49.86.
5. $87\frac{1}{2}$% of \$99.28.
6. 0.7% of 264 ft.
7. 0.07% of 98,000.
8. 125% of 3756.
9. 1.25% of 3756.
10. 400% of $62\frac{3}{4}$.

iv. 1. \$60 is what per cent of \$720? of \$7200? of \$72,000?
2. 9 in. is what per cent of 10 in.? of 10 ft.? of 10 yd.?
3. A pint is what per cent of 1 qt.? of 1 gal.? of 6 gal.?
4. 96 ft. is 8% of what distance? 4% of what distance?
5. 9 ft. 8 in. is 10% of what distance? 8% of what distance?

v. 1. \$12.35 is 5% less than what amount of money?
2. \$204 is 15% less than what amount of money?
3. Find the interest on \$380 for 8 mo. 8 da. at 5%.
4. List price, \$570; discount, 9%; find the net price.
5. A bill of goods amounting to \$825 is allowed discounts of 12%, 8%. Find the net amount.

V. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Write in words the numbers 400.075 and 0.475.

Add, without writing in columns:

2. $8.20 + $5.50 + $7.75 + $6.73 + $4.30 + $25.50.
3. 7 ft. 3 in. + 8 ft. 6 in. + 9 ft. 8 in. + 2 ft. 3 in. + 3 ft. 5 in.

Subtract the following:

4. $790.60 − $488.97.
5. $701.21 − $224.88.
6. $610.00 − $473.29.
7. $912.20 − $337.85.

Multiply or divide as indicated, carrying the quotients to four figures only:

8. 287 × $996.50.
9. 327 × $537.78.
10. 856.6 ÷ 29.4.
11. 119.36 ÷ 3.74.
12. 3.73 × 92.8.
13. 95.9 ÷ 2.13.

Find the values of the following:

14. 27% of $625.
15. $3\frac{1}{4}$% of $550.
16. $4\frac{1}{2}$% of $985.

Find the interest on the following:

17. $272.60, for 2 yr. 2 mo. 10 da., at 4%; at 5%; at $4\frac{1}{2}$%.
18. $591.75, for 2 yr. 8 mo. 10 da., at 5%; at $4\frac{1}{4}$%; at $5\frac{1}{2}$%.
19. $335.50, for 3 yr. 1 mo. 15 da., at 6%; at $4\frac{1}{2}$%; at 5%.
20. $562.75, for 1 yr. 3 mo. 20 da., at 5%; at $4\frac{1}{2}$%; at $4\frac{1}{4}$%.
21. $876.75, for 1 yr. 4 mo. 15 da., at 5%; at $3\frac{1}{2}$%; at $3\frac{1}{4}$%.
22. $928.25, for 2 yr. 5 mo. 10 da., at 6%; at $4\frac{1}{2}$%; at 4%.
23. $827.25, for 1 yr. 1 mo. 16 da., at $4\frac{1}{2}$%; at 5%; at $5\frac{1}{2}$%.
24. $625.40, for 1 yr. 2 mo. 10 da., at 5%; at 6%; at $6\frac{1}{2}$%.
25. $850.70, for 1 yr. 7 mo. 6 da., at $4\frac{1}{2}$%; at 5%; at 7%.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you buy a baseball bat and sell it at a certain rate below cost, how do you find the selling price?

2. If you buy a dog and sell him at a certain rate above cost, and know how much it has cost you to keep him while you owned him, how do you find whether you gained or lost, and the amount?

3. If you know the number of times a baseball player has been at bat during a season and the number of hits he has made, how do you find the percentage of hits?

4. If you know the number of times a baseball player has been at bat during a season and the percentage of runs he has made, how do you find the number of runs?

5. If you know the number of pupils in a school and the number in Grade VII, how do you find the per cent that are in Grade VII?

6. If you know a man's income and the amount which he pays for house rent, how do you find the per cent of his income that he pays for house rent?

7. If you know the list price of an article and the several rates of discount, what are you then able to find, and how do you find it?

8. A man borrowed from an acquaintance a certain sum of money at a certain rate and for a certain length of time. The money was not paid when due, but some payments were made from time to time, each payment being greater than the interest then due. Finally the money was paid. How do you find the amount due at the time of payment?

PROBLEMS FOR COMPLETION

1. A farmer sells 275 lb. of potatoes at $3.20 a hundredweight. Complete the problem and solve it.

Such a problem can be completed in more than one way. The pupil should be allowed to take any way that is reasonable and to solve accordingly.

2. J. H. Knight made the distance of 215 mi. from Cleveland to Bellefonte, Pa., in an airplane in 83 min. Complete the problem and solve it.

Complete and solve the following problems:

3. In an automobile race one car made $\frac{5}{8}$ mi. in $25\frac{1}{2}$ sec.

4. A farmer allows 55 lb. to a bushel of sweet potatoes. He sells 9625 lb. of sweet potatoes to a city dealer.

5. Mr. Monroe's wages were $32 a week, but have now been raised a certain per cent to $34.56.

6. Robert is multiplying 15 by some number and gets 50 for the result.

7. An automobile driver notices that his speedometer has registered 2.4 mi. in 5 min. He is going to a place that is 18 mi. farther on and expects to keep up the same rate.

8. By measuring the distance walked in taking 100 steps, Fred finds that, on the average, he covers 26 in. at a step. He lives $\frac{1}{2}$ mi. from school.

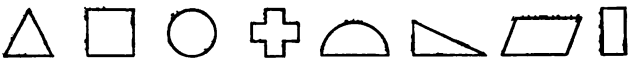
9. A man borrows $170 for 90 da. and pays 5% interest.

10. The Declaration of Independence was signed on a date that we all remember. The armistice in the World War was signed Nov. 11, 1918.

11. A farmer sells 6800 lb. of milk to a creamery in a certain week. The milk contains 4.1% by weight of butter fat.

ALERTNESS TEST

Copy the following and, at a given signal by the teacher, do as you are directed:

1. 

2.

3.

The teacher should ascertain when all the pupils are ready and should have them all begin at the same time. The teacher should prepare three statements, reading the first one and allowing 30 sec.; then the second, allowing 30 sec.; then the third, allowing 30 sec. The statements should not be precisely the same as the following, but should be of this nature:

1. In Ex. 1 write 3 below each figure which has a curve line, write 7 above each figure which contains a right angle, and write 8 inside each figure which contains an acute angle.

2. In Ex. 2 draw various lines above the squares in such a way that these lines shall connect every pair of squares which contain two numbers whose sum is 9, and then draw various lines below the squares in such a way that these lines shall connect every pair of squares which contain two numbers whose sum is less than 7.

3. In Ex. 3 draw a line that will pass above every circle whose number is divisible by 2 and below every circle whose number is not divisible by 2. Put the letter X below every circle whose number is divisible by 3.

The teacher should then score for each pupil (1) the rights and (2) the wrongs, keeping the record for future use. The class can thus be graded with considerable accuracy as to alertness.

These tests are designed to show the school the general nature of psychological tests that are now so extensively used. *Arithmetic is one of the best subjects in the course to make children really alert,* that is, to enable them to acquire the habit of alertness, and this is one of the important reasons for teaching arithmetic in our schools.

Industrial Problems

THE COST OF WASTEFULNESS

1. If a farm wagon that cost $90 is left out in the yard instead of being kept in the shed, it will last about 6 yr., but if kept under cover, it will last about twice as long. How much does a farmer pay for his carelessness per year if he leaves the wagon out?

2. Each of two neighbors bought for his farm work a two-horse cultivator, and each paid $45. One of them took good care of his cultivator and made it last 15 yr. The other left his cultivator out during all kinds of weather and had to buy a new one at the end of every 5 yr. How much did the careless farmer lose in the 15 yr. by neglecting to care for his machine properly?

3. A farmer after threshing his wheat had 16 T. of straw left. A ton of this straw contains 12 lb. of nitrogen, worth 30¢ a pound, and 10 lb. of potash and 2 lb. of phosphoric acid, each worth 5¢ a pound. If the farmer wastes the straw instead of using it on the soil as fertilizer, how much money does he waste?

GARDENING PROJECT

1. Richard's father has rented him a vegetable garden. Richard is to have the profits for a camping trip. On April 16 he plants some early potatoes and counts on his potatoes being ready to dig in 79 da. When must he be home to begin to dig them?

2. If Richard allows 18 da. after the date found in Ex. 1 in which to let all the potatoes mature and to sell them, what is the earliest day on which he can go camping?

3. On April 24 Richard sets out some early cauliflower, and he has to allow 72 da. for this to be ready to sell. When must he be on hand to sell the cauliflower?

4. Richard raised $4\frac{1}{2}$ bu. of early potatoes and sold them at \$1.40 a bushel. How much did he receive for the potatoes?

5. Richard sold 25 heads of early cauliflower at 18¢ a head. How much did he receive for the cauliflower?

6. Richard pays \$1.50 for rent of his garden and for his materials. Make out a statement showing his income and expenses and the amount that he has left for his camping trip.

CHAPTER II

I. PRACTICAL MEASUREMENTS

FINDING HOW MUCH YOU REMEMBER

All work oral

1. Recite the common table of length.

2. How many inches are there in 1 ft.? in 1 yd.? in $\frac{1}{2}$ yd.? in $\frac{1}{4}$ yd.? in $\frac{3}{4}$ yd.?

3. How many feet are there in 1 yd.? in 3 yd.? in $\frac{1}{2}$ yd.? in $2\frac{1}{2}$ yd.? in $5\frac{1}{2}$ yd.?

4. How many rods are there in 1 mi.? in $\frac{1}{2}$ mi.?

5. How many feet are there in 1 mi.? in 0.1 mi.?

6. Recite the common table of square measure, or area.

7. How many square rods are there in $\frac{1}{2}$ A.?

8. A farmer has a field 16 rd. long and 10 rd. wide. How many acres are there in the field?

9. Recite the common table of cubic measure, or volume.

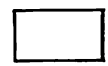
10. Recite the common table of capacity.

11. How many quarts are there in 6 gal.? in 10 gal.?

12. Do you remember any other special measures of length besides those given in the common table? If so, what are they? Possibly you remember about measuring the heights of horses and about measuring depths at sea.

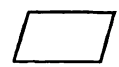
Review of Areas. You have already learned how to find the areas of several kinds of figures, so it is necessary at this time only to repeat certain statements which you have studied.

The area of a rectangle is equal to the product of the base and the height.



For example, if the base is 3 ft. and the height 2 ft., we have 2×3 sq. ft. $= 6$ sq. ft.

The area of a parallelogram is equal to the product of the base and the height.



For example, if the base is 2.4 in. and the height 2 in., we have 2×2.4 sq. in. $= 4.8$ sq. in.

The area of a triangle is equal to half the product of the base and the height.

For example, if the base is $3\frac{1}{2}$ in. and the height $2\frac{1}{2}$ in., we have $\frac{1}{2}$ of $2\frac{1}{2} \times 3\frac{1}{2}$ sq. in. $= 4\frac{3}{8}$ sq. in.

The actual work of the pupils should be as follows:

$$\frac{5 \times 7}{2 \times 2 \times 2} = \frac{35}{8} = 4\tfrac{3}{8}.$$

The area of a trapezoid is equal to half the product of the height and the sum of the parallel sides.

This brief form of stating the rule will now be understood. The longer form, used by beginners, says that *the area of a trapezoid is equal to half the area of a rectangle having the same altitude and having its base equal to the sum of the two parallel sides.*

Volumes. You have learned that *the volume of a rectangular solid is equal to the product of the three dimensions.*

For example, if a box is 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep, we have $3 \times 2 \times 1\frac{1}{2}$ cu. ft. $= 9$ cu. ft.

FINDING AREAS

1. The platform for a teacher's desk is 4 ft. 6 in. long and 3 ft. wide. Find the area in square feet; in square yards.

2. Measure the length and the width of the top of your desk to the nearest half inch. Find the area in square inches.

3. How much will it cost, at \$2.10 a yard, to cover the floor of a room 20 ft. 3 in. wide and 28 ft. 6 in. long with 27-inch carpet, allowing 2 yd. for matching the pattern?

Similar practical problems should be secured by the pupils at home.

4. How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 72 yd. long, each sheet being $2\frac{1}{2}$ yd. long with a top hem $3\frac{1}{4}$ in. and a bottom hem $2\frac{1}{4}$ in., an extra allowance of $\frac{1}{4}$ in. being made for each hem?

Pupils should be told that in buying sheeting the width of the material is often given in quarters of a yard. Sheeting $\frac{6}{4}$ wide means 6 quarters of a yard wide. Sheeting $\frac{6}{4}$ wide is used on single beds, $\frac{8}{4}$ wide on three-quarter beds, and $\frac{10}{4}$ wide on double beds. This need not be memorized.

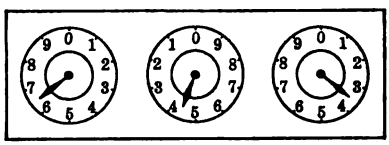
5. A piece of sheeting of $\frac{8}{4}$ width contains 64 yd. How many sheets for three-quarter beds can be made from it if you allow the same measurements as in Ex. 4?

6. A piece of sheeting of $\frac{10}{4}$ width contains 72 yd. How many sheets for double beds can be made from it if you allow the same measurements as in Ex. 4?

The class should study the simple measurements needed in the household, including the measurement of materials used in food and clothing. The cost of furnishing a kitchen, laundry, dining room, living room, and bedroom, with the expense of renovating them, should also be considered. Such work furnishes appropriate projects for a class to consider for a reasonable time. Since, however, they must be based upon local prices and customs, they are best taken without the aid of a textbook.

READING THE GAS METER

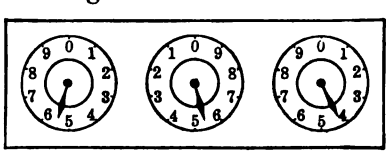
1. Here is a picture of the three dials on a gas meter. The left-hand dial indicates ten thousands, the middle dial indicates thousands, and the right-hand dial indicates hundreds. The dials



show that 64,300 cu. ft. of gas has passed through the meter. If this is the reading for May 1, and the reading for April 1 was 62,300, how much was the gas bill for April at 90¢ per M (1000 cu. ft.)?

In the above case more than 64,300 cu. ft. has passed through the meter, but we read only to the hundred last passed. Notice that the middle dial is read in the opposite direction from the others.

2. Read this gas meter. If the reading was 52,700 a month ago, how much is the gas bill for the month at $1.10 per M?



3. In Ex. 2 suppose that the reading had been 51,500 a month ago. How much would the gas bill have been for the month?

4. If a family used 2800 cu. ft. of gas in January and 2400 cu. ft. in February, how much did the gas cost for the two months at $1.35 per M?

5. If the gas used by a family in January was 2500 cu. ft. and that used in June was 10% less than in January, how much was the gas bill for each of these months at $1.20 per M?

6. A certain gas company deducts 10% from bills paid before the tenth of the following month. If the readings are 83,400 and 85,200 on the first days of two consecutive months, and the rate for gas is $1.25 per M, how much will be saved by paying the bill before the tenth of the month?

7. A company used natural gas in a part of the country where this could be obtained. The meter indicated 72,400 on March 1 and 88,900 on April 1. How much was the gas bill for March at 32¢ per M?

8. A company which uses natural gas is charged 35¢ per M and is allowed a discount of 3% if the bill is paid within 3 da. If the meter indicated 27,700 on Jan. 1 and 47,100 on Feb. 1, and if the bill was paid on Feb. 3, how much did the company pay?

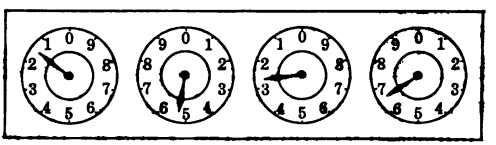
Copy the following and fill in the blank spaces:

	Present Reading of Meter	Previous Reading of Meter	Cubic Feet Consumed	Rate per M before Discount	Amount of Bill Rendered	Cash Discount 5%	Net Amount Due
9.	32,400	31,600		$1.10			
10.	26,700	23,900		.90			
11.	48,700	42,600		1.20			
12.	38,500	33,900		1.00			
13.	67,100	63,800		1.40			
14.	72,300	68,400		.85			
15.	52,200	48,600		1.35			
16.	74,300	69,900		1.05			
17.	65,300	61,600		.95			
18.	90,500	82,700		1.15			

If there is a gas meter in the school or home the pupils should actually read it and compute the amount of a bill for the gas used in 1 mo.

READING THE ELECTRIC METER

1. Mr. Stevens lights his store by electricity. The electricity is measured in kilowatt hours (K.W.H.), and the meter shows thousands, hundreds, tens, and units. He reads the meter in a way similar to that in which he reads the gas meter. Notice that the second and fourth dials are read clockwise and that the first and third are read counterclockwise. Read the meter shown.



The technical meaning of the K.W.H. in the science of electricity need not be considered in elementary problems of this kind.

2. If Mr. Stevens in June uses 50 K.W.H. at 15¢ and 8 K.W.H. at 8¢, how much is his bill for the month?

Companies charge large users different prices according to the number of lights or machines, and usually make lower rates where the electricity is used for power instead of lights. For example, in most places 10 lights would cost more than $\frac{1}{10}$ as much as 100 lights.

3. If Mr. Stevens in the month of May uses in his house 22 K.W.H. at 11¢ and is allowed a reduction of 4% on the bill if paid before the 10th of June, how much is his bill if he takes advantage of the reduction?

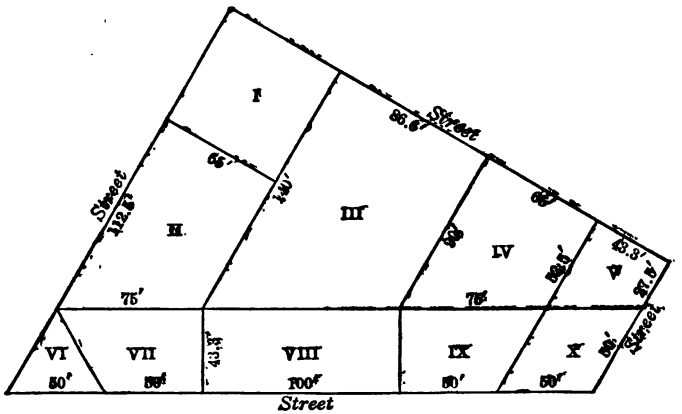
4. Find the cost of 16 K.W.H. at 12¢ and 4 K.W.H. at 8¢, less $\frac{1}{2}$¢ per K.W.H. on account of prompt payment.

5. Find the cost of 32 K.W.H. at $14\frac{1}{2}$¢ and 9 K.W.H. at $11\frac{1}{2}$¢, less $\frac{1}{2}$¢ per K.W.H. on account of prompt payment.

The student should actually read the electric meter in the school or at his home if this can be done conveniently.

MEASURING LAND

1. An irregular city block is divided into lots with measurements correct to one decimal place, as shown in this plan. Find the number of square feet in lot number I, which is a square.



2. Lot number II is a trapezoid. First find the length of the side which is not given in the plan and then find the area.

3. Lot number III is a trapezoid. Find its area.

4. Lot number VI is a triangle. The height is given in lot number VII. Find the area of lot number VI.

Find the area of each of the following lots:

5. V. 6. VII. 7. VIII. 8. IX. 9. X.

10. Find the number of acres in a rectangular field $26\frac{1}{2}$ rd. long and $18\frac{3}{4}$ rd. wide.

11. A field in the form of a right triangle has the sides forming the right angle 30 rd. and 40 rd. Find the area in square rods and also in acres.

MEASURING VOLUMES

1. Find the number of cubic inches in a box 3 ft. 6 in. long, 18 in. wide, and 1 ft. deep.

2. The bottom of an ordinary farm-wagon box is 3 ft. by 10 ft., and the box is from 24 in. to 26 in. deep. Find the contents in cubic feet and in cubic inches for each of these depths.

3. A wagon box 3 ft. by 10 ft. by 24 in. is estimated to contain 50 bu. of shelled corn. How many bushels of shelled corn will such a wagon box 26 in. deep contain?

4. Certain farmers estimate that $2\frac{2}{5}$ cu. ft. of corn in the ear will produce 1 bu. of shelled corn. How many bushels of shelled corn would there be in a crib 20 ft. long, 12 ft. wide, and 8 ft. high, filled with corn in the ear?

5. How many cubic yards of earth must be removed in excavating a rectangular tunnel that is 492 ft. long, 39 ft. wide, and 19 ft. 6 in. high?

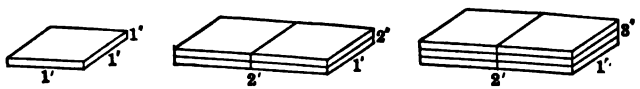
6. A cellar is being excavated near a certain school. The class measured the excavation and found it to be 34 ft. 6 in. long, 24 ft. 6 in. wide, and 7 ft. deep. How many cubic yards of earth were taken out?

7. The ice chamber in a refrigerator is 16 in. by 24 in. by 18 in. Ice weighs 8% less than water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If the ice comes in pieces weighing 10, 25, 50, 100, 150, and 200 lb., which one or which two of these pieces is nearest to the amount of ice required to fill the chamber?

8. In preparing a flower bed in a park it was necessary to fill in a space 64′ by 150′ to an average depth of 1′ 6″. How much earth was used? At $37\frac{1}{2}$¢ a load, how much did it cost?

Remember that 1 cu. yd. is taken as a load.

Measuring Lumber. A board foot (bd. ft.) of lumber means a piece that is 1 sq. ft. on one surface and 1 in. or less in thickness. A board foot is commonly called simply a foot.



1 board foot 4 board feet 6 board feet

A board 2′ long, 1′ wide, and 1″ or less thick contains 2 bd. ft.

A plank 12′ long, 1′ wide, and 3″ thick contains 36 bd. ft.

A board 18′ long, 8″ wide, and 1″ or less thick contains $18 \times \frac{8}{12}$ bd. ft., or 12 bd. ft.

A fraction of a board foot is always counted as 1 bd. ft.

For example, find the number of feet, board measure, in a joist $12' \times 6'' \times 2\frac{1}{2}''$.

We arrange the work as shown at the right, cancel, and find that the joist contains 15 bd. ft.

$$12 \times \frac{1}{2} \times 2\tfrac{1}{2} = \frac{\overset{3}{\cancel{12}} \times 5}{\cancel{2} \times \cancel{2}} = 15.$$

Always use cancellation whenever it shortens the work.

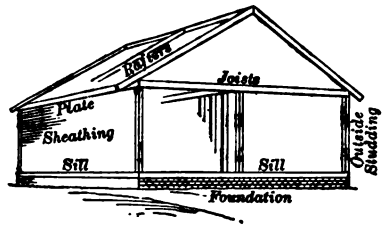
MEASURING LUMBER

Find the number of feet, board measure, in the following:

1. 32 planks 3 in. thick, 16 ft. long, and 8 in. wide.
2. 4 sticks of timber 16 ft. long, 1 ft. wide, and 8 in. thick.
3. 36 joists 3 in. by 4 in. and 16 ft. long.
4. 12 beams 10 in. by 12 in. and 14 ft. long.
5. 60 boards $\frac{7}{8}$ in. thick, 8 in. wide, and 16 ft. long.
6. What is the cost of 30 spruce joists 2 in. by 6 in. and 14 ft. long at $86 per M, that is, per 1000 bd. ft.?

House Building. The ground floor of the house shown in the picture below is to be $27\frac{1}{2}' \times 34'$, outside measure. A builder estimates the cost of materials and labor as stated in the problems. Part of the computation of cost and amount of materials is given in the problems; the rest is left for the class. The specifications for this house are taken from a government report.

From the figure at the right the pupils should be able to tell easily the meaning of the terms used in this project.



BUILDING A HOUSE

1. For sills the builder needs the following pieces. Fill in the parentheses below and find the number of feet, board measure, required for the sills.

3 pieces, $6'' \times 8''$, $16'$ long, 192 bd. ft.
4 pieces, $6'' \times 8''$, $14'$ long, (?) bd. ft.
6 pieces, $6'' \times 8''$, $12'$ long, (?) bd. ft.
1 piece, $2'' \times 8''$, $14'$ long, 19 bd. ft.
1 piece, $2'' \times 10''$, $8'$ long, (?) bd. ft.

2. The builder needs the following joists. Find the number of feet, board measure, required for joists.

30 pieces, $2'' \times 10''$, $16'$ long
18 pieces, $2'' \times 8''$, $12'$ long
13 pieces, $2'' \times 8''$, $6'$ long
27 pieces, $2'' \times 8''$, $16'$ long
27 pieces, $2'' \times 8''$, $12'$ long
27 pieces, $2'' \times 6''$, $16'$ long

3. The builder also needs the following studding. Find the number of feet, board measure, required for the studding.

5 pieces, $4'' \times 6''$, $14'$ long	45 pieces, $2'' \times 6''$, $12'$ long
100 pieces, $2'' \times 4''$, $14'$ long	50 pieces, $2'' \times 4''$, $8'$ long
35 pieces, $2'' \times 4''$, $10'$ long	50 pieces, $2'' \times 4''$, $14'$ long
25 pieces, $2'' \times 4''$, $18'$ long	40 pieces, $2'' \times 6''$, $18'$ long

4. For the roof the builder needs 1050 bd. ft. Show that this, with the amounts in Exs. 1–3, makes a total of 8021 bd. ft.

5. How much will the lumber mentioned in Exs. 1–4 cost at the rate of \$95 per M?

6. The builder also needs 2000 bd. ft. of sheathing at \$90 per M; 1950 bd. ft. of flooring at \$85; 125 bd. ft. of flooring for the porch at \$95; 1500 shingles for the gables at \$10.25 per M; 6500 shingles for the roof at \$12. What will this material cost?

7. He also needs water tables, cornice, corner boards, etc., \$75; porch columns, \$14; 16 doors at an average cost of \$9.75 each; 16 windows at an average cost of \$9.50 each; hardware, \$105; chimney and plastering, \$425; painting, \$280; tin work, \$65.50; the work of four carpenters for 30 da. at \$6 a day each; excavating and foundation, \$325; grading, \$75. What is the total cost of these items?

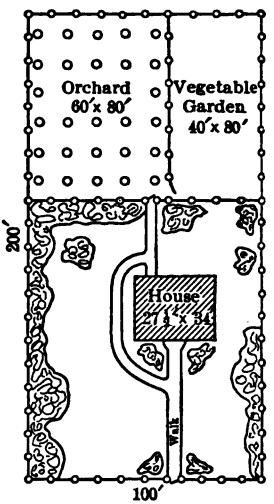
8. He must pay \$475 for the plumbing and \$625 for the heating plant. Taking the costs of the material and labor as found in Exs. 5–8, what is the total cost of the house?

9. How much will the flooring for a house cost, taking the measurement of each floor as $28' \times 32'$ and allowing for two floors, the lumber costing \$80 per M?

One good way of treating the general topic of practical measurements is to work out problems in connection with building and equipping a house that is being erected in the neighborhood.

USING WHAT YOU HAVE LEARNED

1. The house described on page 94 was built on a lot 100 ft. wide and 200 ft. deep. The owner took a piece 40 ft. by 80 ft. for a vegetable garden, and one 60 ft. by 80 ft. for a fruit orchard. How many square feet of land are there in each of these two pieces?



2. Express the area of the whole lot and also of each of the three parts as the decimal of an acre.

3. What is the area of the land occupied by the house?

4. The excavation for the cellar was 36 ft. by 29½ ft. by 5 ft. How much earth was removed?

5. If the excavation in Ex. 4 had been 6 ft. 4 in. deep, how much earth would have been removed?

6. If the owner fences the whole lot, and then fences off the orchard and also the garden as shown in the plan, how many feet of fencing will he need, not allowing for gates? How many rods of fencing?

7. The owner planted a bed of asparagus 8 ft. wide and 36 ft. long; one bed of parsnips and two beds of beets, each 9½ ft. wide and 36 ft. long; three beds of tomatoes and two beds of cauliflower, each 7½ ft. wide and 36 ft. long. How many square feet of land did he plant to each of the vegetables?

8. The shrubs about the lawn, together with the walks, occupy 2200 sq. ft. How many square feet are left for the lawn?

BUILDING A BARN

1. A farmer employs a carpenter to build a barn. The carpenter figures on the following as part of the materials: 4 pieces $2'' \times 10'' \times 18'$; 8 pieces $2'' \times 10'' \times 10'$; 8 pieces $2'' \times 8'' \times 24'$; 36 pieces $2'' \times 10'' \times 14'$; 60 pieces $2'' \times 8'' \times 22'$; 12 pieces $2'' \times 8'' \times 20'$; 10 pieces $2'' \times 8'' \times 18'$; 30 pieces $2'' \times 8'' \times 16'$; 220 pieces $2'' \times 8'' \times 12'$; 20 pieces $2'' \times 8'' \times 10'$; 40 pieces $2'' \times 8'' \times 8'$; 22 pieces $2'' \times 6'' \times 24'$; 8 pieces $2'' \times 6'' \times 12'$; 5000 sq. ft. of siding 1″ thick; 22,000 shingles. Find the number of feet of lumber (board measure), its cost at \$82 per M, and the cost of the shingles at \$9.10 per M.

2. The carpenter also figured on needing 100 lb. of 4d nails, 140 lb. of 8d nails, and 650 lb. of spikes of various sizes. At 9¢ per pound, what was the cost of the nails and spikes?

A 4d nail is called a fourpenny nail, an old name which has nothing to do with the cost. Specimens may profitably be brought to class.

3. The carpenter bought his windows ready-made, needing ten at \$5.60 each, and allowed \$50 for foundation work. He also bought the necessary hardware, such as door hinges, latches, hangers, tracks for the large doors, and window fasteners, the bill amounting to \$32.75. What was the cost of this material?

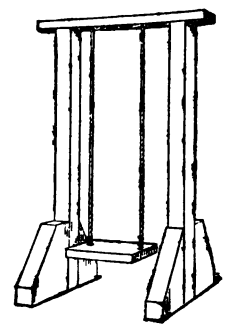
4. The carpenter figured his work at \$5.50 a day and the work of another man at \$4.25 a day, both for a period of 37 da. How much was allowed for this labor?

5. The farmer decided to put in a concrete floor. The floor area was 1860 sq. ft. and the floor was 4 in. thick. How many cubic yards of concrete did he require? If 1 cu. yd. of the concrete required 1.3 bbl. of cement at \$4.25 per barrel, how much would the cement for the floor cost?

In such cases count as a whole barrel any fraction of a barrel in the total.

DRAWING PLANS

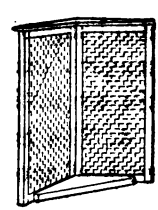
1. Jack made a swing for his sister's doll house. The crosspiece at the top is 7″ long, 1″ wide, and $\frac{1}{2}$″ thick. Make a drawing of the top of this crosspiece, first to full size and then to the scale $\frac{1}{2}$.



It may be necessary to remind the class again that drawing an object to the scale $\frac{1}{2}$ means that every inch of the object is to be represented in the drawing by $\frac{1}{2}$″. All drawings should be made with a ruler and a sharp pencil and should be made to look neat and clean.

2. The seat in the swing is 3″ long, 2″ wide, and $\frac{1}{2}$″ thick. Make three full-size drawings, one showing the seat as seen from the top, one showing it as seen from the front, and one showing it as seen from the end.

3. Our class made a lamp screen like the one shown in the picture below. The screen is $18\frac{3}{4}$″ high and each wing is $9\frac{3}{4}$″ wide. Draw a plan of either wing to the scale $\frac{1}{2}$.



4. What is the total area of the side of the two wings of the screen shown in the figure?

5. The woodwork is filled in with plaited ribbon, the spaces being $7\frac{3}{4}$″ by $16\frac{1}{4}$″. Find the sum of the two areas thus filled in.

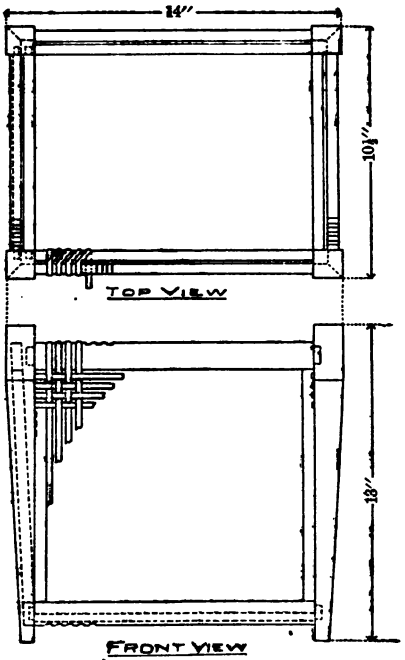
6. A room is 32′ long and 24′ wide. Draw a plan of the floor to the scale of 1″ to 8′.

Using the scale of 1″ to 10′, draw plans of floors having the following dimensions:

7. 14′ by 16′. 8. 12′ 6″ by 16′ 6″. 9. 10′ 8″ by 18′ 6″.

HANDWORK

1. The boys in our class made a wastebasket, these being the working drawings for the top and the front. Measure the drawings and determine the scale used.

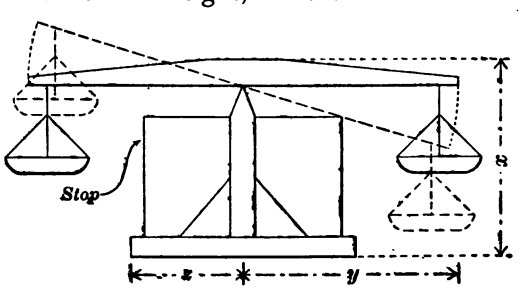


2. Taking the length and the width as given in the working drawings, find the perimeter of the top of the wastebasket, expressing it in feet and inches.

3. The height is greater than the width by what fraction of the width?

4. Measure the area to be filled in by wickerwork in the front of the basket. This is easily done, because the scale was determined in Ex. 1, so you have only to measure the figure carefully, find the width and height, and then find the area.

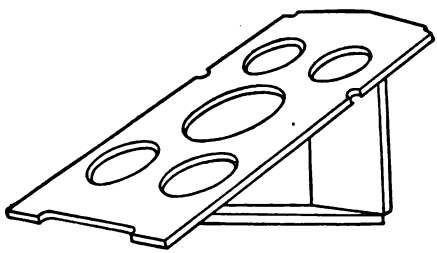
5. The boys also made some wooden balances. These are the drawings, and they are made to the scale $\frac{1}{8}$. Determine the lengths marked x, y, and z.

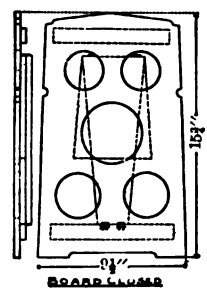


HANDWORK

1. The boys made some bookracks. For each shelf they used a piece of wood 1′ 5″ long. How long a piece of board would they need for 7 shelves?

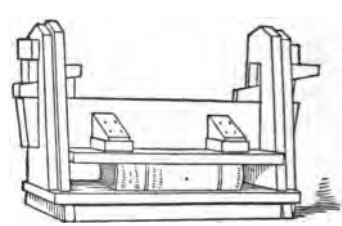
2. The class made this bean-bag board. How long a piece of lumber is needed for the fronts of two such boards?





3. In making this bookpress the class found that 5 pieces of board 14″ long were needed. How many feet long was the board needed to make these pieces?

4. One piece of board $18\frac{3}{8}$″ long and one piece $20\frac{3}{4}$″ long were also needed for making this bookpress. Express the sum of these lengths in feet and inches.

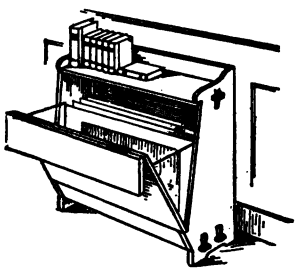


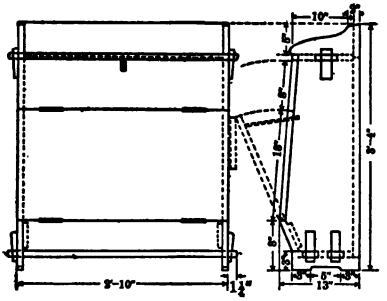
5. Each piece of board is $10\frac{3}{4}$″ wide. This width is what part of a foot? Express the result both as a common fraction and as a decimal.

6. Using 10-inch board $\frac{7}{8}$″ thick for the bookpress, what is the cost of 5 pieces 14″ long, one piece 18″ long, and one piece 20″ long at $125 per M for good oak? What would be the cost at $145 per M? at $105 per M?

HANDWORK

1. The sides of this cabinet are $\frac{7}{8}''$ thick. The plan shows the greatest length and the greatest width. How many feet, board measure, are needed for these two pieces?





2. There are two shelves in the cabinet, each 10″ wide and $\frac{7}{8}''$ thick. On account of the projections shown in the plan the shelves have to be more than 2′ 10″ long. Estimate from the plan the length of each shelf and the number of feet, board measure, needed for the two shelves.

3. For the front of the cabinet three pieces of lumber are used. The dimensions of these pieces are 8″ by 2′ 10″ by $\frac{7}{8}''$, 18″ by 2′ 10″ by $\frac{7}{8}''$, and 6″ by 2′ $8\frac{1}{4}''$ by $\frac{7}{8}''$. Find the number of board feet of lumber needed for the front.

4. For the back of the cabinet three pieces are used, each being 6″ by 2′ $8\frac{1}{4}''$ by $\frac{7}{8}''$. Find the number of board feet of lumber needed for the back.

5. For the back of the top one piece is used $5\frac{1}{2}''$ by 2′ $8\frac{1}{4}''$ by $\frac{7}{8}''$. Find the number of board feet needed for this piece.

In schools where there are workshops this work should be replaced by actual problems found by the pupils. In other schools as many of the problems should be solved as the teacher finds profitable to the class.

DRAWING PLANS

Using a ruler and a sharp pencil, make drawings of the following to any convenient scale:

1. The front of your schoolroom, showing the blackboard and any doors or windows.

This will require measuring the schoolroom. In general, it is better to have the pupils take their own measurements, but if necessary the teacher may give the measurements to the class.

2. The floor plan of your schoolroom, showing the position and size of the platform, if there is one.

3. The school grounds, showing the location and size of the schoolhouse.

The measurements necessary for this drawing may take a whole class period or longer, but the time is profitably spent in this way. The pupils should first make a rough drawing of the plan and should write upon it the necessary measures as they are taken, these measurements having been decided upon before the class goes outdoors.

4. Make a drawing, not to scale, of the front elevation of your schoolhouse, showing the position of the doors and windows.

The teacher should explain that the front elevation is the picture of the front of the building, not in perspective and without shading; that the side elevation is a similar picture of the side of the building; and that the plan means the ground plan. Such terms need not be strictly defined, but they should be used intelligently.

5. Draw a plan of a house that you would like to own and bring it to school tomorrow. Show both the first, or ground, floor, and the floor above it, that is, the second floor.

6. Draw a plan of a football field, showing the location of the goal posts, the two 25-yard lines, and the places for the kick-offs.

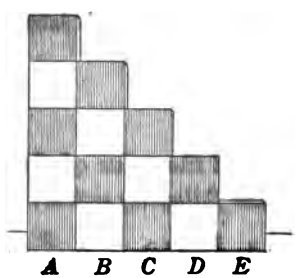
Ratio. The relation of one number to another of the same kind, as expressed by the division of the first by the second, is called the *ratio* of the first to the second.

The ratio of \$2 to \$4 is $\frac{2}{4}$, or $\frac{1}{2}$; the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$; the ratio of 10 mi. to 2 mi. is $\frac{10}{2}$, or 5.

MEANING OF RATIO

All work oral

1. In this picture what is the ratio of E to D?
2. In the same picture what is the ratio of E to C?



Referring to the picture, state the values of these ratios:

3. E to B.
4. D to A.
5. $2E$ to D.
6. $2D$ to B.
7. D to C.
8. C to A.
9. $4E$ to A.
10. $5E$ to A.
11. What is the ratio of any number to itself? to its equal?

Referring to the picture, state the values of these ratios:

12. A to $B + E$.
13. A to $B + C$.
14. B to $C + E$.
15. B to $A + E$.
16. E to A.
17. C to B.
18. $3E$ to C.
19. $2D$ to A.
20. B to $A + D + E$.
21. A to $B + C + D$.
22. $B + E$ to $C + D$.
23. $A + E$ to $B + D$.

Ratio and proportion are now rarely used in solving business problems, since we have better methods. But in measurements like that of the circle it is convenient to speak of ratio, such as that of the circumference to the diameter; hence a brief treatment of ratio and proportion is given.

Writing Ratios. The ratio of 2 ft. to 3 ft. may be written as a fraction, $\frac{2 \text{ ft.}}{3 \text{ ft.}}$, or with the sign of ratio (:), as 2 ft. : 3 ft.

We think of the ratio of 20 ft. to 4 ft. as $\frac{20 \text{ ft.}}{4 \text{ ft.}}$, or 20 ft. : 4 ft., or $\frac{20}{4}$, or 20 : 4, or simply 5. The word *ratio* is used for all.

Ratios always Abstract. Since any number divided by a like number, as feet by feet, has an abstract quotient, we see that

A ratio is always abstract, and the two numbers may be written as abstract numbers.

Teachers should use the fractional form at first because the pupils are already familiar with that form. They should also recall the fact that all number is essentially abstract. In performing work with ratios it is usually better not to label the numbers with their denominations.

STUDY OF RATIOS

All work oral

Find the value of the missing number, designated by x, in each of the following ratios:

1. $\frac{x}{2} = 10.$
2. $\frac{x}{3} = 3.$
3. $\frac{x}{4} = 2.$
4. $\frac{x}{5} = 3.$
5. $\frac{x}{10} = 1.$
6. $\frac{x}{10} = \frac{1}{2}.$
7. $\frac{x}{7} = \frac{3}{7}.$
8. $\frac{x}{6} = \frac{2}{3}.$
9. $\frac{x}{5} = \frac{4}{5}.$
10. $\frac{2}{x} = \frac{1}{3}.$
11. $\frac{20}{x} = 0.4.$
12. $\frac{10}{x} = 1.$

13. What is the number whose ratio to 5 is 6?

14. What is the number to which 75 has the ratio 3?

15. The ratio of the height of a certain door to the width is $3\frac{1}{2}$. If the door is 7 ft. high, how wide is it?

16. The ratio of the height of Fred's father to Fred's height is $\frac{3}{2}$. Fred's father is 6 ft. tall. How tall is Fred?

Separating a Number in a Given Ratio. Required to separate 25¢ in the ratio of 2 to 3.

Since there are to be 2 parts in one group and 3 parts in the other, there will be 5 parts in all.

Since one group has $\frac{2}{5}$ of the whole, it has $\frac{2}{5}$ of 25¢, or 10¢.

The other group has $\frac{3}{5}$ of 25¢, or 15¢.

Check. $10¢ + 15¢ = 25¢$; $10 : 15 = 2 : 3$.

SEPARATING IN A GIVEN RATIO

Numbers 1 to 6, oral

1. Separate 9 in the ratio of 2 to 1; of 1 to 1.

2. What is the ratio of 6 hr. to 24 hr.? of 6 hr. to 3 hr.?

3. In a class of 24 boys and 20 girls what is the ratio of the number of boys to the number of girls?

4. What is the ratio of the length of summer, 3 mo., to the whole year? to the remainder of the year?

5. There were six cloudy days in September, the rest being clear. What was the ratio of cloudy days to clear days?

6. In playing ringtoss Will has a score of 15 and Mollie has one of 18. What is the ratio of Will's score to Mollie's?

7. Divide 100 ft. in the ratio of 4 : 1; of 7 : 3; of 1 : 9.

8. In a year having 165 cloudy days and 200 clear days what is the ratio of cloudy days to clear days?

9. In November the ratio of clear days to cloudy days was 3 : 2. How many days were there of each kind?

10. Four boys arrange a three weeks' camping trip. The first boy stays 18 da.; the second comes late and stays 12 da.; the third is in camp the last two weeks; the fourth stays the three weeks. The total expense is $32.50. What is the share of each?

Proportion. An expression of equality between two ratios is called a *proportion.*

For example, the ratio \$2 : \$3 is equal to the ratio 10 ft. : 15 ft.

Therefore $\$2 : \$3 = 10 \text{ ft.} : 15 \text{ ft.}$

is a proportion. It is read, "\$2 is to \$3 as 10 ft. is to 15 ft." It may, of course, be written simply $2 : 3 = 10 : 15$, or $\frac{2}{3} = \frac{10}{15}$.

Extremes and Means. The first and last terms of a proportion are called the *extremes*; the second and third, the *means.*

In the proportion $\frac{3}{7} = \frac{15}{35}$

we see that $7 \times 15 = 3 \times 35.$

Similarly, we see that in any proportion

The product of the means is equal to the product of the extremes.

We also see that if $x : 21 = 35 : 105,$

or if $$\frac{x}{21} = \frac{35}{105},$$

then $$x = \frac{21 \times 35}{105}.$$

Similarly, *in any proportion the product of the means divided by either extreme is equal to the other extreme, and the product of the extremes divided by either mean is equal to the other mean.*

Illustrative Problems. 1. If $x : 7 = 13 : 21$, find the value of x.

As above, we see that $$x = \frac{7 \times 13}{21} = \frac{13}{3} = 4\tfrac{1}{3}.$$

2. If $31.2 : 14.4 = x : 3$, find the value of x.

Here we see that $$x = \frac{3 \times 31.2}{14.4} = \frac{312}{48} = 6.5.$$

USING WHAT YOU HAVE LEARNED

1. A miller uses 18 bu. of wheat for making 4 bbl. of flour. How many barrels of flour can he make from 207 bu. of wheat?

Show that $x : 4 = 207 : 18$.

2. If 4 men can do a piece of work in 10 da., how many men would it take to do the work in 8 da.?

In all such problems the words "at the same rate" are evidently to be understood.

It will take more men to do the work in fewer days. Therefore x must be greater than 4, and the proportion is $x : 4 = 10 : 8$.

3. A certain circle has a circumference of 366.66 in. What is the circumference of a circle the radius of which is to the radius of the first as $3.4 : 5.82$?

The longer the radius the longer the circumference in the same proportion. Therefore $x : 366.66 = 3.4 : 5.82$.

If the terms "circumference" and "radius" are not understood, explain them informally.

4. How long will it take a train to go 148 mi., at the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min.?

5. If 9 men can complete some work in 20 hr., how many men could complete it in 16 hr. more time?

6. If a man can skate 75 yd. in $8\frac{1}{3}$ sec., how long would it take him to skate 200 yd.?

7. If a man can skate 220 yd. in 19 sec., how long would it take him to skate 5 mi. if he could maintain this rate?

8. If 750 lb. can be transported 100 mi. for a certain sum, how far should $\frac{1}{2}$ T. be transported for the same sum?

9. A prize of \$90 was divided between two persons in the ratio 4 : 5. How much did each receive?

PROPORTION

Find the value of x in each of the following proportions:

1. $x : 51 = 12 : 17$.
2. $x : 17 = 54 : 51$.
3. $41 : 11 = x : 22$.
4. $x : 250 = 35 : 50$.
5. $27 : x = 243 : 36$.
6. $x : 1001 = 8 : 143$.
7. $3 : 20 = 24 : x$.
8. $x : 18.2 = 9.1 : 7.3$.
9. $82.7 : 82 = x : 4.1$.
10. $x : 21.7 = 2.4 : 3.3$.
11. $x : 1.37 = 2.4 : 13.7$.
12. $3.21 : x = 6.6 : 4.12$.
13. $7.21 : 1.8 = x : 0.32$.
14. $3.17 : x = 4.28 : 3.2$.

15. If you are thinking of a number which has to 17 the ratio 35 : 119, what is the number?

16. If a boat crew is rowing at the average rate of 1 mi. in 5 min. 30 sec., how long will it take to row 1 mi. 1540 yd.?

17. If a man can swim 45 yd. in 23 sec., how long would it take him to swim 2 mi. if he could maintain this rate?

18. A prize of \$60 was divided between two persons in the ratio of $\frac{2}{3}$ to $\frac{5}{8}$. How much was the share of each?

19. A map is drawn so that 1 in. represents 0.8 mi. on the surface of the earth. How many square miles of land are represented by a portion of the map 3 in. square?

20. When a sum of money is divided equally among seven persons, each receives \$16.80. How much would each receive if the same sum were divided equally among eight persons?

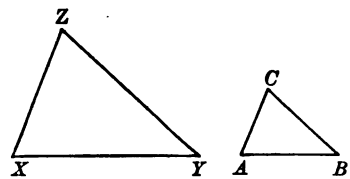
21. One man can do a piece of work in 10 da., and another can do the same work in 9 da. If the wages of the first are \$5.40 a day, what should be the wages of the second?

22. If a farmer needs 26 lb. of seed for $\frac{1}{2}$ A., how many pounds does he need for 19.9 A.?

Proportional Numbers. Numbers that are in proportion are called *proportional numbers* and are said to be *proportional.*

Similar Figures. Figures that have exactly the same shape are called *similar figures.*

Proportional Sides. *The corresponding sides of similar figures are proportional.*



For example, the sides XY, YZ, AB, BC, in the two triangles here shown, are proportional; that is, $XY : YZ = AB : BC$.

PROPORTIONAL SIDES

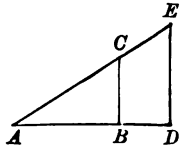
All work oral

1. The two triangles shown above are similar. If XY is twice AB, how does ZX compare with CA?

2. In the triangles above, if the ratio $ZX : XY$ is equal to $\frac{3}{4}$, what is the value of the ratio of $CA : AB$?

3. In the triangles above, if $ZX : XY = 3 : 4$, and if $XY = 1$ in. and $AB = \frac{1}{2}\, XY$, how long is CA?

4. In this figure what proportion exists among AB, AD, AC, and AE? If $AB = \frac{2}{3}\, AD$, what is the ratio of AC to AE?



5. In the same figure, if $AD = 1\frac{3}{16}$ in., $AC = 1\frac{0}{16}$ in., or $\frac{5}{8}$ in., and $AB = \frac{9}{16}$ in., what is the length of AE?

6. In the same figure, if DE represents the height of a man 6 ft. tall, BC the height of a boy, AD the shadow cast by the man, and AB the shadow cast by the boy, show how to find the height of the boy from measuring the shadows.

Measuring Heights and Distances. The height of a tree, of a high building, or of a church spire can conveniently be measured by proportion, as shown below.

The man holds a right triangle in which $AB = BC$. He holds AB level and walks to the spot from which he just sees the top of the tree along AC. Then because

$$AB = BC,$$

and

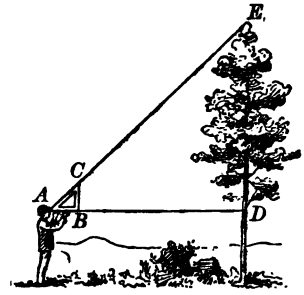
$$AB : BC = AD : DE,$$

the height above D will be equal to the distance AD.

MEASURING HEIGHTS AND DISTANCES

1. What is the height of the tree in the picture if the triangle is 5 ft. 4 in. above the ground and AD is 23 ft. 8 in.?

In all such problems remember to have the pupils estimate the result in advance. This is a valuable check on the accuracy of the work.



2. Suppose that a triangle is used in which AB is equal to twice BC, that AD is 75 ft., and that the point D is 5 ft. 8 in. above the ground; find the height of the tree.

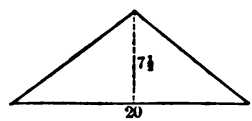
3. Suppose that a triangle is used in which $AB = 2$ ft. and $BC = 1$ ft. 6 in. If AD is 66 ft. and the point D is 5 ft. 6 in. above the ground, what is the height of the tree?

4. Using the triangle of Ex. 3, find the height of the tree if AD is 61 ft. and the point D is 5 ft. 3 in. above the ground.

5. Using the triangle of Ex. 3, find the height of the tree if AD is 70 ft. and the point D is 5 ft. 2 in. above the ground.

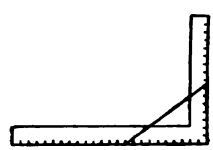
PROPORTION

1. A carpenter is constructing the roof here shown, the house being 20 ft. wide and the gable $7\frac{1}{2}$ ft. high, and wishes to know the length of the slope. He draws a plan, representing 20 ft. by 8 in. How many inches in the plan represent the "rise," $7\frac{1}{2}$ ft.? How can the carpenter, by measuring the plan, find the length of the slope of the roof?

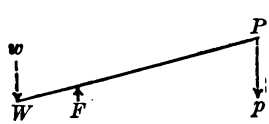


2. The carpenter finds that the plan of Ex. 1 is not convenient, so he draws a right triangle showing half the plan, and represents the "run," 10 ft., by a line 10 in. long. Draw such a plan and measure the slope. How long is the slope of the roof?

3. In practice the carpenter would take his square, and on the tongue, or short arm, he would take a point $7\frac{1}{2}$ in. from the corner; then on the blade, or long arm, he would take a point 10 in. from the corner. He would then measure the distance between these points, and the number of inches would be the same as the number of feet in the slope of the roof. Draw the figure and write out the reason involved.



4. In a lever PW, if sufficient power (p) is applied downwards at P, a weight (w) can be lifted upwards at W. If F is the fulcrum, it is known that the ratio $p : w$ is equal to the ratio $FW : FP$ of the distances. If w is 90 lb., FW $1\frac{1}{2}$ ft., and FP $4\frac{1}{2}$ ft., what is the value of p?



5. A boy weighing 100 lb. just balances a seesaw by sitting 4 ft. from the stick on which the plank rests. If the plank is 9 ft. long and we neglect the weight of the plank, how much does the boy on the other end weigh?

SHADOW RECKONING

1. The boys in the picture find that the post is 3 ft. 6 in. high and casts a shadow 3 ft. 2 in. long. The shadow of the tree is 34 ft. 10 in. long at the same time. Find the height of the tree.

An old name for this kind of application is "shadow reckoning."

2. If a tree casts a shadow 60 ft. 6 in. long when a post 3 ft. 3 in. high casts one 5 ft. 6 in. long, how high is the tree?

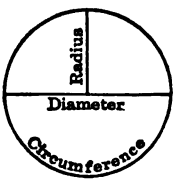
3. How high is a flagpole if it casts a shadow 100 ft. long when a gatepost 9 ft. high casts a shadow 11 ft. 3 in. long?

4. How high is a building if it casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long?

5. How high is a tree if it casts a shadow 30 ft. long when a post 6 ft. 3 in. high casts a shadow 3 ft. 9 in. long?

6. Joseph and Willard wished to find the height of the school building. Joseph stood a yardstick upright, measured its shadow, and measured the shadow of the school building. The shadow of the yardstick was 4 ft. long, and the shadow of the building was 36 ft. long. How high was the school building? Draw a picture to illustrate the problem.

Circumference, Diameter, and Radius. The distance around a circle is called the *circumference.* A straight line drawn through the center and terminated at each end by the line of the circumference is called a *diameter.* A straight line drawn from the center to the line of the circumference is called a *radius.*



Although the pupils have used the word "circle" throughout the grades, the teacher may find it necessary to explain it in some cases.

A circumference is like the length of a tire of a wheel; a radius is like a spoke of the wheel; a diameter is like two opposite spokes. The word "circumference" is used for the line which forms the circle, as well as for the length of the line. The radius is half the diameter.

Ratio of Circumference to Diameter. By measuring several circles and dividing each circumference by its diameter the circumference will be found to be about $3\frac{1}{7}$ times the diameter.

It is proved in geometry that this ratio of the circumference to the diameter is more nearly 3.1416. We shall use $3\frac{1}{7}$, or $\frac{22}{7}$, instead of 3.1416, unless the contrary is stated.

Illustrative Problems. 1. What is the circumference of a circle the diameter of which is 21 in.?

We have $\frac{22}{7} \times 21 \text{ in.} = 66 \text{ in.}$

2. Find the circumference of a circle, the radius being 7 in.

The diameter is 2×7 in. Therefore the circumference is

$$\frac{22}{7} \times 2 \times 7 \text{ in.} = 44 \text{ in.}$$

3. Find the diameter, the circumference being 2860 in.

Since the circumference is the product of the diameter by $\frac{22}{7}$, we divide the circumference by $\frac{22}{7}$ to find the diameter.

Then $$\frac{2860 \text{ in.}}{\frac{22}{7}} = \frac{7}{22} \text{ of } 2860 \text{ in.} = 910 \text{ in.}$$

CIRCUMFERENCES AND DIAMETERS

1. Rob measures the diameter of the drive wheel of a locomotive and finds it to be 5 ft. 3 in. What is the circumference?

2. What radius must be used in drawing a pattern for a wheel in order that the circumference shall be $91\frac{1}{7}$ in.?

3. If an iron pipe has a circumference of 4.2 in., what is the diameter of the pipe?

Find the circumferences, given the diameters as follows:

4. 68.2 in. 6. 5.11 ft. 8. 4.69 ft. 10. 13 ft. 6 in.
5. 423 in. 7. 6.37 in. 9. 6.02 in. 11. 64.4 in.

Find the diameters, given the circumferences as follows:

12. 132 ft. 13. 176 in. 14. 68.2 in. 15. 198 in.

Using 3.1416 instead of $\frac{22}{7}$, find the circumferences, given the diameters as follows:

16. 17 in. 17. 6.28 in. 18. 3.42 in. 19. 6 ft. 2 in.

It is not necessary to teach pupils in this grade that, given a diameter to hundredths of an inch, we cannot expect the result of the computation for the circumference to be accurate to more than a hundredth of an inch. Pupils should, however, be told that a result to the nearest hundredth is sufficient, this being the general rule followed thus far in this book. Thus, 53.41 in. is a better answer to Ex. 16 than 53.4072 in.

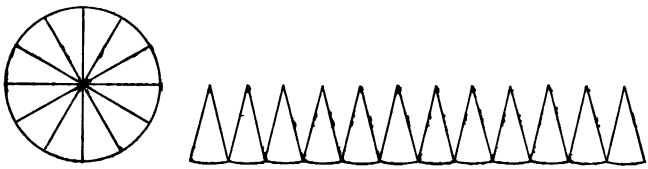
Using 3.1416, find the diameters, given the circumferences as follows:

20. 53.4072 in. 21. 97.3896 in. 22. 182.2128 in.

Using $\frac{22}{7}$, find the circumferences, given the radii as follows:

23. 49 in. 24. 91 in. 25. 9.8 in. 26. 15.4 ft.

Area of a Circle. A circle can be separated into figures which are nearly triangles. The height of each triangle is the radius, and the sum of the bases is the circumference. If these were



exact triangles the area would be $\frac{1}{2} \times$ radius $\times$ circumference. It is proved in geometry that this is the true area.

The above should be explained informally at the blackboard.

Given the Radius, to find the Area. Because the circumference is $\frac{22}{7} \times$ diameter, the area must be

$$\frac{1}{2} \times \text{radius} \times \frac{22}{7} \times \text{diameter},$$

or $$\frac{1}{2} \times \text{radius} \times \frac{22}{7} \times 2 \times \text{radius},$$

or $$\frac{22}{7} \times \text{square of radius.}$$

The area of a circle is $\frac{22}{7}$ times the square of the radius.

Explain to the class, if necessary, that the *square* of a number is the product obtained by multiplying the number by itself.

Pupils need not be required to give the above explanation. The teacher may take it up at the blackboard, but the essential thing is that the pupils learn this important statement and understand its meaning.

Illustrative Problems. 1. Required the area of a circle whose radius is 5 in.

Since the square of 5 is 25, we have

$$\text{area} = \frac{22}{7} \times 25 \text{ sq. in.} = \frac{550}{7} \text{ sq. in.} = 78\frac{4}{7} \text{ sq. in.}$$

2. Find the area of a circle when the diameter is 8 in.

Since the diameter is 8 in., the radius is 4 in. Then the square of 4 is 16, and $\frac{22}{7} \times 16$ sq. in. is the result.

MEASUREMENT OF CIRCLES

Find the area in each case, given the radius as follows:

1. 7 in. 2. 70 in. 3. 2.1 ft. 4. $3\frac{1}{2}$ in. 5. 1.4 in.

6. If you use a radius half as long as a given radius, the area of the circle is what part as great? Make and solve a problem to illustrate your answer.

7. If you double the length of the radius, the area of the circle is how many times as large? Make and solve a problem to illustrate your answer.

8. Find the area of the base of a tank 42 ft. in diameter.

9. A horse is tethered to a post by a rope. Over how many square feet of ground can the horse graze if he is tethered so that he can feed anywhere within 17 ft. 6 in. of the post?

10. A flagpole has a circumference of 24.2 in. at the base. Find the diameter, the radius, and the area of the cross section.

11. What is the area of the cross section of a circular iron beam the circumference of which is 31.416 in.?

Have the pupils proceed as in Ex. 10, but use 3.1416.

12. The external diameter of an iron water pipe is 4 in. If the iron is $\frac{1}{4}$ in. thick, what is the internal circumference?

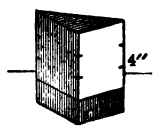
13. The external diameter of an iron water pipe is 3 ft. 6 in. If the iron is 1 in. thick, how many square inches of iron are there in the cross section of the pipe?

14. The globe in a schoolroom has a diameter of 30 in. How long is the equator on the globe?

15. If Texas has 4,660,000 inhabitants, and 25% of them live within 100 mi. of Dallas, find to the nearest unit the average population per square mile in this circle.

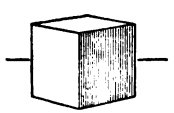
Prism. A solid having two parallel faces exactly alike and all the rest of the faces parallelograms is called a *prism.*

The solid shown in the picture is a prism, the parallel faces in this case being triangles. The *base* of a prism is one of the two parallel faces, and the *height*, or *altitude*, of the prism is the distance between these faces. If all the faces of a prism are rectangles, as in a crayon box, the prism is called a *rectangular prism.*



Volume of a Prism. Contractors, builders, and mechanics frequently have to find the volume of prisms of various kinds.

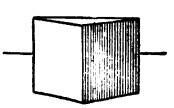
For example, a contractor computing the amount of earth to be removed in excavating a canal uses the rule for the volume of a prism. To find this rule first consider the following questions:



If the kind of prism here shown, a cube, is 1 in. on an edge, what is its volume?

If a cube is 2 in. on an edge, what is its volume? If a cube is 3 in. on an edge, what is its volume?

This figure represents half of the above cube. If the cube is 1 in. on an edge, what is the volume of this half?



If the cube is 4 in. on an edge, what is the volume of this half?

If the area of the base of the first figure on this page is 5 sq. in., what is the volume of the lower shaded part, which is 1 in. high? What is the total volume?

We have now found the volume of a triangular prism. The volume of any other prism is found in the same way as that of a triangular prism.

The volume of a prism is equal to the area of the base multiplied by the height.

MEASUREMENT OF PRISMS

1. In making an excavation for a canal the contractor speaks of removing a prism of earth. If the cross section has an area of 2700 sq. ft. and the prism is 723 ft. long, how many cubic yards of earth are removed?

The figure should be roughly sketched on the blackboard.

Find the volumes of prisms with bases and heights as follows:

2. 375 sq. in., 29 in.
3. 67.9 sq. in., 4.8 in.
4. 61.3 sq. ft., 2.91 ft.
5. 1.28 sq. ft., 3.2 in.
6. 42.6 sq. in., 7.3 in.
7. 62.8 sq. in., 3.17 in.

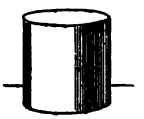
Find the heights of prisms with volumes and bases as follows:

8. 243 cu. in., 29 sq. in.
9. 140 cu. in., $17\frac{1}{2}$ sq. in.
10. 783 cu. in., 87 sq. in.
11. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

Find to the nearest tenth the volumes of rectangular solids with dimensions as follows:

12. 42 in. long by 6.8 in. wide by 3.5 in. high.
13. 2.9 in. wide by 3.8 in. long by 6.4 in. high.
14. 3.91 ft. wide by 4.27 ft. long by 6.8 ft. deep.
15. 2.8 ft. deep by 3.25 ft. wide by 4.75 ft. long.
16. 6.5 ft. long by 4.75 ft. wide by 3.25 ft. high.
17. 21.3 in. wide by 29.2 in. long by 3.7 in. deep.
18. What is the volume of a cube of edge 17.2 in.?
19. A rectangular solid with a base $12\frac{1}{2}$ in. square is twice as high as it is wide. Find its volume.
20. A prism has a square base, a volume of 1100 cu. in., and a height of 11 in. Find the lengths of the edges.

Volume of a Cylinder. If the base of a cylinder is 4 sq. in., and we put a volume of 1 cu. in. on each square inch as a base, we shall have a cylinder 1 in. high with a volume 4 cu. in. If the cylinder is 2 in. high, the volume is 2 × 4 cu. in.



In general we may say that

The volume of a cylinder is equal to the area of the base multiplied by the height.

The teacher should explain informally the meaning of *cylinder*.

If the cylinder is the common one with a circular base, the area of the circle is $\frac{22}{7}$ × the square of the radius. Therefore

Volume = $\frac{22}{7}$ × square of the radius × height.

VOLUME OF A CYLINDER

1. Find the volume of a cylinder if the base is 600 sq. ft. and the height is 3.5 ft.

2. Find the volume of a cylindric water tank 40 ft. high and 40 ft. in diameter.

3. How many cubic feet are there in a cylindric stone shaft of height 16 ft. and diameter 1 ft. 11 in.?

4. How many gallons (231 cu. in.) will a cylindric tank hold if the height is 32 ft. and the diameter is 30 ft.?

5. Find the number of cubic feet in a boiler of length 15 ft. and diameter 3 ft. 6 in., internal measure.

6. Find the number of cubic inches in a pipe of length 18 ft. and external diameter 4 in., the metal being $\frac{1}{4}$ in. thick.

Find the volumes of cylinders with these bases and heights:

7. 9 sq. in., 4 in.
8. 8 sq. ft., 9 ft.
9. $6\frac{1}{2}$ sq. in., 3 in.
10. 7 sq. in., $2\frac{1}{2}$ in.
11. $16\frac{1}{2}$ sq. in., $3\frac{1}{2}$ in.
12. $18\frac{2}{3}$ sq. ft., 27 in.

MEASUREMENT OF CYLINDERS

1. These boys measure the circumference and the height of this water tank and find them to be 33 ft. and 18 ft. respectively. Find the diameter, the area of the base, and the volume.

Find the volumes of cylinders whose heights and areas of base are respectively as follows:

2. 13 in., 27 sq. in. 3. 15 in., 88 sq. in. 4. 43 in., $48\frac{1}{2}$ sq. in.

Find the heights of cylinders whose volumes and areas of base are respectively as follows:

5. 22.5 cu. ft., 7.5 sq. ft. 6. 197.6 cu. ft., 30.4 sq. ft.

Formerly the volume in Ex. 5 would be given as 22 cu. ft. 864 cu. in.

Find the areas of base of cylinders whose volumes and heights are respectively as follows:

7. 324 cu. ft., 18 ft. 8. 33.15 cu. ft., 2 ft. 2 in.

Find the volumes of cylinders whose heights and radii of base are respectively as follows:

9. 27 in., 4 in. 10. 32 in., 8 in. 11. 2 ft. 6 in., 2 in.

Curve Surface of a Cylinder. If we slit the curve surface of a cylinder and spread it out flat, what will it become? How may we find the area of this surface? Try it with a paper cylinder.

What is the area of the curve surface of a cylinder of height 6 in. and circumference 8 in.?

It is the same as that of a rectangle 6 in. by 8 in., or 48 sq. in.

We see that *the area of the curve surface of a cylinder is equal to the circumference multiplied by the height.*

That is, area = circumference × height,

or area = $\frac{22}{7}$ × diameter × height.

SURFACE OF A CYLINDER

Numbers 1 to 12, oral

State the areas of the curve surfaces of cylinders with heights and circumferences as follows:

1. 6 in., 10 in.	**5.** 40 in., 15 in.	**9.** $66\frac{2}{3}$ in., 48 in.
2. 60 in., 70 in.	**6.** 20 in., 20 in.	**10.** $12\frac{1}{2}$ in., 24 in.
3. 25 in., 40 in.	**7.** 25 in., 30 in.	**11.** $16\frac{2}{3}$ in., 36 in.
4. 50 in., 100 in.	**8.** 60 in., 50 in.	**12.** $33\frac{1}{3}$ in., 33 in.

13. How many square inches are there in the curve surface of a wire of circumference 1 in. and length 200 ft.?

14. How much sheet iron will be needed for a pipe 8 ft. long and 6 in. in circumference, allowing 1 sq. ft. for overlapping?

15. A tin cup is 7 in. in circumference and 3 in. high. How many square inches of tin are needed for the curve surface?

In Exs. 15 and 16 the measurements include the overlapping.

16. How many square feet of galvanized iron are needed for a water pipe 10 ft. long and 9 in. in circumference?

USING WHAT YOU HAVE LEARNED

1. Write a rule for finding the circumference of a circle, given the radius; also for finding the area of the circle.

2. How many square inches of surface are there on a wire 50 ft. long and $\frac{1}{8}$ in. in circumference?

3. If the height and diameter of a solid cylinder are each 8 in., what is the total area of surface?

The teacher should explain that to the cylindric surface should be added the areas of the circles forming the bases.

4. How many square inches of tin are needed for a cylindric cup 4 in. high and 4 in. in diameter?

5. How many square feet are there in the curve surface of a water tank that is 40 ft. high and 127.3 ft. in circumference?

6. How many square feet of surface are there on the outside of a smokestack 30 ft. high and 2 ft. in exterior diameter?

7. The curve surface of a granite cylindric shaft 3 ft. 6 in. in diameter and 22 ft. 3 in. long is to be polished. How many square feet of surface are to be polished?

8. In a certain factory there is a room heated by 210 ft. of steam pipe of diameter 2 in. Find the radiating surface; that is, find the area of the curve surface which radiates the heat.

9. A large suspension bridge has 4 cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In letting the contract for painting these cables it is necessary to know their surface. Compute the area to be covered.

10. In making metal bedsteads some iron rods 0.7 in. in diameter are covered with thin rolled brass. Suppose that a shop needs 6000 ft. of such rods, how many square feet of rolled brass will be needed, not allowing for waste?

II. PROBLEMS OF DAILY LIFE

HELPING AT THE MARKET

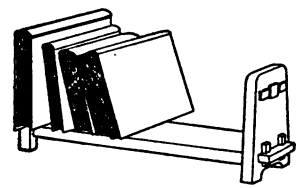
1. Henry helps his father at the market. They have been buying grapefruits, 64 to a box, at \$5.50 a box, and selling the grapefruits at 12¢ each, averaging two boxes a day. Another dealer sells two boxes a day of larger grapefruits, costing \$6.50 per box of 48, at 20¢ each. Henry thinks it would be better to deal in the larger fruit. Is he right? What is the difference in net income on the grapefruits per day?

2. They buy lemons at \$8.10 for a box of 360, selling them at 35¢ a dozen. How much do they gain on a box? What per cent do they gain on the cost? on the selling price?

3. Henry's father pays \$28 a month for the rent of his stall in the market, \$6.25 a month for light, and \$9.25 a day for help, besides allowing Henry 15¢ an hour. Henry works 12 hr. a week. There being 306 market days in a year, how much are the expenses for a year?

INDUSTRY IN THE SCHOOL

1. The class made this bookrack. If the rack will hold 12 books $1\frac{1}{2}$ in. thick, how many books $\frac{3}{4}$ in. thick will it hold?



2. The inside length of the shelf is 18 in., each end piece is $1\frac{1}{4}$ in. thick, and the shelf projects $1\frac{1}{4}$ in. at each end. What is the total length?

3. The end pieces are $6\frac{1}{2}$ in. high. The shelf is $4\frac{1}{2}$ in. wide, and the greatest width of the end pieces is $5\frac{7}{8}$ in. Taking the total length of the shelf as found in Ex. 2, what length of board 6 in. wide will be needed for the entire rack, allowing $\frac{3}{8}$ in. for waste in sawing and dressing?

4. The price of 1-inch lumber of this kind is \$160 per M. This means, as we have already learned, that 1000 sq. ft. of boards 1 in. thick cost \$160. At this rate, how much will 1000 sq. ft. of boards $1\frac{1}{4}$ in. thick cost?

5. If a shelf requires a board 32 in. long and 6 in. wide, how many square feet will such a board cover?

6. If two boards, each 16 ft. long and 6 in. wide, are required to make a set of shelves, what is the cost of the lumber at \$140 per M if the boards are 1 in. thick? if they are $1\frac{1}{4}$ in. thick?

7. If a shelf is $4\frac{1}{2}$ in. wide and 18 in. long, the width is what fractional part of the length? what per cent of the length?

8. If an end piece is $6\frac{1}{2}$ in. high and $5\frac{7}{8}$ in. wide, the width is what fractional part of the height? what per cent of the height?

9. If an end piece is $1\frac{1}{4}$ in. thick and $6\frac{1}{2}$ in. high, the thickness is what fractional part of the height? The height is what fractional part of the thickness? what per cent of the thickness?

HOUSEHOLD ECONOMICS

1. For making a certain kind of cake a cook uses 3 eggs at 68¢ a dozen, $\frac{1}{2}$ cup of butter at 56¢ a pound, $\frac{1}{2}$ cup of milk at 16¢ a quart, 1 cup of sugar at 14¢ a pound, 2 cups of flour at 8¢ a pound, and 6 cents' worth of other materials. If she is careless in baking and the cake is so burned that she throws it away, how much money is wasted through loss of these materials?

Allow 2 cups of butter, or of sugar, to the pound, 4 cups of flour to the pound, and 4 cups of milk to the quart.

2. If a family is careless and wastes a cup of cereal every week, the cereal costing 12¢ a pound and $2\frac{2}{3}$ cups weighing a pound, how much money is wasted in this way each year?

3. If each of the 250,000 families of a certain state wastes as much as the family of Ex. 2, what is the waste in a year?

4. In making apple sauce and pies a woman pares the apples so carelessly that she loses 15% of the usable part of the fruit. If she buys apples costing 4¢ each and uses on an average $\frac{1}{2}$ doz. apples every fifth day in the year, how much does she waste in this way every year?

5. If a cook throws away the water used in boiling meats and vegetables, thereby losing the materials needed for soup; and if, as a result, the family buys during the year 28 cans of soup at 15¢ a can, 2 doz. cans of soup stock at $1.25 a dozen, and soup meat costing $6.80, how much money is wasted in this way during the year?

6. If each of the 200,000 homes in a certain state wastes $\frac{1}{8}$ cup of milk a day, how many quarts are wasted in a year?

7. In Ex. 6 what would be the cost of this waste in a year when the average price of milk was 8.6¢ a quart?

HOUSEHOLD INDUSTRY

All work oral

1. For 4 cups of cocoa these girls use 3 cups of milk, 8 tablespoons of hot water, 2 tablespoons of cocoa, and 2 tablespoons of sugar. How much of each should they use for 50% of this recipe? How much of each should they use if they wish 50% more cocoa than the recipe makes?

2. How much of each ingredient mentioned in Ex. 1 should they use if they wish to make 18 cups of cocoa?

3. A recipe for 6 pop-overs calls for 1 cup of milk, 1 egg, 1 cup of flour, and $\frac{1}{2}$ teaspoon of salt. If a woman makes 3 doz. pop-overs, how much of each ingredient must she take?

4. If 21 girls are to make 2 pop-overs each, how much of each of the ingredients mentioned in Ex. 3 must be provided?

5. If some girls are to make 6 cups of cocoa in a saucepan, how large a pan must they use, estimating 2 cups to a pint and allowing an extra pint in the capacity of the pan?

HOUSEHOLD ECONOMICS

1. If a woman can buy corn at 18¢ a can or $1.90 per dozen, how much does she save on 24 cans in buying by the dozen?

2. If a woman can buy soup at 24¢ a can or $2.50 per dozen, how much does she save on 36 cans in buying by the dozen?

3. A dealer sells coffee in half-pound packages at 24¢ a package or in 5-pound cans at $2.15 a can. If a woman wishes 5 lb., how much does she save in purchasing by the can?

4. A woman can buy Dutch cocoa in ¼-pound boxes at 28¢ a box or in 5-pound cans at $4 a can. If she wishes 5 lb., how much does she save in purchasing by the can?

5. A woman can buy maple sirup at 85¢ a quart or in gallon cans at $2.85 a can. If she wishes 4 qt., how much does she save in purchasing by the can?

6. A woman buys a pound each of mustard seed, allspice, and cayenne pepper, paying 90¢ for the three. The mustard seed costs 18¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

7. A woman bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost?

8. If flour costs $15.70 a barrel (196 lb.) or 9¢ a pound, how much does a housekeeper save in purchasing flour by the barrel if she can use the amount in a barrel?

9. If a family's ice bill averages $2 a month, and ice costs 40¢ per 100 lb., how many pounds on the average does the family use per month? If with a better ice box only 0.9 as much ice is used, how much money would be saved per month?

PROBLEMS ABOUT A SILO

1. A farmer finds that he needs for a herd of ten cows for 6 mo. a cylindric silo of the kind shown in the picture, the silo being 10 ft. in diameter and 25 ft. high. How many cubic feet of silage will such a silo hold?

All silos referred to on pages 128 and 129 are cylindric.

2. For a herd twice as large as in Ex. 1 a silo 12 ft. in diameter and 32 ft. high is needed. Has this twice the volume of the other silo? If not, how much does it lack?

Teachers should state that, owing to the packing down of the silage as the height increases, there is not need for as much space.

3. To build a concrete silo that will hold 71 T. of silage the cost in a recent year was $3.10 per ton. How much did such a silo cost? If the price has gone up 90% on account of the war, what is the total cost today?

4. The internal diameter of a silo being 12 ft. and the height 25 ft., how many cubic feet will the silo hold?

5. The outside diameter of a silo is 14 ft. 7 in. How many feet of band iron will be needed to make 12 hoops for the silo, allowing 8 in. on each hoop for joining?

6. How many feet (board measure) of inch lumber will be needed to line the inner wall of a silo that is 21 ft. in diameter and 28 ft. high?

7. How much will it cost for a cement floor for a silo that is 21 ft. in diameter, at $1.90 per square yard?

8. How many cubic feet of silage will a silo hold that is 17 ft. 6 in. in diameter and 20 ft. high?

9. Allowing 40 lb. of silage to the cubic foot, how many tons of silage can be put into a silo that is 18 ft. 8 in. in diameter and 32 ft. high?

Corn silage weighs from 25 lb. to 50 lb. per cubic foot, depending upon the place in the silo from which it is taken and upon the amount of moisture it contains. A fair average is 40 lb.

10. Allowing 40 lb. daily per cow, how many tons of silage will be needed for a herd of 32 cows for 188 da.?

11. Find the total number of tons of silage that are needed during a period of 180 da. for 32 cows at 50 lb. daily per cow, 16 cows at 40 lb. daily per cow, and 22 calves at 16 lb. daily per calf.

12. A silo 40 ft. high and 10 ft. in diameter will hold 70 T. How many tons will a silo 40 ft. high and 20 ft. in diameter hold?

How does doubling the diameter affect the area of the base?

13. A silo 40 ft. high and 10 ft. in diameter will hold silage enough for 15 dairy cows for 240 da. How many dairy cows can be fed for 240 days from a silo of the same height and of twice the diameter?

PROBLEMS ABOUT CONCRETE

1. A cubic yard of "1 : 2 : 3 concrete" contains approximately $1\frac{1}{2}$ bbl. of cement, $\frac{1}{2}$ cu. yd. of sand, and $\frac{3}{4}$ cu. yd. of crushed rock. How much of each of these materials will be needed for a job requiring $6\frac{1}{2}$ cu. yd. of concrete?

It will be noticed that the sum of the volumes of the different materials is considerably more than 1 cu. yd. But they make only 1 cu. yd. of concrete, because the smaller particles of the sand and cement, when mixed with the crushed rock, fill in around the pieces of rock and so add very little to the volume of the mixture.

2. A cubic yard of $1 : 2\frac{1}{2} : 3$ concrete contains approximately $1\frac{1}{2}$ bbl. of cement, 0.6 cu. yd. of sand, and 0.7 cu. yd. of crushed rock. How much of each of these materials will be required for 180 sq. yd. of pavement 9 in. thick?

3. How many square feet of 1 : 2 : 3 concrete sidewalk can be made for each barrel of cement used if the concrete is 6 in. thick?

How many cubic yards of concrete will 1 bbl. of cement make? How many cubic feet?

4. How many cubic yards of $1 : 2\frac{1}{2} : 3$ concrete will 15 bbl. of cement make, the necessary sand and gravel being added?

5. A farmer plans to build three concrete watering troughs for his cattle, using a 1 : 2 : 3 mixture. The inside of each trough is to be 8 ft. long, 3 ft. wide, and 1 ft. 8 in. deep, the walls are to be 6 in. thick, and the bottom 10 in. thick. How much of each of the materials named in Ex. 1 will be required?

First find to the next larger whole cubic yard the amount of concrete required for the three troughs.

6. In Ex. 5 find the total cost of the material needed if the cement costs \$3 per barrel, the sand \$1.50 per cubic yard, and the crushed rock \$1.75 per cubic yard.

PROBLEMS OF THE FARMER

1. A farmer asks his son to measure a field. The boy finds the field to be 20 rd. by 80 rd. How many acres does it contain?

2. In measuring the field the boy finds that on an average a strip 3 ft. wide, next to the fence inclosing the field mentioned in Ex. 1, is not cultivated, and that 1 ft. of this strip could be cultivated as easily as the rest of the field. How many square feet of land are wasted in this way?

3. The boy has a garden 60 ft. by 100 ft. which he must water on account of the dry weather. If a cubic foot of damp clay loam contains on an average 18 lb. of water, what is the weight in tons of water in the garden to the depth of a foot when the soil contains as much water as damp clay loam?

4. The farmer wishes to fence in the garden mentioned in Ex. 3 with a four-strand barbed-wire fence. How many pounds of barbed wire will he need, allowing 1 lb. of wire to a rod?

5. On the farm is an artesian well producing $2\frac{1}{4}$ cu. ft. of water per minute. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how long will it take the well to produce the water required for the garden as found in Ex. 3?

6. A farm wagon costing \$120 depreciates in value each year about $6\frac{2}{3}\%$ of its cost if handled with reasonable care and kept in a shed when not in use. If left out in the yard it depreciates about twice as much. In 3 yr. what would be the loss by leaving it outdoors over the depreciation if it were carefully kept?

7. A farmer allows 350 cu. ft. of solidly packed hay to the ton. How many tons in a mow containing 4375 cu. ft.?

8. If corn fodder loses 60% of its weight in curing, how much will 28 T. of green corn fodder weigh after it is cured?

HEATING THE HOUSE

1. Maude's father put a steam-heating plant in his house last winter. In a season it consumes 15 T. of chestnut coal at $11.50 per ton. How much does the coal cost?

2. The heating plant in Ex. 1 cost $950 and will last only a certain number of years, after which it must be replaced. It is the custom in business to set aside each year for depreciation a certain part of the original cost of equipment, so that when it is worn out there will be money to replace it. If the heating plant lasts 10 yr., what per cent of the cost should be added to the expenses of heating the house each year in order to take care of this depreciation? Taking the cost of the coal as found in Ex. 1, what is the annual cost of heating the house?

In this example do not consider interest on the money set aside.

3. The house in Ex. 1 has 10 rooms and was heated for 212 da. What was the average cost per day of heating the house? What was the average cost per room per day?

4. Only part of coal is turned into heat, 15% being lost in ashes. How many pounds of the coal which was used in Ex. 1 are lost in ashes? How many pounds are turned into heat?

5. Maude's uncle thought it would be cheaper for him to put in a hot-water heater costing $1400, which consumes 12 T. of coal per year at $11.50 per ton. If this heater lasts 10 yr. and he allows for depreciation, was he right in thinking the annual cost less than that of the plant in Ex. 1? Find the difference.

Problems on the cost of heating the home or the school building should be made up from actual data. If possible, some boy should weigh the coal put into the furnace during a day and weigh the ashes taken out so as to give the class a practical knowledge of the amount of coal used in heating the house and the per cent of coal that actually goes into heat.

PLASTERING AND PAPERING

1. How many square yards of plastering are there in the walls and ceiling of a room 15 ft. by 21 ft., and 9 ft. high, allowing 14 sq. yd. for openings and baseboard?

The unit for estimating the cost of plastering is usually the square yard. The rule varies greatly in different parts of the country. Teachers should ask the pupils to ascertain the method in common use in their vicinity.

2. How much will it cost, at 60¢ a square yard, to plaster a schoolroom 34½ ft. long, 18 ft. wide, and 12 ft. high, making an allowance of 30% of the total surface to be plastered for blackboards, baseboard, and openings?

3. The walls of a room are 8 ft. 6 in. high above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, how many rolls of paper, 18 in. wide and containing 8 yd. of paper, will be needed? Find the cost of the paper at 45¢ a roll.

The standard width for estimating wall paper is 18 in., although it varies greatly. A single roll is considered to contain 8 yd. of paper. In estimating the number of rolls needed for a room, paper hangers usually count 3 rolls to 100 sq. ft. Parts of rolls cannot be bought.

4. How many rolls of plain paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, the paper being 24 in. wide, allowing 2 rolls for windows and doors? At 55¢ a roll, what will the paper cost?

5. The walls of a room are to be papered with plain paper worth 60¢ a roll. They are 9 ft. high above the baseboard, and the total length of the walls, allowing for windows and doors, is 60 ft. Allowing 3 rolls to 100 sq. ft., what will the paper cost?

Pupils may be asked to take the necessary measurements of rooms at home and submit problems on the cost of papering them.

WALLS AND FLOORS OF A HOUSE

1. A bathroom 9 ft. long and 6 ft. wide has a tile floor. The lower part of the walls also is tiled to a height of 3 ft. Deducting 12 sq. ft. for door and fixture openings, find the number of square feet of tiling used.

2. The rest of the walls and the ceiling in Ex. 1 are plastered and painted. If the room is 9 ft. high, find the number of square yards of painted surface, deducting 3 sq. yd. for openings.

3. Allowing 3 rolls of paper to 100 sq. ft., how many rolls of paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, deducting $1\frac{1}{2}$ rolls for openings?

4. How many strips of plain carpeting $\frac{3}{4}$ yd. wide are needed for a room 18 ft. wide?

5. What is the cost, at \$2.10 a yard, of 27-inch plain carpeting required for a room 20 ft. 3 in. long and 15 ft. wide? for a room 16 ft. 3 in. long and 13 ft. 6 in. wide? In each case lay the carpet the more economical way.

6. A carpet has an 18-inch pattern. Will there be any loss in matching the strips if they are 15 ft. long? if they are 14 ft. long? Write the reason in each case, using a plan drawn to scale if this will help you.

In allowing for waste in matching the pattern, no allowance need be made on the first strip, since the other strips are cut to match that one.

7. How many yards of carpet, $\frac{1}{2}$-yard pattern, are needed for a room 17 ft. 6 in. long and 14 ft. wide, the carpet being 27 in. wide and laid lengthwise of the room. How much is wasted in matching the pattern? Draw a plan.

8. In Ex. 7 would it be more economical if the strips run across the room? If so, how many yards would be saved?

SEWING AT HOME

1. If a sheet is $2\frac{1}{2}$ yd. long and $5\frac{1}{2}$ in. is allowed for hemming the ends, how many sheets can be cut from 54 yd. of the right width? How much material, if any, will be left over?

2. In buying sheeting, the width of the material is often given in quarters of a yard. If sheeting is $\frac{5}{4}$ (5 quarters) wide, what is the width in yards and a fraction? in inches?

The teacher may refer to the notes on page 87 if necessary.

3. How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 64 yd. long, each sheet being $2\frac{1}{2}$ yd. long, allowing 6 in. of cloth extra on each sheet for hemming?

4. If it takes $2\frac{1}{2}$ yd. of thread to stitch 1 yd. of cloth, how many spools of 50 yd. each will a woman have to buy if she wishes to stitch 75 yd. of cloth?

5. At $3\frac{7}{8}$ yd. to a skirt, how many yards of ruffling will be needed for four skirts?

6. Dish toweling can be bought for \$10 per roll of 50 yd., or at the rate of 24¢ a yard. What per cent is saved in purchasing 50 yd. by the roll instead of by the yard?

7. Allowing 27 in. for each towel, how many dish towels can be made from a piece of toweling 39 yd. long?

8. A housekeeper wishes to make two curtains for each of four windows. The windows are 6 ft. 4 in. high, and the curtains are to be the same length. She wishes to put a hem 3 in. wide at the bottom of each curtain and another hem $1\frac{1}{2}$ in. wide at the top, allowing $\frac{1}{4}$ in. for finishing each hem. She can buy a 20-yard piece of curtain material for \$10, losing what she does not need, or she can purchase exactly the amount she needs at 58¢ a yard. Which plan is the cheaper and how much cheaper?

CORN AS BEEF AND PORK

1. On a farm when corn was selling at 60¢ a bushel it was decided to feed the corn to cattle. It was estimated that the increase in the value of the cattle, from the corn alone, was 80¢ for each bushel used for feed. What was the per cent of increase in the value of the corn by using it as feed?

2. On another farm corn was selling at 62¢ a bushel. Estimating that it was worth $1.24 when fed to cattle, find the per cent of increase in the value of the corn by using it as feed.

3. A record of the result of feeding corn to hogs was kept on several farms. On one farm, when corn was selling at 70¢ a bushel, it was found that the increase in the value of the hogs was equivalent to 1.57\frac{1}{2}$ per bushel of corn fed. What was the per cent of increase in the value of the corn by using it as feed?

4. On another farm the figures of Ex. 3 were 64¢ a bushel for corn when sold and $1.28 a bushel when used as feed. What was the per cent of increase in the value of the corn?

PAVING THE CITY STREETS

1. John saw that the workmen, in preparing a street for paving, took out dirt to the average depth of 12 in. The street is 36 ft. wide and $\frac{1}{2}$ mi. long. How many cubic yards of dirt are removed and how much does it cost at 65¢ per cubic yard?

2. John finds that they put in a foundation of 7 in. of crushed rock costing $1.80 per cubic yard. How much does this cost?

3. John finds that the foundation is to be covered either with wood blocks at $1.95 per square yard or with asphalt at $2.40 per square yard. What is the difference in cost for the street?

4. The curbing for the street costs $1.10 per running foot, and there is a curbing all along both sides, except where there are cross streets. John finds that 480 ft. is to be deducted on each side for intersecting streets. How much does the curbing cost?

5. John's father's lot fronts 66 ft. on the street. If his father has to pay half the cost of the paving to the middle of the street and half the cost of the curbing in front of his lot, how much must he pay if wood blocks are used for paving?

PROBLEMS OF THE PAY ROLL

1. A mason in a certain city receives 90¢ an hour, and a plasterer $72\frac{1}{2}$¢ an hour. How much will eight masons and four plasterers receive for 8 hr. of work?

2. The following is last week's pay roll of a manufacturer:

PAY ROLL OF K. P. THORNE & CO.

For the week ending * *, 19* *

No.	Name	M.	T.	W.	T.	F.	S.	Total Time	Wages per Hour	Total Wages	
1	J. P. Driscoll	8	8	8	$6\frac{1}{2}$	8	4	$42\frac{1}{2}$	70¢	$29	75
2	R. L. Bennett	8	$7\frac{1}{2}$	8	8	8	4	*	70¢	*	*
3	R. F. Purvis	7	$7\frac{1}{4}$	8	$7\frac{1}{2}$	8	4	*	60¢	*	*
4	B. J. Mead	8	7	6	8	$7\frac{1}{2}$	3	*	65¢	*	*
5	R. K. King	8	8	7	$7\frac{3}{4}$	8	$3\frac{1}{2}$	*	55¢	*	*
6	M. L. Drake	8	6	8	0	7	$3\frac{3}{4}$	*	$37\frac{1}{2}$¢	*	*
		*	*	*	*	*	*	*		*	*

Fill out each space that is marked with an asterisk (*).

Make out pay rolls (inserting names) when the numbers, the hours per day, and the wages per hour are as follows:

3. No. 1: 8, 8, 8, 8, $7\frac{1}{2}$, 4, $72\frac{1}{2}$¢; No. 2: 8, 8, 8, 7, 6, 4, $67\frac{1}{2}$¢; No. 3: 8, $7\frac{1}{2}$, 8, 8, 8, 4, $58\frac{1}{4}$¢; No. 4: 8, 8, 8, 8, $7\frac{1}{2}$, 4, $76\frac{2}{3}$¢; No. 5: 8, 7, 8, 8, $6\frac{1}{2}$, 4, 82¢.

4. No. 1: 8, 7, 6, 8, 8, 4, $77\frac{1}{2}$¢; No. 2: 8, 7, 6, $7\frac{3}{4}$, $7\frac{3}{4}$, 4, 70¢; No. 3: 8, 6, 6, 8, $7\frac{3}{4}$, 4, 72¢; No. 4: 7, 8, $7\frac{3}{4}$, $6\frac{1}{2}$, 8, 4, 67¢; No. 5: 8, 6, 7, $7\frac{1}{2}$, $6\frac{1}{4}$, 4, 58¢.

5. No. 1: 8, 8, 8, 8, 8, 4, $72\frac{1}{2}$¢; No. 2: 8, 8, $7\frac{3}{4}$, $7\frac{1}{2}$, $7\frac{3}{4}$, $3\frac{3}{4}$, 70¢; No. 3: 8, 8, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, $3\frac{3}{4}$, 68¢; No. 4: $7\frac{1}{2}$, $7\frac{1}{4}$, $7\frac{1}{2}$, 8, $7\frac{3}{4}$, $3\frac{3}{4}$, 72¢; No. 5: 8, 8, 8, 8, 8, 4, $73\frac{1}{3}$¢.

6. It is also customary for the manufacturer to have a change sheet showing the amount of change needed to pay each man. Fill out the following change sheet to provide for the pay roll in Ex. 2, using as large denominations as possible:

No.	Name	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1	J. P. Driscoll	2	1	2		1	1			
2	R. L. Bennett									
3	R. F. Purvis									
4	B. J. Mead									
5	R. K. King									
6	M. L. Drake									
	Totals									

7. Make out a change sheet for each pay roll in Exs. 3–5.

8. The cashier of the company having the pay roll mentioned in Exs. 2 and 6 will send to the bank a change memorandum showing the amounts of each denomination required. This memorandum will be of some such form as the one here shown. Copy this form and fill in the rest of the blanks from the change sheet in Ex. 6. Check the result by comparing the total amount with the total of the pay roll in Ex. 2.

CITIZENS' BANK
BAY CITY, MICH.

K. P. Thorne & Co.
require on Feb. 6, 1924

	Number	Amount	
Pennies	10		10
Nickels			
Dimes			
Quarters			
Halves			
$1			
$2			
$5			
$10			
Total			

9. Make out a change memorandum for each pay roll in Exs. 3–5 from the change sheets made in Ex. 7. Check each one as in Ex. 8.

III. LITTLE EXAMINATIONS

i. 1. Two men hire a pasture for $132. One puts in 7 cattle and the other 15. How much rent should each pay?

2. How much will 360 ft. of iron pipe cost if listed at $1.50 a foot, a discount of 40%, 10%, being allowed?

3. How much will 4 gross of jars of mixed pickles cost at $6 a dozen, a discount of 20%, 10%, being allowed?

4. A man's salary of $2100 a year was increased 10% and later was decreased 10%. How much was it then?

5. If the ratio of cream to the rest of the milk of a certain quality is 1:5, how many pounds of cream in 750 lb. of milk?

6. If $\frac{3}{8}$ in. on a map represents a distance of 750 mi., how many miles will $1\frac{1}{4}$ in. represent?

ii. 1. A triangular gable to a house has a base of 36 ft. and a height of 12 ft. 8 in. Find the area of the gable.

2. If a tree casts a shadow 45 ft. long when a post 3 ft. high casts a shadow 3.6 ft. long, how high is the tree?

3. If 72 reams of paper are used in printing 3800 books, how many reams of paper will be needed for printing 9800 books, each $\frac{1}{2}\frac{9}{8}$ as large?

4. If the ratio of butter fat to the rest of the milk of a certain quality is 1:24, how many pounds of butter fat in 1050 lb. of milk of that quality?

5. If the ratio of solid matter to the rest of the milk of a certain quality is 33:217, how many pounds of solid matter in 375 lb. of milk of that quality?

6. After deducting a discount of $33\frac{1}{3}$%, 25%, and paying $5 cartage and $19.50 freight, 30 stoves cost $504.50. At what price was each stove listed?

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

Add and also subtract the following:

1.	2.	3.	4.
\$427.32	\$932.01	\$870.23	\$900.01
198.56	687.56	592.68	48.75

Multiply the following:

5.	6.	7.	8.
\$423.78	\$573.81	\$609.98	23 ft. 8 in.
65	74	127	15

Divide the following, in turn:

9. 864 by 16, 32, and $\frac{4}{5}$.
10. 9.72 by 0.36 and 36.
11. 58 ft. 8 in. by 8 and 4.
12. 64 lb. 2 oz. by 9 and 12.

Express the following as decimals and also as per cents:

13. $\frac{1}{2}$. 14. $\frac{3}{4}$. 15. $\frac{5}{8}$. 16. $\frac{2}{3}$. 17. $\frac{5}{6}$.

18. Find the area of a circle of radius 4.9 in.
19. What is the area of a circle of radius 6.3 ft.?
20. Find the curve surface of a cylinder 18.9 in. high and 1 ft. in diameter.
21. Find the volume of the cylinder in Ex. 20.
22. Find the entire surface of the cylinder in Ex. 20.
23. Find the circumference of a circle of diameter $2\frac{5}{8}$ in.
24. Find the radius of a circle of circumference 22 in.

Add these numbers without writing them in columns:

25. \$3.35 + \$2.60 + \$3.55 + \$4.75 + \$9.35 + \$19 + \$0.72.
26. \$12.55 + \$15.85 + \$22.60 + \$33.75 + \$19.60 + \$18.45.
27. \$13.75 + \$16.92 + \$17.08 + \$55.40 + \$32.90 + \$45.65.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you know the number of cups of flour needed for a certain number of loaves of bread, how do you find the number of cups of flour needed for a certain other number of loaves?

2. If you know how much water flows through a pipe in a minute, how do you find the amount of water that will flow through the pipe in a whole day, at the same rate?

3. If a boy wishes a piece of board to make a bookshelf, how do you find the cost of the board, knowing the size of the shelf and the cost of the lumber per M?

4. If you know how much per foot an iron rod expands in length when heated from the ordinary temperature to red heat, how do you find the amount of expansion of an iron rod a given number of inches in length when heated to red heat?

5. If you know the number of hours per day that a man works, for each day of the week, and his rate of wages per hour, how do you find the amount of money due him at the end of the week?

6. How would you calculate the cost of a concrete walk in front of the schoolhouse?

7. Suppose that your school has a playground in the form of a rectangle, how would you find the area in acres?

8. If you know the weight of two volumes of a substance, and one of the volumes, how do you state the proportion?

9. How can you find the height of your school building by means of shadows and the height of a pole?

10. How can you find the height of a tree by sighting over the top of a pole of known length?

PROBLEMS FOR COMPLETION

1. A man bought 5 loads of coal containing respectively 2475 lb., 2830 lb., 1970 lb., 2150 lb., 1950 lb. The coal cost him $11.45 a ton. Complete the problem and solve it.

See the note after Ex. 1, page 81.

Complete and solve the following problems:

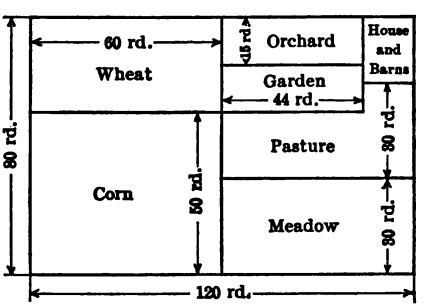
2. A grocer sold Mrs. Pratt the following items: $4\frac{1}{4}$ lb. butter @ 52¢; 3 bunches celery @ 18¢; 12 lb. beans @ 16¢; 8 cans corn @ 18¢; and 2 bottles olives @ 60¢.

3. An electric meter indicated 198 K.W.H. on May 1 and 251 K.W.H. on June 1. Electricity costs $11\frac{1}{2}$¢ per K.W.H.

4. A bill of goods amounts to $170. There are discounts of 10% and 8%.

5. A cow gave 675 lb. of milk in a month. The milk tested 4.2% butter fat.

6. A farm is 120 rd. long and 80 rd. wide. It is laid out as here shown. The sum of the areas of the various parts is equal to the area of the entire farm.



7. A farmer has a cylindric water tank which is 10 ft. in diameter and 8 ft. high. There are 231 cu. in. in 1 gal.

8. A grain bin is 18 ft. long, 14 ft. wide, and 6 ft. deep. There are $1\frac{1}{4}$ cu. ft. in 1 bu.

9. A city lot is in the form of a rectangle. The frontage of the lot is 22 ft. 6 in. The area of the lot is 1080 sq. ft.

ALERTNESS TEST

Copy the following on paper as rapidly as possible, canceling in each case the numbers (if any) that should be omitted so that the numbers shall run along in a regular order according to some kind of rule or law. For example, Ex. 1 should appear as 9 8 7 6 5 4 ~~2~~.

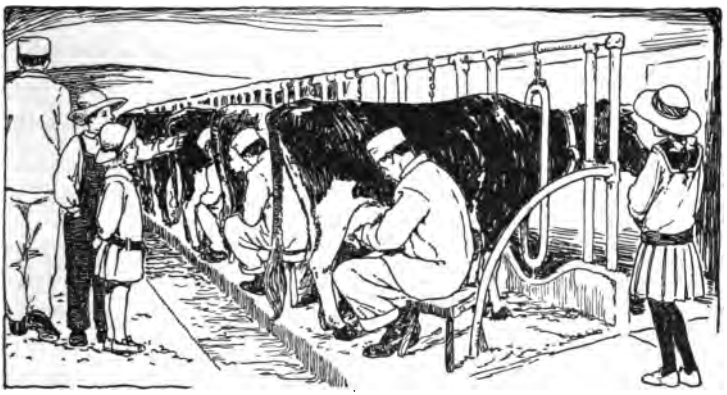
1. 9 8 7 6 5 4 2.
2. 1 7 2 3 4 5 6.
3. 3 4 6 8 10 12 14.
4. 4 7 10 12 13 16 19.
5. 1 2 4 8 10 16 32.
6. 64 32 24 16 8 4 2.
7. 7 11 15 19 23 27 30.
8. 8 4 3 2 1 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{12}$.
9. 25 20 15 12 10 5 0.
10. 3 9 27 81 92 243 729.
11. 42 40 38 36 34 32 30.
12. 29 30 32 31 32 33 34.
13. 40 34 28 22 16 14 10.
14. 2 6 18 54 162 486 1024.
15. 93 83 73 68 63 53 43.
16. 40 20 10 8 5 $2\frac{1}{2}$ $1\frac{1}{4}$.
17. 81 27 9 3 2 1 $\frac{1}{3}$.
18. 7 7 6 7 7 7 7 7 7 7.
19. 2 8 32 128 512 729 2048.
20. 100 92 84 76 68 62 60.

The teacher should have all the pupils begin at the same instant and should call all papers in at the end of 4 min. The teacher should then score for each pupil (1) the rights and (2) the wrongs and should repeat the test at least once a month during the term to see if the score is improved. To have the test at its best, the teacher should write it on the blackboard, with the order of the lines changed and with occasional new combinations, so that it cannot be worked out in advance.

As in the case of all these alertness tests, it is in the discretion of the teacher to omit the work if the class does not have the time for taking it. Nevertheless, it should be understood that such a test not only furnishes an interesting game but also furnishes a valuable exercise in becoming familiar with number relations. Not many of the pupils in an ordinary class should be expected to complete any given test in the time specified.

It is such tests that enabled the army to rank its men according to mental alertness and that are coming to be used in determining the fitness of applicants for positions requiring alertness, speed, and accuracy.

Industrial Problems



PROBLEMS ABOUT MILK WASTE

1. It is estimated that we have 23,000,000 homes in this country. We are not very careful about avoiding waste of such foods as milk. If each home wastes on an average only $\frac{1}{8}$ of a cup of milk a day, and 1 cup is equal to $\frac{1}{2}$ pt., how many quarts of milk does the country waste per day? per year?

2. If the average yield of milk per year is 2300 qt. for each cow, how many cows would it take to supply the annual waste of milk as found in Ex. 1?

3. If it requires one person to care for 8 cows, how many persons would it take to care for the number found in Ex. 2?

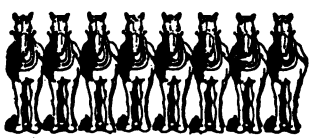
4. Under recent conditions it is claimed that it costs 5.86¢ to produce 1 qt. of milk. At this rate, what is the annual loss from waste of milk as found in Ex. 1?

The school fails in its full mission if such facts as these and many others given in this book do not lead the pupils to new ideas of thrift. We are all partners in this great country of ours.

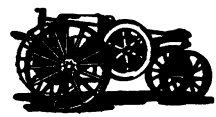
PROBLEMS ABOUT TRACTORS

1. If a tractor which costs $1225 is delivered and paid for a month early, a 3% discount is given. Find the discount.

2. An "8–16" horse-power tractor has about the same power as 8 work horses, but since horses cannot be used continuously, a tractor can do as much in 100 da. as 8 horses can do in 1 yr. The relative cost of maintenance is as follows:



8 HORSES FOR 1 YR.



AN 8–16 H.P. TRACTOR FOR 100 DA.

$53.76	Interest on investment		$43.50
80.00	Depreciation		145.00
18.16	Harness depreciation		
6.32	Shoeing or repairs		36.25
701.76	Feed or fuel		154.00
	Lubricating oil		35.00
156.56 (Feeding, etc.)	Labor	(Oiling, etc.)	25.00
4.24	Miscellaneous		

Find the expense of 8 horses for 1 yr.; of the tractor for 100 da.

3. If an 8–16 horse-power tractor is used, how much is saved in the cost of maintenance as found in Ex. 2, on work requiring the use of 8 horses for 1 yr.?

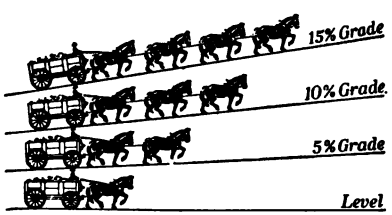
4. When the tractor is not in use the only expenses are depreciation and interest on investment. If the tractor in Ex. 2 is used only 100 da. in the year, what is the yearly cost of fuel and lubricating oil? How much is this per day? What is the cost of feed per day for the 8 horses? Find the difference.

HAULING BY HORSE POWER

1. The table below shows the power required to haul 1 T. on various types and grades of roads. A 5% grade is one that rises

Grade	Macadam Road	Gravel Road	Sand or Earth Road
Level	50 lb.	140 lb.	240 lb.
1%	70 lb.	160 lb.	260 lb.
2%	90 lb.	180 lb.	280 lb.
5%	150 lb.	240 lb.	340 lb.
10%	250 lb.	340 lb.	440 lb.
15%	350 lb.	440 lb.	540 lb.

5 ft. in every 100 ft. measured horizontally. This illustration depicts graphically the increase of power necessary to haul the same load on various grades of a gravel road. From the table find out whether the illustration is correct.



Of course, in determining the number of horses that are needed for hauling a given load on a certain grade, we must consider any fraction of a horse as a whole horse.

2. The power required to haul 1 T. on level macadam is what part of the power required on a 10% macadam grade? If a horse can haul 5000 lb. on level macadam, how much can it haul with the same effort up a 10% macadam grade?

3. A good horse can haul about 2000 lb. on an improved level gravel road. At 56 lb. to the bushel, how many bushels of corn can two horses haul over this road?

THE FARMER'S INCOME

1. James's father owns two farms of the same size and value. One he runs himself and the other he lets on shares. James asked his father if it was worth while to learn any more, and his father said: "Let us see what brains do on our property. Here is the total income of the farm we let on shares, and there are my books for last year. Now see the result." This is what James found:

	Home Farm	Farm Rented
Dairy products	$348.60	$193.75
Wool	21.75	10.43
Eggs and poultry	332.60	180.04
Domestic animals	695.75	388.17
Crops	1075.30	607.22

Find the total receipts on each farm.

2. James's father told him that the tenant was not a careful manager, and he asked James to find the per cent of increase of each of their items over the tenant's. How much was this?

3. He told James that the expenses of the two places showed poor management by the tenant, and showed him these figures:

	Home Farm	Farm Rented
Labor	$575.45	$502.43
Fertilizers	86.75	10.00
Feed	52.60	47.13
Maintaining buildings	22.60	49.72
Maintaining equipment	18.75	39.78
Taxes and miscellaneous	85.70	83.03

Find the total expenses, the difference in each pair of items, the difference in the total expenses, and the net gain of each farm.

CHAPTER III

I. BANKS AND BANKING

Functions of a Bank. In order that money may be kept safely until they wish to use it, people often deposit it in a bank. There are several kinds of banks, and most people at some time have business with one kind of bank or another. When money is deposited only a part of it is kept by the bank; the rest is lent to people whose financial reputation is good. The bank charges interest for the money that it lends.

Bank Account Essential. One thing that is essential at some time to everyone who hopes to succeed is a bank account. A reliable person may "open an account," as it is called, as soon as he begins to save even a small amount.

Certain kinds of banks, such as savings banks and trust companies, not only guarantee to take care of all the money deposited but also pay a certain per cent of interest to the depositor. National banks also generally allow interest on what is called an *inactive account*, which is a deposit that remains undisturbed for some time, or on an *active account*, when the depositor keeps a certain amount on deposit at all times.

Many schools have found it interesting and profitable to organize school banks, electing the officers and carrying on a regular banking business with imitation money. Many schools have done banking business with small amounts of real money, placed on deposit by pupils and transferred by the teacher to some bank or trust company. Such exercises should not, however, interfere with the work in computing.

Saving. A boy who puts 1¢ each day into a toy bank will save enough money in six months to buy a fielder's glove or two baseball bats. A girl who saves 10¢ a day will save enough in two months to buy a pair of shoes. A man who puts aside $1 a day may save enough in a year to buy a building lot in some place where he may care to live. We get the things we need for comfortable living or the things that give us legitimate pleasure by saving a little at a time.

Begin with a brief discussion of the need and value of saving money. There are many of us who never learn how to save money wisely. Many of us prefer to gratify our immediate desires rather than to provide for the future. Why is this a bad plan? On the other hand, there are others of us who, in order to save for the future, deny ourselves the things which it would be real economy to buy. There is always the temptation to live extravagantly. Extravagance includes not only living beyond our means but also spending money foolishly. Every boy and girl should begin to save early, no matter if it be but a few dollars or a few cents a year. Investments in government loans have done much to lead people to save money.

Savings Bank. When a person wishes to deposit money he goes to a bank, opens an account, and leaves his money with the officer in charge. The officer gives him a book in which is written the amount deposited, and the depositor writes his name in a book or on a card, for identification. When he wishes to draw out money he takes his book to the bank, signs a receipt or a check for the amount he desires, and receives the money. The customs vary in different states.

Pupils should be told of the advantages of opening even small accounts at a savings bank. A boy who deposits $1 a week for 10 yr., in a bank paying 2% every 6 mo. and adding it to the account, will have $631.54 in 10 yr., and a man who deposits $10 a week will have about ten times as much, or $6317.16.

The class should be told about trust companies, which take charge of funds, manage estates, and pay interest on deposits.

SAVING MONEY

1. If a boy, beginning when he is 16 yr. old, saves 25¢ a day for 300 da. a year and deposits it in a savings bank, how much will he have when he is 21 yr. old, not counting interest?

2. If a father gives his daughter $1 when she is 1 yr. old, $2 on her next birthday, and so on until she is 21 yr. old, depositing the money for her in a savings bank, how much will she have when she is 21 yr. old, not counting interest?

3. If a man saved $1 a week for 50 wk. of the year in which he was 21 yr. old, and increased his savings each year by 10% of the amount he saved during the preceding year, how much would he save during the year in which he was 29 yr. old?

4. A merchant saves $1125 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30% over the third year. How much does he save in the four years?

5. A man works on a salary of $45 a week for 50 wk. in a year. His expenses are $35 a month for house rent, 60% as much for clothing, three times as much for food as for clothing, and 30% as much for other necessary expenses as for food. How much of his salary can the man reserve for depositing in the savings bank?

6. A clerk in a store had a salary of $28 a week two years ago, and a commission of 2% on the goods he sold. That year he worked 48 wk. and sold $6350 worth of goods. Last year his salary was increased 25%, his rate of commission remaining the same. He worked 49 wk. and sold $7750 worth of goods. How much was his income increased?

Interest at Savings Banks. Savings banks usually pay interest every six months or every three months. This interest is added to the principal, and the total amount draws interest.

Compound Interest. If interest when it is due is added to the principal and the total amount draws interest, the interest is called *compound interest.*

The method of finding compound interest is substantially the same as that used in simple interest.

For example, find the amount of $2000 in 2 yr. at 4%, compounded semi-annually. How much is the compound interest?

Arranging the work as here shown, we find the amount to be $2164.87 and the interest $164.87.

The compound interest in this example was found exactly, but savings banks pay interest only on the dollars and not on the cents, and in this case would pay $164.84 interest.

$2000 = first principal
0.02
$40 = int. first 6 mo.
2000
$2040 = amt. after 6 mo.
0.02
$40.80 = int. second 6 mo.
2040.
$2080.80 = amt. after 1 yr.
0.02
$41.62 = int. third 6 mo.
2080.80
$2122.42 = amt. after $1\frac{1}{2}$ yr.
0.02
$42.45 = int. fourth 6 mo.
2122.42
$2164.87 = amt. after 2 yr.
2000.
$164.87 = int. for 2 yr.

Disregard a fraction less than $\frac{1}{2}\,¢$ in any interest; call a fraction $\frac{1}{2}\,¢$ or greater a full cent.

Simple interest in the example above would be $160, or $4.87 less than the compound interest. If simple interest is collected when due and at once reinvested, the investor practically receives compound interest.

Savings Bank Account. Savings banks usually pay interest semiannually, adding this to the principal on deposit. This is, therefore, a case of compound interest. The following is a specimen account at a bank paying 4% interest, payable semiannually, on the smallest balance on deposit in an interest period.

Date		Deposits		Interest		Payments		Balance	
1923									
July	1	600	50					600	50
July	20	75						675	50
Sept.	6					120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1924									
Jan.	1			11				561	50
May	9	200						761	50
July	1			11	22			772	72

The term "balance" is familiar from the work with cash accounts in Book Two, and its meaning as applied to a bank account is the same.

In this account the smallest balance during the first interest period is \$550.50, and interest is computed on the dollars only, the cents being neglected. At 2% the interest on \$550 is \$11. In the second period the smallest balance is \$561.50, the interest being 2% of \$561, or \$11.22.

Some banks allow interest from the first of each month; others from the first of each quarter; others, as above, from the first of each half year. The interest is computed on the smallest balance on deposit between this day and the next interest day, and is usually added every half year.

In the above account if the interest period had been 3 mo. and the interest credited every 6 mo., the first interest credit would have been:

First quarter, 1% of \$555 = \$5.55.
Second quarter, 1% of \$550 = 5.50. Credit \$11.05.

SIMPLE AND COMPOUND INTEREST

At simple interest, and also at interest compounded annually, find the amount of principal and interest:

1. \$2500, 4 yr., 5%.
2. \$4000, 3 yr., 6%.
3. \$3500, 3 yr., 4%.
4. \$2250, 4 yr., 3%.
5. \$1750, 3 yr., $3\frac{1}{2}$%.
6. \$625.50, 4 yr., $3\frac{1}{2}$%.
7. \$10,000, 6 yr., 4%.
8. \$425.50, 4 yr., $4\frac{1}{2}$%.

Find the amount of principal and interest, the interest being compounded semiannually:

9. \$200, 3 yr., 4%.
10. \$650, 2 yr., 6%.
11. \$700, 3 yr., 3%.
12. \$500, 2 yr., 4%.
13. \$1000, 2 yr., $4\frac{1}{2}$%.
14. \$2000, 2 yr., $3\frac{1}{2}$%.

15. John has \$500 on deposit in a savings bank on Jan. 1. How much interest is due on the following July 1 if the bank pays 4%? if it pays $3\frac{1}{2}$%? if it pays $3\frac{1}{4}$%? if it pays $3\frac{3}{4}$%?

16. A school deposits in a savings bank as one account the savings of all the pupils, and a committee of the pupils keeps accounts for all the children. If the school deposits \$475 before July 1, and the bank pays 4%, how much interest will be due on the following Jan. 1?

17. Edwin has \$50 in a savings bank on July 1, and deposits \$10 on the last day of each month for 6 mo., beginning July 31. The bank pays 4% per annum on all sums on deposit the first day of each month, compounding the interest on Jan. 1 and July 1. If Edwin does not draw any money out of the bank, how much will he have on deposit on Jan. 1 and how much interest will be due at that time?

18. Which brings the more interest in 2 yr., \$1250 drawing $4\frac{1}{2}\%$ simple interest, or \$1250 deposited at the beginning of a year in a savings bank which pays 4% interest, compounded semiannually? How much is the difference?

Remember that interest on all savings-bank deposits is paid only on the dollars and not on the cents.

19. If a man on Jan. 1 deposits \$2000 in a savings bank and leaves it for 2 yr., the interest at 4% being compounded semiannually, how much interest does he receive?

20. Mrs. Hall deposits \$40 each month in a savings bank that pays 4%, compounded on Jan. 1 and July 1, on all sums on deposit on the first day of each month. If she deposits the money just before the first of the month, beginning the last of January, what will be her balance on the following Jan. 1?

21. In Ex. 20, if the bank credits interest Jan. 1 and July 1 on all sums on deposit on Jan. 1, Apr. 1, July 1, and Oct. 1, what would be Mrs. Hall's balance on Jan. 1?

22. A man deposits \$950 in a savings bank on Jan. 2, 1923. His further deposits are as follows: Mar. 22, 1923, \$40; Apr. 12, 1923, \$65; Sept. 10, 1923, \$25; Mar. 8, 1924, \$125. He draws out money as follows: Feb. 1, 1923, \$35; Aug. 2, 1923, \$75; Sept. 7, 1923, \$125; Dec. 13, 1923, \$95; Jan. 2, 1924, \$385. The bank pays 4% interest on the smallest balance on deposit in an interest period, compounding the interest on Jan. 1 and July 1. Find his balance on July 2, 1924.

23. If a man deposits \$200 in a savings bank on Jan. 1, \$300 on Feb. 1, \$100 on May 1, \$400 on Aug. 1, and \$350 on Nov. 1, and the bank allows 4% interest from the first of each month, compounding it on July 1 and on Jan. 1, how much will the man have to his credit on the Jan. 1 following these deposits?

Postal Savings System. The United States government conducts a savings bank, known as the *Postal Savings System*, in connection with the post office. Although every savings bank is carefully regulated and inspected by its state government, many persons are willing to take the smaller rate of income from the Postal Savings System, because our national government guarantees the payment of their money at any time.

Any person of the age of 10 yr. or over may open an account and deposit at any time any number of dollars, except that the balance must not exceed $2500, exclusive of accumulated interest.

Interest is paid by the government at the rate of 2% for each full year that the money remains on deposit, beginning on the first day of the month following the one in which the deposit is made. Interest is not paid for any fraction of a year. The government does not pay compound interest on a postal savings account, but if the interest when due is withdrawn and deposited, practically the depositor will receive compound interest.

A person may exchange his deposits for United States postal savings bonds, bearing 2½% interest, issued in denominations of $20, $100, and $500, such exchange to be considered as made on Jan. 1 and July 1 of each year.

Postal Savings Certificate. The depositor receives a *postal savings certificate of deposit* for the amount of each deposit made. These certificates are issued in denominations of $1, $2, $5, $10, $20, $50, $100, $200, and $500.

Postal Savings Card. Since no deposit of less than $1 is allowed in the Postal Savings System, the government has arranged to sell *postal savings stamps* at 10¢ each, to be put upon a *postal savings card.* One of these cards can be bought for 10¢ and has a stamp already upon it. With nine more stamps affixed it is worth $1, and may be used in making deposits.

POSTAL SAVINGS SYSTEM

Numbers 1 to 15, oral

Find the interest for 1 yr. at 2% on the following deposits:

1. \$20. 2. \$50. 3. \$100. 4. \$400. 5. \$800.

Find the interest for 2 yr. at 2% on the following deposits:

6. \$10. 7. \$40. 8. \$200. 9. \$700. 10. \$1000.

Find the interest for 1 yr. received by a holder of $2\frac{1}{2}\%$ bonds for:

11. \$20. 12. \$100. 13. \$300. 14. \$500. 15. \$1000.

16. Using as large denominations as possible, what are the denominations of the certificates of deposit which you will receive for a deposit of \$87?

17. A boy saved 10¢ each week day, filling his postal savings card as fast as he could and then buying postal certificates of deposit. If there were 312 week days in that year, how much money did he have on deposit with the post office at the end of the year?

18. If you deposit \$4 on June 7 and \$16 on June 15, 1921, how much interest will be due on July 1, 1922?

19. If you deposit \$10 the last day of each month for 1 yr., beginning Jan. 31, 1922, how much do you have on deposit at the post office on Jan. 1, 1923? What is the first date on which any interest is due you and how much is then due?

20. If in Ex. 19 you make no withdrawals of interest, how much interest is due you on Feb. 1, 1924?

21. Mr. Watson had \$500 in postal savings certificates of deposit. On July 1 he exchanged his total deposit for a $2\frac{1}{2}\%$ bond. Find the amount of the interest which was due on this bond on the following Jan. 1.

Commercial Bank Account. A bank which permits deposits to be withdrawn on an order called a *check* is often called a *commercial bank* to distinguish it from a savings bank. A business man will send a check instead of sending cash from one place to another. An account in such a bank is often called a *checking account*, and if small does not draw interest. The convenience and safety of such an account is more important.

Deposit Slip. When a man makes a deposit of money or of checks in a commercial bank he fills out a *deposit slip*, as shown here, and hands it in with his deposit. Most banks request that the name of the bank on which a check is drawn be listed as shown.

DEPOSITED FOR CREDIT OF
.....James K. Rogers.....
IN THE
FIRST NATIONAL BANK
OF NEW ORLEANS
.....Jan. 8, 1924

Bills	21	
Gold		
Silver	2	75
Checks:		
Union Bank	28	73
Pitt Trust Co.	17	21
Market Bank	7	84
	77	53

DEPOSIT SLIPS

All work oral

State the sums of each of the following lists of deposits:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
$15.50	$10.	$12.75	$22.50	$23.80	$45.	$12.	$16.75	$62.50	$75.50
3.75	5.75	2.25	2.50	12.70	5.75	13.75	10.	2.50	4.50
	1.25		5.		1.25		3.25		20.

DEPOSIT SLIPS

Fill out deposit slips for the following deposits, inserting in each case the name of the depositor and of some bank in your vicinity:

1. Bills, $254; silver, $40; check on First National Bank, $87.50; check on Traders Bank, $127.50.

2. Bills, $423; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60.

3. Bills, $135; check on Chemical National Bank, $325; check on Third National Bank, $63.73.

4. Bills, $1726; gold, $125; silver, $100; check on Traders Bank, $335.50; check on Second National Bank, $175.40.

5. Bills, $262; checks on Garfield National Bank, $96.50, $200; check on Jefferson and Lee National Bank, $325.

6. Gold, $50; checks on First National Bank, $27.62, $41.75, $32.80; check on Lincoln Trust Co., $37.42.

7. Bills, $146; silver, $275.50; check on Miners National Bank, $43.50; check on Rocky Mountain National Bank, $250.

8. Bills, $145; silver, $350.75; gold, $480; check on Merchants National Bank, $255; check on Farmers Trust Co., $162.50; check on Second National Bank, $275.50.

9. A merchant deposited $175.80 in cash today, a check for $176, and a check in payment for $18\frac{3}{4}$ yd. of velvet at $3.15 a yard less $33\frac{1}{3}\%$. Make out a deposit slip, inserting the names of the banks on which the checks are drawn.

10. A jeweler received cash for 3 doz. forks @ $26.50; $4\frac{1}{2}$ doz. teaspoons @ $25; a watch @ $42.75; and 2 clocks @ $9.75. He also received a check on the First National Bank for 3 doz. dessert spoons @ $27.75 and 9 nutcrackers @ $14 a dozen. He deposited all this in a bank. Make out a deposit slip.

Check. A *check* is an order written by the depositor directing the bank to pay out money from his account. A *check book* containing blank checks with *stubs* annexed is furnished by the bank. The stubs are left in the book and show what checks have been issued. Usually there is also a space on the stub to keep a record of the *balance*, or the money left in the bank.

No. 896		
Jan. 6, 1924		
To M. P. Jones		
For Groceries		
Bal. forward	678	62
Am't this check	82	75
Bal. forward	595	87

Stub

FIRST NATIONAL BANK

New Orleans, La., Jan. 6, 1924

Pay to the order of Myron P. Jones $82.75

Eighty-two 75/100 Dollars

No. 896 James K. Rogers

Check

The person to whom a check is payable, as Myron P. Jones in the check above, is called the *payee*. A check may also be made payable to "Self," in which case the drawer alone can collect it; or to "Bearer" or to "Cash," in which cases anyone who presents the check to the bank can collect the money.

If the check is made payable to the order of the payee, as in the one shown above, the payee must *indorse* it; that is, he must write his name across the back of the check, as shown here.

Myron P. Jones

The teacher should explain the nature of checks and the various methods of making them payable. The pupils should write or fill out checks payable to "Cash," to "Self," or to the order of a specified payee, and give their reasons for making the checks payable as they do.

DEPOSITS AND CHECKS

1. On Jan. 7 Mr. Rogers, a page of whose check book is shown on page 160, makes out check No. 897 to the Haines Rug Co. in payment of a bill of $35.65. Write the check and fill in the stub, showing his balance after paying this bill.

2. A man deposits $350 in the bank on the first of the month. During the month he pays the following bills by check: rent, $40; telephone, $3.50; milk, $4.27; groceries, $22.17; meat, $17.67; light, $3.86; coal, $72.20; insurance, $18.76. What is his balance on the last day of the month?

3. If your deposits in a bank have been $68.45, $92.30, $47.60, and $96.70, and you have drawn checks for $8.55, $23.65, $8.58, $48.75, and $34.60, what is your balance?

4. A merchant having $980.75 in the bank deposits during the next week $185.50, $97.85, $135.50, $86.85, and $236.80. He gives checks for $89.65, $37.20, $93.60, $15.20, $248.70, and $39.80. What is his balance?

5. A man having $825.60 in the bank deposits checks for $75.80, $126.75, $234.80, and $42.80. He gives checks for $128.75, $125.80, and $24.75. What is his balance?

6. A grocer having $828.50 in the bank deposits $567.80, $245.50, $89.65, $482.86, $429.50, and $376.50, and draws checks for $427.50, $38.95, $67.82, $568.70, and $122.58. He also pays by check a bill for $86 less 4%. What is his balance?

7. A restaurant keeper having $1026.92 in the bank deposits $488.75, $928.75, $386.48, $442.80, $196.85, and $327.75, and draws checks for $96.75, $286.75, $342.80, and $438.50. He also pays back by check a loan of $230 with interest for 4 mo. at 5%. What is his balance?

Pass Book. When an account is opened, the bank usually gives the depositor a *pass book*, in which is entered the amount deposited. When he makes a deposit, he hands the book to the receiving teller, who enters the amount and returns the book.

FIRST NATIONAL BANK

OF NEW ORLEANS

James K. Rogers
16 Day St., City

Statement for the month of JANUARY, 1924

Checks		Deposits	Day	Balance
		Balance brought forward	1	786.72
108.10			4	678.62
		77.53	8	756.15
82.75	35.65		9	637.75
53.54		95.00	15	679.21
32.70	6.74		18	639.77
		167.87	25	807.64
12.00	4.66		31	790.98

Statement. While some banks balance the pass book, most large banks simply use it as the customer's receipt for money deposited and, at the end of each month, send the depositor a *statement* of his account in some such form as the one shown above. The deposits and other credits are added to the balance from the previous month's statement, and the checks are deducted as they are paid by the bank. The column at the right shows the daily balance, and the last entry in this column shows the balance to be carried to the next month's statement.

The depositor checks this statement from the bank with his own record, which he keeps in his check book.

BANK STATEMENTS

1. The bank statement for October shows that a firm has a balance of $7632.96 on Oct. 31. The pass book shows the following deposits in November: 6, $3758.60; 9, $2843.85; 12, $2976.42; 16, $4826.75; 20, $2928.85; 24, $3174.30; 29, $2672.29. The check book shows the following checks issued in November: 2, $689.85; 3, $392.90, $167.75; 5, $3228.45, $7.90, $654.68; 9, $850, $525.35; 14, $796.80, $2285.40; 20, $948.65; 22, $3689.85; 29, $34.80, $340, $68.38. What is the firm's balance on Nov. 30?

2. In Ex. 1 make out the statement, like the one on page 162, which the bank sends the firm on Nov. 30. Consider that the checks issued were paid by the bank on the day after they were drawn. Use the name of some bank and firm in your vicinity.

Of course such an assumption as to the date of payment of the checks is entirely arbitrary and is made only for the purposes of the problem and to show that the bank usually pays a check some time after it is drawn.

3. A merchant's bank balance was $14,296.35 on Oct. 1. Deposits were made during the month as follows: 8, $2978.30; 14, $2285.85; 18, $2862.75; 22, $2438.70; 25, $3162.62; 28, $1657.37. Checks were drawn as follows: 6, $697.28, $1128.67, $231.27; 13, $268.92, $765.43, $1.47; 16, $1428.83, $647.26, $2786; 19, $11.28, $350, $591.23; 25, $1100; 29, $6.27, $285, $528.92. Make out the bank statement for October as in Ex. 2.

4. Make out a bank statement for December as in Ex. 2, using the following items. Balance, Dec. 1, $2385.60. Deposits: 2, $495.65; 3, $229.86; 6, $308.75; 8, $445.65; 10, $638.80; 28, $1275.30. Checks: 3, $50, $74.28, $61.97; 7, $221.62, $45; 8, $4.27, $940, $78.26; 14, $16.30, $28.16, $97.80; 18, $1225, $87.30; 25, $1.47, $6.89, $3.26; 29, $41.60, $75.

Promissory Note. A paper signed by a borrower, agreeing to repay a specified sum of money on demand or at a specified time, is called a *promissory note*, or simply a *note*.

The sum borrowed is called the *principal*, or, if a note is given, the *face of the note*.

The sum of the principal (or face) and the interest is called the *amount* of the note.

A note should state the date, face, rate, person to whom payable, and date of maturity (date when it is due to be paid), and should state the fact that it has been given for value received.

The following sample shows a simple form of a *time note*:

$75.\frac{40}{}$ TOPEKA, KANSAS, February 7, 1922

Sixty days after date, for value received, I promise to pay to John Williamson or order,

Seventy-five $\frac{40}{100}$ Dollars,

with interest at 5 %

Frank Francis

The following sample shows a simple form of a *demand note*:

$50.\frac{00}{}$ DALLAS, TEXAS, May 2, 1922

On demand, for value received, I promise to pay to

Robert J. Jones or order,

Fifty $\frac{00}{100}$ Dollars,

with interest

James P. Poole

Parties to a Note. The person named in a note as the one to whom it is payable is called the *payee.* The person who signs a note is called the *maker.*

Indorsing a Note. If the payee wishes to sell the note he must, when it is payable to himself or order, *indorse* it.

A note is indorsed by the payee's writing his name across the back. The indorser must pay the note if the maker does not.

A note payable to John Johnson or *bearer* may be sold without indorsement. Such notes are not common.

If the payee wishes to sell the note without being responsible for the payment in case the maker should fail to pay it, he may write the words "without recourse" across the back and write his name underneath. This means that he relinquishes all title to it, but that the buyer cannot come back (have recourse) on him. The following are common forms of indorsement:

John Johnson	*Pay to the order of John Roberts John Johnson*	*Without recourse John Johnson*
Indorsement in blank	**Indorsement in full**	**Limited indorsement**

Teachers should explain fully the meaning of these several indorsements and should have the pupils indorse notes properly.

Rate of Interest. The United States borrowed money before the war at about 2%. During the war the rate began at $3\frac{1}{2}$% and increased to $4\frac{3}{4}$%. Savings banks pay depositors about 4% or 5%. In cities, on good security, borrowers usually pay 4% to 6%.

When a note bears interest, but the rate is not specified, it bears interest at a certain rate fixed by the law of the state. In many of the states this rate is 6%. If a note falls due on a legal holiday, it is payable on the next business day.

USING WHAT YOU HAVE LEARNED

1. Mr. Brownell borrowed $2750 from a note broker, who charged a commission of 1% of the face of the note for arranging the loan. He paid the principal with interest at 6% in 3 mo. How much did the loan cost him?

Teachers should explain that note brokers make a business of finding persons who wish to lend money and those who wish to borrow it, and arranging loans.

During the war the government began to require the makers to buy revenue stamps and put them on notes. Since the law will probably soon be changed, at least in some of its features, this textbook can merely suggest that the teacher mention the matter in connection with a problem like this, stating the temporary conditions if necessary.

2. A merchant borrowed $900 on May 4 and $1150 on June 7, giving his note for each loan. He paid both notes the first of the following October with interest at 4½%. Find the total amount of the notes on that date.

3. A purchaser of an automobile listed at $2400 is offered the option of paying half cash and giving his note for the rest due in 4 mo., at 5%, or of paying $2300 in cash. How much less is the bill if he pays cash?

4. A salesman starting on a trip on Jan. 5 borrowed $750, giving a demand note for this sum with interest at 5%. He returned March 9 and immediately paid his note. Write the note and find the amount of it on the date paid.

5. A manufacturer had a note signed by a company for $3500 with interest at 6% from May 16, 1923, to Feb. 26, 1924. The company failed and paid only 40% on its debts. Meanwhile the manufacturer had put the note in the hands of an agency, which was to receive 5% of the amount collected. How much did the manufacturer receive after paying the agency?

Borrowing Money at a Bank. If Fred lends a baseball bat to Frank, it is evident that Frank borrows the bat from Fred. The same is true of lending and borrowing money. Lending by one person means borrowing by another, so that the arithmetic of the one is the same as that of the other.

Business houses usually have need of large sums of money at certain times of the year, and it is better for them to borrow it for two or three months than to keep such an amount on hand for a whole year. Borrowing money is, therefore, often good business policy for the merchant, contractor, or manufacturer, although it is poor policy for any person to borrow money simply to gratify some desire for extravagance.

It is the custom for business houses and private individuals to borrow money at banks. The banks obtain the money from depositors, who find it safer and more convenient to keep their money in a bank than to lock it up in their homes or offices. The banks are paid for their work by charging borrowers a higher rate of interest than they pay the depositors.

If Peter Barlow wishes to borrow money from a bank, he can do so if he has a first-class financial standing in business, or if he has some securities (such as stocks and bonds, explained later), commonly called *collateral*, which he can leave with the bank to sell in case he does not pay the note, or if he can get some responsible person to indorse his note, thereby agreeing to pay it if he does not.

The teacher should explain to the class that some city banks have about a third of their loans secured by collateral, the rest being on notes with or without indorsers. Money is often borrowed on a note which has no other indorser than the maker himself, the bank having faith in the integrity and financial standing of the borrower. The good business reputation of a man or a corporation is a very valuable asset. Start life with a good reputation and never lose it. See the note on page 73.

Time Note. If John Brown wishes to borrow $1000 for 2 mo., and if he is doing a safe business and his financial standing is good, the bank with which he does business will probably lend him the money, he paying the interest in advance. Mr. Brown would then fill out a note in some such form as the one shown below.

$1000.00 LOUISVILLE, KY., May 15, 1922

.....Two months.....after date.....I.....promise to pay to the order of

.....myself.....

One thousand 00/100 ~~~

Demand Note. Below is shown the essential part of a note payable on demand and secured by the deposit of collateral.

$4500.00 NEW YORK, October 1, 1922

On demand, FOR VALUE RECEIVED, the undersigned promises to pay to the order of

SECOND NATIONAL BANK OF THE CITY OF NEW YORK

(hereinafter called the Bank), at its Banking Office in New York City,

Forty-five hundred $\frac{no}{100}$ Dollars,

in United States gold coin or its equivalent, with interest from the date hereof, at the rate of .....3..... per cent per annum, having deposited with the Bank, as collateral security for the payment of this note, or any note given in extension or renewal thereof, as well as for the payment of any other obligation or liability, direct or contingent, of the undersigned to the Bank, due or to become due, whether now existing or hereafter arising, the following property, viz.:

.....6 M....N.Y. State $4\frac{1}{4}$% Canal bonds. Nos. 4286–4291

of a market value estimated by the undersigned at $.....5160.00.....; and the undersigned agrees to deliver to the Bank additional securities, or to make payments on account to its satisfaction, should the market value of the said securities, as a whole, suffer any decline. The undersigned hereby gives to the Bank a lien for the amount of all such obligations and liabilities upon all the property or securities now or at any time hereafter given unto or left in the possession of the Bank by the undersigned, whether for the express purpose of being used by the Bank as collateral security, or for any other or different purpose, and also upon any balance of the deposit account of the undersigned, with the Bank.

Peter Barlow

Since the pupils have not studied stocks and bonds, the teacher may state that these are valuable securities which will be considered later. Demand notes like the above usually run for only a short time. The rate of interest is ordinarily low, but it is often subject to large and rapid variations. Banks usually require that the face of a note shall be not greater than about 80% of the market value of the collateral, to allow for a possible decline in the value of the security.

Review of the Six-Per-Cent, or 60-Day, Method. The Six-Per-Cent Method mentioned on page 75 is so commonly used in banking, where the notes often bear 6% and run for 30 da., 60 da., or 90 da., that it is here reviewed. Repeating the important part of one of the rules there given, we have,

To find the interest for 60 da. at 6%, move the decimal point two places to the left; for 6 da., three places to the left.

For example, the interest at 6% for 60 da. on the amounts specified are as follows:

Amount	Interest	Amount	Interest
$75	$0.75	$1275	$12.75
$250	$2.50	$15,250	$152.50

For 30 da. the interest is half as much as for 60 da.; for 90 da. it is half more than for 60 da. Thus, the interest on $1200 at 6% for 30 da. is $\frac{1}{2}$ of $12, or $6; for 90 da. it is $12 + $6, or $18.

DRILL ON THE SIX-PER-CENT METHOD

Find the interest at 6% on notes for the following amounts:

1. $120, for 90 da.
2. $270, for 90 da.
3. $525, for 30 da.
4. $2575, for 90 da.
5. $3625, for 90 da.
6. $4250, for 30 da.
7. $3850, for 60 da.
8. $4000, for 60 da.
9. $7500, for 90 da.
10. $16,000, for 30 da.
11. $450, for 2 mo. 10 da.
12. $725, for 2 mo. 15 da.
13. $850, for 2 mo. 20 da.
14. $550, for 1 mo. 15 da.
15. $1275, for 1 mo. 20 da.
16. $6250, for 1 mo. 10 da.
17. $7500, for 3 mo. 15 da.
18. $8000, for 3 mo. 20 da.
19. $6500, for 2 mo. 15 da.
20. $8500, for 1 mo. 15 da.

Six-Per-Cent Method for Other Rates. We may use the Six-Per-Cent Method to find interest at the other rates in common use.

Thus, since 5 is $\frac{1}{6}$ less than 6, the interest at 5% on a given principal is $\frac{1}{6}$ less than the interest at 6%. Similarly, the interest at $5\frac{1}{2}$% is $\frac{1}{12}$ less than the interest at 6%, the interest at $4\frac{1}{2}$% is $\frac{1}{4}$ less than the interest at 6%, the interest at 4% is $\frac{1}{3}$ less than the interest at 6%, and so on.

For example, find the interest on \$1260 for 90 da. at 5%.

Interest for 60 da. at 6% is 1% of \$1260 . . .	\$12.60
Interest for 30 da. at 6% is $\frac{1}{2}$ of \$12.60 . . .	6.30
Total interest at 6%	\$18.90
Deduct $\frac{1}{6}$	3.15
Total interest at 5%	\$15.75

SIX-PER-CENT METHOD FOR OTHER RATES

Find the interest at 5% on the following amounts:

1. \$780, 30 da.
2. \$840, 60 da.
3. \$86.40, 60 da.
4. \$79.20, 30 da.
5. \$1200, 90 da.
6. \$1560, 90 da.

Find the interest at 4% on the following amounts:

7. \$240, 30 da.
8. \$630, 60 da.
9. \$68.80, 60 da.
10. \$72.24, 30 da.
11. \$1800, 90 da.
12. \$2436, 90 da.

Find the interest at $5\frac{1}{2}$% on the following amounts:

13. \$720, 30 da.
14. \$960, 60 da.
15. \$60.72, 60 da.
16. \$97.68, 30 da.
17. \$5600, 90 da.
18. \$4048, 90 da.

Find the interest at $4\frac{1}{2}$% on the following amounts:

19. \$160, 30 da.
20. \$920, 60 da.
21. \$54.80, 60 da.
22. \$62.48, 30 da.
23. \$6000, 90 da.
24. \$8648, 90 da.

Bank Discount. When a man borrows from a bank on a time note he usually pays the interest in advance. Interest is not then mentioned in the note, because it has already been paid.

In some parts of the country banks do not require the interest to be paid in advance. Teachers should be guided by the local custom.

Interest paid in advance on a note is called *discount.*

Teachers should call the attention of the pupils to the fact that the same word is used for bank and commercial (trade) discount, explaining that the process of finding the discount is the same in both cases; that is, it is simply finding some per cent of a number.

In the work on this page call 30 da. a month.

Proceeds of a Note. The face of a note less the discount is called the *proceeds of the note.*

For example, what are the discount and proceeds of a note for \$400 for 3 mo. at 5%?

The discount (interest) on \$400 for 1 yr. is 5% of \$400, or \$20.

The discount for 3 mo. is $\frac{1}{4}$ of \$20, or \$5.

The proceeds are \$400 − \$5, or \$395.

$$\frac{\overset{1}{\cancel{4}} \times \overset{100}{\cancel{\$400}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{100}}} = \$5$$

BANK DISCOUNT

Find the discount at 6% and the proceeds on the following:

1. \$75, 1 mo.
2. \$90, 30 da.
3. \$50, 2 mo.
4. \$250, 3 mo.
5. \$275, 3 mo.
6. \$425, 4 mo.
7. \$1500, 60 da.
8. \$1800, 63 da.
9. \$4500, 93 da.

Find the discount at 5% on the following:

10. \$40, 1 mo.
11. \$70, 2 mo.
12. \$80, 90 da.
13. \$175, 3 mo.
14. \$250, 4 mo.
15. \$900, 5 mo.
16. \$1250, 1 mo.
17. \$2275, 60 da.
18. \$4750, 90 da.

NOTES AND DISCOUNTS

1. Make out a note for \$1200, dated today, payable on demand to self or order at a bank in your vicinity, and sign it by the initials X. Y. Z. so that it shall not be a real note.

2. Make out a 30-day note for \$675, dated today, payable to Richard Roe's order at some bank. If the note is discounted today at 6%, what are the proceeds?

3. Make out a 90-day note for \$1250, dated today, payable to James Jameson's order at some bank of which you know. If the note is discounted today at 6%, what are the proceeds?

4. A man's bank account shows deposits of \$37.50, \$82.50, \$75, \$100, \$50, \$300, \$40, \$125, and \$500; checks drawn, \$13.75, \$62, \$5, \$125.50, and \$82.75. He needs \$2000 to start him in business and wishes to keep about \$500 in the bank. How much money, to the nearest \$100, should he borrow?

5. If the man in Ex. 4 gives a note for this sum for 90 da., how much is the discount at 6%? What are the proceeds?

6. A merchant has to pay a debt of \$1275 less 10%. His balance in the bank is \$672.80 and he has \$107.50 in cash in his safe. He wishes to leave about \$300 in the bank and about \$65 in his safe. How much, to the nearest \$50, must he borrow? If he gives a 90-day note for this sum and the note is discounted the same day at 6%, the proceeds are how much more or how much less than the exact sum which he needs?

Find the discounts and the proceeds on the following notes:

7. \$875, 30 da., 6%.
8. \$975, 90 da., 5%.
9. \$425, 30 da., 6%.
10. \$2565, 90 da., 5%.
11. \$3250, 90 da., $5\frac{1}{2}$%.
12. \$4250, 30 da., $4\frac{1}{2}$%.

Discounting a Note. A dealer may buy goods for the fall trade and instead of paying cash may give the manufacturer a note which bears interest and is due after the holidays. If the manufacturer needs the money he may sell the note to a bank for the amount less the discount. Such notes are called *commercial paper.*

For example, if you give a manufacturer your note for $500, dated Sept. 1 and due Jan. 1, with interest at 5%, and he discounts the note at a bank Sept. 1 at 6%, what are the proceeds?

Face of the note	$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	$508.33
Discount for 122 da. at 6%	10.34
Proceeds	$497.99

In computing the discount the bank usually counts the actual number of days a note has to run, excluding the day of discount but including the day the note is due, as shown in the problems on this page.

The manufacturer may not need the money Sept. 1 and may keep the note, letting it draw interest. But if he needs the money Sept. 16 he then discounts the note, and we have

Face of the note	$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	$508.33
Discount for 107 da. at 6%	9.07
Proceeds	$499.26

Federal Reserve Bank. If the banks wish to borrow money they may do so, subject to certain restrictions, from the *federal reserve banks* established by the government.

The technical details of such banks do not concern the schools, being too remote from the ordinary interest of pupils and of most citizens.

DISCOUNT ON NOTES

Find the discounts and the proceeds on the following notes, the notes drawing no interest:

1. \$750, 60 da., 5%.
2. \$225, 60 da., 5%.
3. \$500, 60 da., 6%.
4. \$450, 60 da., 5%.
5. \$225, 90 da., 5%.
6. \$5575, 90 da., 6%.
7. \$7500, 30 da., 5%.
8. \$4750, 30 da., 6%.
9. \$42.50, 30 da., 6%.
10. \$125.50, 60 da., 6%.
11. \$275.25, 60 da., 6%.
12. \$375.50, 90 da., 6%.
13. \$457.50, 30 da., 5%.
14. \$296.50, 30 da., 5%.
15. \$287.60, 90 da., 5%.
16. \$375.40, 90 da., 5%.
17. \$2750, 90 da., $4\frac{1}{2}$%.
18. \$3275, 90 da., $4\frac{1}{2}$%.

19. The discount at 6% on a note for 90 da. is \$18.75, and on another note for 30 da. the discount is \$32.50, neither note drawing interest. What is the face of each note?

20. A dealer bought \$1750 worth of goods, giving his note Oct. 4 for 90 da. at 5%. On Oct. 10 the holder of the note discounted it at 6%. Find the proceeds.

Find the discounts and the proceeds on the following notes:

	Face of Note	Date of Note	Note Due	Rate of Interest	Date of Discount	Rate of Discount
21.	\$300	July 1	July 31	4%	July 16	6%
22.	\$800	Sept. 1	Oct. 31	5%	Sept. 11	6%
23.	\$550	Oct. 1	Dec. 30	4%	Dec. 2	6%
24.	\$325	Mar. 1	Apr. 30	5%	Mar. 17	6%
25.	\$1000	Nov. 1	Dec. 1	6%	Nov. 1	6%
26.	\$1250	June 1	Aug. 30	5%	July 23	5%

USING WHAT YOU HAVE LEARNED

1. Find the interest on \$1750 for 60 da. at 6%, first by the Six-Per-Cent Method, then by cancellation, and finally by the ordinary method of finding the interest for 1 yr. and then for the fractional part of a year. Write a statement telling the advantage of the Six-Per-Cent Method over the other methods.

2. Using the three methods mentioned in Ex. 1, find the interest on \$675 for 90 da. at 6%; on \$1425 for 30 da. at 6%.

3. A firm wishes to borrow about \$5000 for 60 da. The bank offers to lend it at 6%. If the firm gives its note for \$5100 and discounts this, how much more than \$5000 will the firm actually receive after the discount is paid?

4. A real-estate dealer buys some property for \$15,000. He pays \$4800 down and borrows the balance at his bank, giving a 90-day note at 6%. How much discount must he pay?

5. How much greater, if any, is the discount on a note for \$1250 discounted for 60 da. at 6% than the discount on a note for \$2500 discounted for 30 da. at 6%?

6. A man borrows \$4875. If he gives a note for \$4900 for 30 da. at 6%, will the proceeds be more or will they be less than the amount he needs? How much more or how much less?

7. On Apr. 1 a dealer buys 16 automobiles at \$1285, paying \$8060 cash and giving a 90-day note at 6% for the balance. If the manufacturer who holds the note discounts it at 6% on Apr. 16, what are the proceeds of the note?

8. If a 2% discount is allowed for cash on a bill of \$1850, and the bank will discount a 30-day note for \$1850 at 6%, how much does a merchant save by borrowing the money and taking advantage of the cash discount?

Draft. A written order by which one person directs another to pay a certain sum of money is called a *draft.*

Explain to the class that a check is a kind of draft. Usually the word "check" is used for an order on a bank, signed by any person except a representative of another bank; the word "draft" is used for an order other than a check. A draft may therefore be made by an individual on a company, by one bank on another, by a company on an individual or firm, and so on. A draft made by a cashier on his own bank is called a *cashier's check.*

Commercial Draft. A draft made to secure the payment of a debt is called a *commercial draft.*

The following is a form of commercial draft payable at sight:

DAYTON, OHIO, *Feb. 15,* 1923

AT SIGHT pay to the order of.....

.....*The First National Bank of Dayton*.....$*735.*$^{80}_{-}$

Seven hundred thirty-five $\frac{80}{100}$ ~~~~~~~~~~~~~~~~Dollars

To .....*The Cash Register Co.*

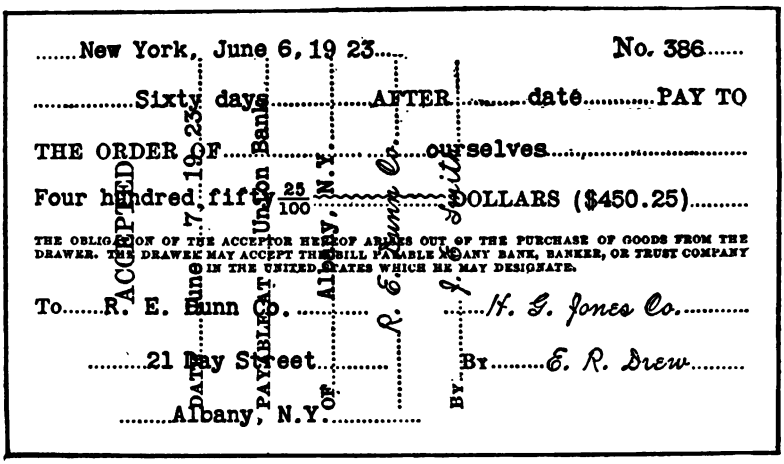
James X. Wye..... .....*Per J. K. S.*.....

Cleveland, Ohio.....

Parties to a Draft. Here the Cash Register Company is the *drawer*, Mr. James X. Wye is the *drawee*, and the First National Bank is the *payee.*

Explain to the class that the Cash Register Co. deposits this draft with the First National Bank, which sends it to some bank in Cleveland. The Cleveland bank collects the money from Mr. Wye and sends its own draft to the Dayton bank. The Dayton bank then notifies the company that the draft on Mr. Wye is paid, and the amount, called the *proceeds* of the draft, is added to the company's account in the First National Bank. This is a convenient method of collecting money. The bank usually charges the drawer a small commission for this work.

Trade Acceptance. There has recently come into extensive use the acceptance of a draft payable some time in the future; that is, a *time draft.* If this is for goods purchased it is called a *trade acceptance.* The following is a common form for such a draft:


New York, June 6, 19 23
No. 386
Sixty days AFTER date PAY TO
THE ORDER OF Union Bank
ourselves
Four hundred fifty 25/100 DOLLARS ($450.25)
THE OBLIGATION OF THE ACCEPTOR HEREOF ARISES OUT OF THE PURCHASE OF GOODS FROM THE DRAWER. THE DRAWEE MAY ACCEPT THIS BILL PAYABLE AT ANY BANK, BANKER, OR TRUST COMPANY IN THE UNITED STATES WHICH HE MAY DESIGNATE.
To R. E. Dunn Co.
21 Bay Street
Albany, N.Y.
ACCEPTED
June 7, 19 23
PAYABLE AT Union Bank
OF Albany, N.Y.
R. E. Dunn Co.
BY
H. G. Jones Co.
By E. R. Drew

Explain to the class that a wholesale dealer will send such a draft with the invoice. If at once accepted by the drawee, he signs the acceptance across the face. The draft may then be discounted by the dealer, who will thus get his money immediately, less the discount, even though the goods were sold on 30 da. or 60 da. credit.

We found on page 174 that notes sold to a bank at a discount are called commercial paper. In the same way drafts and trade acceptances that are discounted in this way are also called *commercial paper.*

Trade acceptances have a great advantage over the old method of letting accounts run for a long time. By this method a manufacturer does not have to carry a large number of accounts on his books, but transfers all these details to his bank.

DRAFTS AND ACCEPTANCES

1. A miller in Seattle bills some flour at " 30 da. net, covered by trade acceptance." The bill is for $500, and the buyer writes his acceptance on a 30-day draft, sending it back to the miller. The miller discounts the acceptance, that is, the accepted draft, at $\frac{1}{2}\%$ per month, the same day. What are the proceeds?

Pupils are not expected to learn all the details of these commercial transactions, but a single problem of this kind will give some idea of the general plan of collecting by the use of accepted drafts.

2. A draft dated Aug. 8, at 30 da. sight, for $975, is accepted Aug. 10 and is discounted Aug. 16 at 6%. What are the proceeds of the draft?

To find the period of discount subtract the number of days from Aug. 10 to Aug. 16 from the number of days that the draft runs.

3. A draft dated Sept. 1, at 30 da. sight, for $350, is accepted Sept. 4 and is discounted Sept. 7 at 6%. What are the proceeds of the draft?

4. A draft dated Apr. 2, at 60 da. sight, for $725, is accepted Apr. 7 and is discounted Apr. 12 at 6%. What are the proceeds of the draft?

5. A draft dated Oct. 15, at 60 da. sight, for $1225.75, is accepted Oct. 19 and is discounted Oct. 25 at 6%. What are the proceeds of the draft?

6. A draft dated Feb. 7, at 90 da. sight, for $1675.50, is accepted Feb. 14 and is discounted Mar. 16 at 5%. What are the proceeds of the draft?

7. A manufacturer sells a bill of goods amounting to $725.50 at list prices, discounted at 10%, 5%, and draws on the buyer at 60 da. sight. The buyer accepts the draft, and it is discounted 10 da. later at 6%. Write the draft and find the proceeds.

COLLECTING BY DRAFT

1. The Westinghouse Linen Company draws on R. T. Dow for $150. The banks charge 0.2% for collection. What are the proceeds? Write the draft.

2. Robertson Bros. draw on J. P. Shipley for $37.50. The bank charges 10¢ for collection. The charge for collection is what per cent of the face of the draft?

3. A. F. Rowe owes M. S. Stanton for 2 doz. suits @ $271.50; $1\frac{1}{2}$ doz. suits @ $234; $7\frac{1}{2}$ doz. pairs trousers @ $40. Stanton draws on Rowe for the money, and the bank charges him $1.25 for collecting. What rate is this?

4. Suppose William Bentley of Winnipeg owes you $250, and you draw on him for this amount. Write the draft, payable to the order of some bank near your home. If the bank charges $\frac{1}{10}\%$ for collecting, what are the proceeds?

5. A manufacturer sells goods amounting to $1672.25 at list prices, discounted at 8%, 3%, and draws on the buyer at 60 da. sight. The buyer accepts the draft, and it is discounted 5 da. later at 5%. Write the draft and find the proceeds.

It may be stated informally to the pupils that the new use of trade acceptances tends to do away with chain discounts like 8, 3; that is, 8%, 3%. Manufacturers are coming to list their goods at "30 da. net, subject to trade acceptance," or "$\frac{1}{2}\%$ for cash by return mail," or in some similar way. The prices are thus more easily computed and the manufacturer's bookkeeping is much simplified. Problems like Ex. 5 will, however, long continue to be practical.

6. A jobber sells goods amounting to $2737.50 at list prices, discounted at 15%, 8%, 3%, and draws on the buyer at 90 da. sight. The buyer accepts the draft, and it is discounted 30 da. later at 6%. Write the draft and find the proceeds.

Paying Bills at a Distance. If you owe money to a merchant living in another place from whom you have ordered a baseball, tennis racket, book, or other article, you may send the money by a registered letter. You will be more likely, however, to pay the bill by a money order. A man who had an account at a bank would be apt to pay the bill by a check or by a draft.

Exchange. The payment of money by means of checks, money orders, or drafts is called *exchange.*

Paying by Check. If a man sends his check for the amount owed, the creditor will deposit the check in his bank. This bank will send it to the debtor's bank for collection, and may charge a small amount for the trouble.

Paying by Money Order. A money order may be purchased at the post office, or from an express company, or, at considerable expense, money may be telegraphed. Postal or express orders may be sent to the creditor, who can then obtain the money at his post office or express office. The extra cost of postal money orders is as follows:

For sums not exceeding \$2.50, 3¢; from \$2.51 to \$5, 5¢; \$5.01 to \$10, 8¢; \$10.01 to \$20, 10¢; \$20.01 to \$30, 12¢; \$30.01 to \$40, 15¢; \$40.01 to \$50, 18¢; \$50.01 to \$60, 20¢; \$60.01 to \$75, 25¢; \$75.01 to \$100, 30¢.

POSTAL MONEY ORDERS

All work oral

Using the above list, state the total cost of money orders for:

1. \$31.50.	4. \$86.50.	7. \$16.30.	10. \$60.
2. \$19.90.	5. \$92.30.	8. \$90.50.	11. \$80.
3. \$52.75.	6. \$69.95.	9. \$75.50.	12. \$100.

Paying by Bank Draft. One of the common methods of paying a debt due in another place is by means of a bank draft.

MERCHANTS NATIONAL BANK OF AUSTIN

No. *48104* AUSTIN, TEXAS, *July 6*, 19*23*

Pay to the order of ..... *John Roberts* ..... $*78.*$\frac{75}{}$

Seventy-eight $\frac{75}{100}$ ~~~

PREMIUMS ON DRAFTS

1. How much will a draft for \$300 cost at $\frac{1}{5}\%$ premium?

2. How much will a draft for \$3200 cost at 0.1% premium? a draft for \$2500 at $\frac{1}{2}\frac{1}{0}\%$ premium?

3. How much will a postal money order for \$37.50 cost? for \$62.75? for \$14.30? for \$86.50? for \$75.40?

4. How much will a Camden merchant pay for a draft on Chicago for \$4000 at 40¢ premium per \$1000?

5. When the government charges 30¢ per \$100 for a money order, what per cent premium does it charge?

6. A draft cost a merchant \$2752.75, including 0.1% premium. Find the face of the draft and the premium.

7. A draft cost a merchant \$3751.50, including the premium of 40¢ per \$1000. What was the face of the draft?

8. If a man owes bills of \$250, \$150, \$100, and \$350 to different jobbers from whom he bought goods, what is the total cost of drafts for these amounts at 0.1% premium?

9. Which is cheaper for you if you owe \$75 for some goods, to send a money order or a draft on which you have to pay 15¢ premium? How much cheaper?

10. It costs 10¢, besides the regular postage, to register a letter. If the letter is not delivered the government will pay the loss, up to \$50. If you owe \$22, is it cheaper for you to send it by registered letter or by money order? How much cheaper?

11. If you owe \$100 to a manufacturer at a distance, from whom you have bought goods, in what manner would you make the payment? Tell why you would make it in that way and how much it would probably cost. The premium on a draft may be taken at the common rate of 0.1%.

Rate of Exchange. A money order is always sold at a slight premium, and this is usually the case with a bank draft, as already stated. In each case the rate of the variation from the face is called the *rate of exchange.*

If the exchange is added to the face, exchange is said to be *at a premium*, as we have already seen. If there is no exchange, exchange is said to be *at par.* If the exchange is deducted from the face, exchange is said to be *at a discount.*

For small sums, say for $500 or less, New York, Chicago, or Philadelphia exchange usually sells at a premium of about 0.1%. This is to pay the bank for its trouble and for the expense of shipping the money when its balance in these cities gets low. Banks usually buy such drafts from their depositors at the face value, thus making no charge for cashing them.

For large sums the rate of exchange varies. If the Chicago banks owe the New York banks $2,000,000, they must send that amount by express, an expensive proceeding. If a man in Chicago at that time wished to buy a draft on New York for $30,000, they would charge him more than usual because they would have to express that much more to New York; but if a man in New York wished to buy a draft on Chicago, he might buy it for less than $30,000 because the bank would get its money at once, and the risk and expense of transmitting it would be saved.

The premium or discount is usually quoted at a certain per cent of the face of the draft, varying at present from about $\frac{1}{40}\%$ to $\frac{1}{8}\%$; but sometimes as so much on $1000, a fraction of $1000 counting as $1000. In the latter case the quotation of $\frac{1}{40}\%$ premium is the same as that of 25¢ premium.

The explanation of the *Clearing House*, a place where bank officials of a city meet daily to exchange drafts and checks and to pay balances due one another, if given at all, should be given orally.

EXCHANGE ON DRAFTS

Find the cost of the following drafts:

1. \$3756.70, at par.
2. \$750, 0.2% premium.
3. \$2450, 0.1% premium.
4. \$3500, 0.1% discount.
5. \$6750, $\frac{1}{20}$% premium.
6. \$17,500, 0.2% discount.

7. What is the cost of a draft on San Francisco for \$5200 at $\frac{3}{4}$% discount? Write the draft.

8. What must be paid in Rahway for a draft on Chicago for \$3400, exchange being at $\frac{1}{4}$% premium?

9. A draft for \$4800 was bought for \$4794. Was exchange at a premium or at a discount? What was the rate?

10. When a Boston draft for \$35,000 can be bought in New Orleans for \$34,930, is exchange at a premium, at par, or at a discount? What is the rate?

11. If J. R. Glover draws on J. B. Thornton for \$250, and the banks charge 0.1% for collection, what are the net proceeds that Glover will receive? Write the draft.

12. My agent in Toronto sells a house for me for \$2500. He charges 2% commission, and the bank charges \$2.50 premium for a draft for any amount between \$2000 and \$2500. What sum does he remit to me?

13. A telegraphic money order costs twice the rate for a ten-word message, plus 1% premium on the face. A ten-word message from Trenton to Kansas City costs 50¢. How much will a telegraphic money order for \$375 cost?

14. Mr. Edgcomb of Denver owes Mr. Nourse in that city \$3243.24. Mr. Nourse has gone to New York on business and asks that the money be sent, less the cost of exchange. If exchange is at 0.1% premium, what is the face of the draft?

Foreign Money. If an importing house or a private individual receives a bill of goods from abroad, this bill must ordinarily be paid in the money of the country from which it came. This may be done by postal money order, up to a certain amount; by express money order, issued by an express company; by telegraphic money order; or, as is usually the case, by bank draft. Foreign drafts are often called *bills of exchange.*

Bills of exchange, like domestic drafts, may be above par (when exchange is at a premium), at par, or below par (when exchange is at a discount).

The most important units of value in countries whose currency, like that of the United States, may be called international are as follows:

In the British system

1 pound (£) = 20 shillings (s. or /) = 240 pence (d.)
= $4.8665 = about $5

In the French, Swiss, and Belgium system

1 franc (fr.) = 100 centimes (c.) = 19.3¢ = about 20¢

The Italian system is the same except that the franc is called a lira (lē'rä), and the centime is called by its Italian name. The same system of value is used in Spain and Greece, the names being changed.

1 German mark (M.) = 100 pfennigs (pf.) = 23.8¢ = about 25¢

1 Japanese yen (¥) = 1 Mexican peso (₱) = 49.8¢ = about 50¢

British exchange, often called sterling exchange, is quoted at dollars to the pound (4.88 meaning that $4.88 will buy a bill of exchange for £1). If English exchange is above 4.8665, exchange is at a premium; if below, it is at a discount.

French exchange is quoted either at francs to a dollar (5.15 meaning that $1 will buy a bill of exchange for 5.15 fr.) or at cents to the franc (19.7 meaning that 19.7¢ will buy a bill of exchange for 1 fr.).

War conditions have seriously affected the above relative values, and it may be some years before they will again be normal.

FOREIGN MONEY

1. An importing house buys a bill of goods in Sheffield, England, amounting to £180 10s. Exchange being 4.80, how much will a draft cost for this amount?

Take £180.5 for £180 10s., and notice that a draft for £1 costs $4.80.

2. A merchant buys a quantity of silk in Lyons, France, amounting to 7850 fr. 20c. Exchange being 5.05, how much will a draft cost for this amount?

Take 7850.2 fr. for 7850 fr. 20c., and notice that at the given rate of exchange a draft for 5.05 fr. can be bought for $1.

3. How large a draft on London can be bought for $1205 when exchange is 4.82?

4. How large a draft on Paris can be bought for $12,500 when exchange is 5.28?

5. A merchant imports goods costing £348 6s. What is the cost of a bill of exchange at 4.82?

6. An importer buys a shipment of Lyons silk to the value of 54,750 fr. What will a bill of exchange cost at 5.20?

Taking 25¢ to the shilling, express approximately in our money:

7. 15s. 6d. 8. 17s. 9d. 9. 35s. 8d. 10. 12s. 4d.

Taking $5 to the pound, express approximately in our money:

11. £17 8s. 12. £24 9s. 13. £62 6s. 14. £125 10s.

Taking 5 fr. to the dollar, express approximately in our money:

15. 75 fr. 50c. 16. 19 fr. 20c. 17. 72 fr. 75c. 18. 34,870 fr.

Taking 2 ¥ to the dollar, express approximately in our money:

19. 350 ¥. 20. 7500 ¥. 21. 37,280 ¥. 22. 52,725 ¥.

Other Banking Information. The information already given relating to banks and banking covers all the facts that the citizen ordinarily needs and all that have arithmetical features of importance. There are, however, a few other facts which, although not relating to arithmetic, the pupil may care to know.

Certified Check. If a man wishes to pay by check, and there is any doubt as to his having the money in the bank with which to pay the check when it is presented, he may have the check *certified.* To do this he takes it to the bank on which it is drawn and has the cashier, or some other officer who is empowered to do so, stamp across the face the word "Certified" and place the date and the officer's signature below. This means that the bank guarantees the payment of the check.

Letter of Credit. If a person is traveling, either in this country or abroad, it is necessary for him to obtain money in various places; otherwise he would have to carry a large amount of money with him and run the risk of theft or loss. One of the methods employed is that of carrying a *letter of credit.* For example, the person may deposit $5000 with a bank that sells letters of credit, and this bank will, for a reasonable compensation, usually 1% or less, give him a letter, signed by the bank officials and addressed to a large number of other banks telling them that it guarantees the payment of any drafts drawn by him up to $5000.

When a draft is drawn on a letter of credit, the bank that pays the money writes on the letter a statement of the amount drawn. The holder of the letter is identified by his signature, which is either on the letter or in a book of instructions accompanying the letter.

Express money orders and bankers' checks are often used in place of the letter of credit, but the principle is the same.

REVIEW PROBLEMS

1. Find the amount of principal and interest of \$500, compounded semiannually at the rate of 4%, for 5 yr.

2. A man deposits \$750 in a savings bank on July 1, 1921. His further deposits are as follows: Aug. 1, 1921, \$30; Sept. 15, 1921, \$25; Dec. 31, 1921, \$100; June 15, 1922, \$50. He draws out money as follows: Oct. 12, 1921, \$40; June 20, 1922, \$75; Aug. 2, 1922, \$20; Aug. 15, 1922, \$30; Sept. 12, 1922, \$25. Every three months, on Jan. 1, Apr. 1, July 1, and Oct. 1, the savings bank pays 1% on the smallest balance for the preceding quarter. Find his balance on Jan. 1, 1923.

3. Make out a deposit slip for the following deposits, naming some bank in your vicinity: bills, \$575; silver, \$72.25; checks, \$48.25, \$72.50, \$61.30, \$227.52, and \$86.95. If your balance was \$326.42 before the deposit, how much was it after?

4. A merchant's balance the first of the month was \$1725.60. He has since deposited \$37.50, \$296.75, \$21.30, \$572.32, and \$217.65. He has given checks for \$128.92, \$27.36, \$273.96, \$78.76, and \$18.92. How much is his present balance?

5. Make out a 90-day note for \$1250, dated today and payable to R. P. Foster, or order, at some bank in your vicinity. If this note is discounted today at 6%, what are the proceeds?

6. Which is the greater, the proceeds of a note for \$750 for one year, drawing interest at 6% and discounted on the day of its date at 5%, or the amount of a note for \$700 for one year at 6%? How much greater?

7. Mr. Woodhull lends Mr. Baker \$750 on June 5, 1921, at 6%, due in 6 mo., taking his note. On Sept. 5, 1921, Mr. Woodhull discounts the note at a bank at 5%. Find the proceeds.

8. Mr. Collins goes abroad with a letter of credit for £1000. When in London he needs £50 and finds that on that day exchange is quoted at $4.35 to the pound. Find the value in our money of the indorsement on his letter of credit.

It will be noticed that this rate, $4.35 to the pound, is lower than the normal rate of gold exchange, which is approximately $4.87. Since the war most European rates of exchange have been below normal. This is partly because these countries are heavily in debt to us for money borrowed.

9. Suppose that you send to New York for some uniforms for the Boy Scouts or the Girl Scouts, and the bill is $72.60; and suppose that you wish to pay the bill by sending an express money order to the dealer. The rate charged by the express company is 25¢ on sums from $60.01 to $75. If the postage is 4¢ on the two letters that you write, and the express is $2.30, how much will the uniforms cost?

10. Write a draft for $275 payable 15 da. after date, addressing it to someone who, we may suppose, owes you that amount of money, and make the draft payable to the order of some bank with which we may suppose you to do business.

11. Write a draft for $750 payable 10 da. after date, addressing it to some dealer in a distant city who, we may suppose, has bought goods from you to that amount, and make the draft payable to the order of some bank.

12. Find the proceeds of a 30-day trade acceptance for $1250 discounted for the entire time at 5%.

13. Find the proceeds of a 60-day trade acceptance for $2500 discounted for 50 da. at 6%.

In this review, or earlier in this chapter, pupils should be encouraged to tell what plans they have for saving money and in what form of investment they would put their money if it amounted to a considerable sum. The subject is discussed more fully on page 237.

II. OUR DUTY TO THE GOVERNMENT

What we owe to the State. When we speak of what we owe to the State we mean not only to that one of forty-eight states of the United States in which we live but also to the whole country. The United States and our own particular state educate us, protect us, build our roads for us, erect our schoolhouses, keep our public buildings in repair, and make the conditions of our life such as to give us a chance in the world.

In return we owe it to the State to help support it, to give money, service, and, if necessary, our health and even our lives to make it strong and safe not merely for ourselves but for those who come after us.

One of the ways in which we may help in supporting the State is through the payment of taxes.

Meaning of Taxes. Some boys in school wanted to organize a baseball team. They found that for $4 a month they could rent a piece of land for a baseball field, and decided that they needed $5.50 at once for balls and bats. Each boy bought his own suit, but they had to raise $9.50 for the first month. They decided that ten boys should be allowed to join and that each should pay his share, which was 95¢ for the first month. This 95¢ was a *tax* upon each member of the club.

A class wished to buy a picture for the schoolroom. There were 30 pupils in the class, and the picture they desired cost $3.90. Each pupil agreed to pay his share. A committee was appointed to raise the money, and each member of the class was taxed 13¢. Here the *total tax* on the class was $3.90, the tax on each pupil was 13¢, the committee *levied a tax* of 13¢ on each pupil, and a *collector* collected the money. These are the terms that are used in connection with taxes in general.

Taxes in a Village. A village needed $15,000 for building a new schoolhouse, and its citizens voted to raise the amount by a tax. The sum was so large that it would not be fair to make each citizen pay the same amount, so each one was required to pay according to the value of his property. In this way those who had much property were taxed more than those who had little property. A man who owned $10,000 worth of property was required to pay twice as much tax as one who owned $5000 worth of property and to pay half as much tax as one who owned property to the value of $20,000.

Taxes in a City. It costs a large amount of money to pay all the expenses of a city. The streets have to be cleaned, lighted, and kept in repair; the policemen have to be paid; the fire department has to be kept up; the city buildings have to be cared for; the city officers have to be paid; the schools have to be supported; the parks have to be kept in good condition and besides all these there are many other expenses.

For example, suppose that all the expenses of a certain city amount to $50,000,000 a year. If the property of the citizens is *assessed* (valued) at $2,500,000,000, each dollar of real property (land, buildings, trees, etc.) and each dollar of personal property (money and movable goods) ought to pay an amount equal to $50,000,000 ÷ 2,500,000,000, or $0.02. That is, for every dollar that a citizen owns he should pay a tax of 2¢.

Tax Rate. Upon the assessed valuation a certain *tax rate* is fixed by the city authorities. The words "tax rate" are often used to designate the *number of cents* of tax on each dollar of assessed valuation of the property, or the *number of mills* of tax on each dollar of assessed valuation, or the *number of dollars* on each $100 or on each $1000 of assessed valuation.

Finding the Tax Rate. If a village with an assessed valuation of $3,200,000 must raise $16,800, what will be the tax rate?

$16,800 \div 3,200,000 = \$0.005\frac{1}{4}$.

That is, for every dollar of property that a person living in the village owns he must pay $5\frac{1}{4}$ mills of tax. Although, as we have seen, the mill is not coined, it is sometimes used for such purposes as this.

$$
\begin{array}{r}
\$0.005\frac{1}{4} \\
32\text{ØØØØØ}\,\overline{)\,\$0.168\text{ØØ}} \\
\underline{160} \\
8
\end{array}
$$

$\frac{8}{32} = \frac{1}{4}$

Finding the Tax. After the tax rate has been determined for the village, city, county, or other political division, the tax of each taxpayer can easily be found.

If a man's property is assessed at $12,000, for example, and the tax rate is $5\frac{1}{4}$ mills, he must pay $5\frac{1}{4}$ mills on every dollar; that is, he must pay $12,000 \times 5\frac{1}{4}$ mills, or, what is the same thing, he must pay $0.005\frac{1}{4} \times \$12,000$, which is $63.

The method of collecting taxes varies so much in different states that the teacher should have the pupils inquire about it at home.

TAX RATES AND TAXES

1. If the tax rate is 8 mills on $1, what is the tax of a man whose property is assessed at $5000?

2. If the tax rate is $2.18 on $100, what is the tax of a man whose property is assessed at $8000?

Find the taxes paid on the following, at the rates specified:

3. $8500, $7\frac{1}{2}$ mills on $1.

4. $16,500, $1.46 on $100.

5. The assessed valuation of a city is $75,000,000 and the total tax levied is $1,845,000. Find the tax rate, expressing it as so many dollars and cents on each $1000 of assessed valuation.

COMPUTING TAXES

1. The tax rate is 9.6 mills. What is the tax on property assessed at $6800?

2. The tax rate is $2.20 on $100. What is the tax on property assessed at $22,800?

3. The tax rate is $12.30 on $1000. What is the tax on property assessed at $12,500?

Find the tax at $5\frac{1}{2}$ mills on each of these amounts:

4. $6500.	7. $7675.	10. $24,400.	13. $23,250.
5. $8750.	8. $12,000.	11. $23,500.	14. $18,875.
6. $4875.	9. $16,500.	12. $31,100.	15. $45,250.

Find the tax at $21.40 on $1000 on each of these amounts:

16. $4500.	20. $7000.	24. $6750.	28. $32,500.
17. $6500.	21. $4550.	25. $8500.	29. $38,500.
18. $8000.	22. $8750.	26. $12,000.	30. $45,250.
19. $4600.	23. $9750.	27. $25,000.	31. $75,750.

32. If the state tax is 1.1 mills, the county tax 3.6 mills, and the village tax 4.2 mills, what tax must a man pay on $8000?

33. A man died, leaving an estate valued at $23,750, of which $22,000 was willed to his children and the rest to his nephew. Under the laws of his state there is no inheritance tax to be paid by children, but 4% must be paid by the nephew. What inheritance tax must be paid by the nephew?

The details of the inheritance tax vary in the different states.

34. If a man's tax is $195 and the rate is 13 mills on a dollar, find the assessed valuation of his property. What is the assessed valuation of the property of a man whose tax is $234?

Tax Table. To assist them in their work of calculating taxes after the tax rate has been determined, assessors usually prepare a *tax table* similar to the following table, which is arranged for 17.8 mills on a dollar, or \$1.78 on \$100, or \$17.80 on \$1000:

TAX TABLE. RATE 17.8 MILLS ON \$1

	0	1	2	3	4	5	6	7	8	9
0	.000	.0178	.0356	.0534	.0712	.0890	.1068	.1246	.1424	.1602
1	.178	.1958	.2136	.2314	.2492	.2670	.2848	.3026	.3204	.3382
2	.356	.3738	.3916	.4094	.4272	.4450	.4628	.4806	.4984	.5162
3	.534	.5518	.5696	.5874	.6052	.6230	.6408	.6586	.6764	.6942
4	.712	.7298	.7476	.7654	.7832	.8010	.8188	.8366	.8544	.8722
5	.890	.9078	.9256	.9434	.9612	.9790	.9968	1.0146	1.0324	1.0502
6	1.068	1.0858	1.1036	1.1214	1.1392	1.1570	1.1748	1.1926	1.2104	1.2282
7	1.246	1.2638	1.2816	1.2994	1.3172	1.3350	1.3528	1.3706	1.3884	1.4062
8	1.424	1.4418	1.4596	1.4774	1.4952	1.5130	1.5308	1.5486	1.5664	1.5842
9	1.602	1.6198	1.6376	1.6554	1.6732	1.6910	1.7088	1.7266	1.7444	1.7622

In this table the first figure of the number of dollars assessed is shown in the column at the left, and the second figure is shown in the row at the top. Thus, at 17.8 mills on \$1, the tax on \$100 is \$1.78; the tax on \$2500 is \$44.50.

Illustrative Problems. 1. How much tax must a man pay on property assessed at \$8500, at 17.8 mills on \$1?

By the table the tax on \$8500 is \$151.30. This is found at the right of 8, the first figure in 8500, and under 5, the second figure in 8500.

2. How much tax must a man pay on property assessed at \$17,500, at 17.8 mills on \$1?

By the table the tax on \$17,000 is \$302.60 and the tax on \$500 is \$8.90. Hence the tax on \$17,500 is \$302.60 + \$8.90, or \$311.50.

USE OF THE TAX TABLE

1. The tax rate is 17.8 mills. What is the amount of the tax on property assessed at $7800?

Have the pupils use the table on page 195.

2. The tax rate is $0.0178. What is the amount of the tax on property assessed at $22,800?

3. The tax rate is $1.78 on $100. What is the amount of the tax on property assessed at $12,500?

4. The tax rate is $17.80 on $1000. What is the amount of the tax on property assessed at $9500?

Using the table on page 195, find the taxes on property assessed at the following amounts:

5. $7000.	10. $15,000.	15. $55,000.	20. $125,000.
6. $7500.	11. $22,500.	16. $75,000.	21. $675,000.
7. $8000.	12. $25,000.	17. $75,500.	22. $275,000.
8. $9000.	13. $32,500.	18. $80,000.	23. $350,000.
9. $9500.	14. $42,500.	19. $87,500.	24. $875,000.

25. Taxes in a certain city are payable in semiannual installments, May 1 and Nov. 1 respectively. If the whole tax is paid May 1 a discount of half a year's interest at the rate of 4% per annum is allowed on the second payment. How much is due on a piece of property assessed at $3000 if the rate is $0.0178 and the whole tax is paid on May 1?

26. A company owns four parcels of land in four different cities, one parcel assessed at $35,000, another at $8000, a third at $20,000, and a fourth at $40,000. If the tax rates in these cities are, respectively, $21.40, $17.80, $22.60, and $19.90 on $1000, what is the total amount of taxes for the year?

TAXING FOR GOOD ROADS

1. The assessed valuation and highway tax rate on a certain farm in Wisconsin for four successive years were as follows:

In the Year	Assessed Valuation		Highway Tax Rate per $1000
	Land	Buildings	
1918	$4600	$1300	$1.56
1919	4600	1300	1.78
1920	5000	1450	2.32
1921	5000	1450	1.96

The increase in the valuation of the land and buildings was due to the construction of good roads. If the roads had not been improved and the valuation of the farm had remained unchanged, what would the total highway tax have been for the four years at the 1918 rate? How much would the owner of the farm have saved on his highway taxes?

2. At the end of 1921, after paying the year's taxes, the owner sold the farm for its assessed valuation in that year as given in Ex. 1. If the assessed valuation in 1918 represented fairly the value of the farm on that date and we consider the increase in value as due wholly to the building of good roads, did the owner gain or lose by their construction? Determine the gain or loss.

3. In the same county in Wisconsin in which the farm in Ex. 1 was located, it cost $24,000 to maintain 60 mi. of macadam road and $1000 for the maintenance of 55 mi. of concrete road. Find the cost per mile of maintaining each type of road.

4. In Ex. 3 the cost of maintaining a mile of the macadam road was what per cent greater than the cost of maintaining a mile of the concrete road?

5. A certain county borrowed $1,500,000 for building good roads, paying the interest and part of the principal each year. For the purpose of making these payments a special tax of 50¢ on each $100 of assessed valuation of the property in the county was voted. If the farms in the county were assessed as worth $25 an acre, how much was this highway tax per acre?

6. If land referred to in Ex. 5 is rented for $6.25 an acre, what per cent of the rent must be paid each year to meet this special highway tax?

7. If a farm in the county referred to in Exs. 5 and 6 was rented at $5 an acre before the good roads were built and at $6.25 an acre thereafter, what was the per cent of gain on the rent of the farm? Considering the special tax paid in Ex. 5, what was the per cent of net increase in the rent of the farm?

8. In the case of the farm referred to in Ex. 7 the taxes per $100 of assessed valuation were as follows: school tax, 30¢; county, state, and township tax, 20¢; total highway tax, $62\frac{1}{2}$¢. The $1.25 increase per acre in rent of the land was how many times as great as the total tax per acre?

9. In a county where the highway tax is $1.62 per $1000, how much must a man pay who owns 160 A. of land assessed at $60 an acre and 80 A. assessed at $75 an acre?

10. In Ex. 9 what is the average highway tax per acre on the entire property?

11. A certain county borrowed $1,750,000 for building good roads. In order to pay the interest on the loan and gradually to pay back the principal, it was decided to levy a tax of $2.50 on each $1000 of assessed valuation of property. If the assessed valuation of Mr. Johnson's land is $6250 and that of his buildings is $2500, how much is his total highway tax?

Expenses of the United States Government. The expenses of the United States government naturally vary from year to year, and the enormous increase due to the war has made it impossible to tell what the expenses will be in the next few years. Whatever they are, it is our duty to raise the money to pay the debts of the government and to join in the general movement on the part of civilized nations to see to it that the entire world agrees to give up war in the future.

Recently the ordinary disbursements of our government averaged about $41,000,000 a day, or about $15,000,000,000 a year. This represents an enormous increase over our expenses before the war. These disbursements will naturally be reduced from year to year. Some items of our income and expenditures were recently as follows:

INCOME, EXCLUDING MONEY BORROWED

Customs (duties on imported goods)	$184,000,000
Internal revenue (income tax, etc.)	3,019,000,000
Miscellaneous	1,943,000,000

EXPENDITURES, EXCLUDING CERTAIN WAR EXPENSES

War Department	$10,225,000,000
Navy Department	1,573,000,000
Pensions	220,000,000
Salaries and other civil expenses	2,232,000,000

Teachers are advised to ascertain some of the amounts of income and expenses for the current year, or to have the pupils do so. The methods of raising money by bonds and taxation should also be stated. Manifestly, no arithmetic can furnish this information, since the methods of taxation, the income of the government, and its expenses will not be settled for years to come.

Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue*, *tariff*, or *duty*.

Customhouse. Customs revenue is collected at *customhouses*. These are situated at *ports of entry*.

Classes of Goods. Goods imported may be on the *free list* and not subject to duty, as raw silk; subject to *ad valorem* (on the value) duty, a certain per cent on the value at the place of purchase, as books, on which the duty is 15% ad valorem; subject to *specific* duty, a certain amount per unit of measure, as apples, on which the duty is 10¢ per bushel of 50 lb.; subject to both ad valorem and specific duty, as perfumery, on which the duty is 40¢ per pound and 15% ad valorem. The rates of duty are subject to change by act of Congress.

DUTIES ON IMPORTS

All work oral

State the duty on the following:

1. 700 lb. of firecrackers, duty 6¢ per pound.
2. 1 T. of Roman candles, duty 10¢ per pound.
3. $100 worth of beads, duty 35%.
4. $800 worth of buttons, duty 25%.
5. $200 worth of dolls, duty 35%.
6. $700 worth of furs, duty 40%.
7. $500 worth of fans, duty 50%.
8. $2000 worth of jewelry, duty 50%.
9. $1250 worth of pearls, duty 20%.
10. 250 doz. pairs of gloves, duty $1 per dozen.

USING WHAT YOU HAVE LEARNED

1. How much is the duty at 20% on needles costing $4350? Is this specific duty or ad valorem duty?

2. How much is the duty on 3500 bu. of barley at 15¢ per bushel? Is this specific duty or ad valorem duty?

3. Find the duty on 2750 lb. of hops at 16¢ a pound.

4. How much is the duty at 10% on horses costing $7565?

5. How much is the duty on 9750 bu. of oats at 6¢ a bushel?

6. A dealer imports 375 lb. of maple sugar from Canada. How much is the duty at 3¢ per pound?

7. A grocery house imports from Canada some cheese costing $1250. How much is the duty at 20%?

8. A dealer imports 2750 lb. of butter. How much is the duty at $2\frac{1}{2}$¢ per pound?

9. The duty on honey is 10¢ per gallon. If a dealer imports 975 gal., how much is the duty?

10. A dealer imported 475 T. of straw, paying duty at the rate of 50¢ per ton. How much duty was paid?

11. How much is the duty at 15% on 15 sets of Thackeray's works which cost £7 a set in England, allowing the normal rate of gold exchange, $4.87 to the pound?

12. How much is the duty at 15% on 2 doz. sets of Scott's works which cost £4 10s. a set in England, allowing $4.87 to the pound as in Ex. 11?

13. How much is the duty at 50% on 2000 yd. of carpet invoiced at $1.50 a yard?

14. The duty on cigars is $4.50 per pound, plus 25% ad valorem. If the consignment weighs 275 lb. and costs $825, how much is the total duty?

Income Tax. The chief part of the money needed to run the United States government is raised by a tax on incomes. Every citizen of the United States, whether residing at home or abroad, and every person residing in the United States must pay the government a certain per cent of his income above a certain amount. This is called the *normal rate.* Persons whose incomes are large have to pay excess taxes, the rate of taxation increasing as the amount of income increases.

The World War made such enormous demands upon the resources of our government that the rate of taxation on incomes has increased very rapidly within the last few years, and it is not probable that the rate will become standardized again for a number of years to come.

For example, if a man has an income of $7500 a year, of which $2000 is exempt from taxation on account of his being the head of a family, and if the normal rate is 2%, his normal income tax is 2% of $5500, or $110.

$7500
2000
$5500
0.02
$110

Various states also levy income taxes.

The details of the income tax change so rapidly and the rates vary so greatly that the school cannot be expected to give attention to them. All that is essential is that the pupils shall understand why such a tax is raised, and in a general way how it is computed. The rates given in the exercise are mere possibilities. It is not necessary to go into details concerning surtaxes or exemptions. These details do not affect the mathematical treatment, and the pupil would not understand them, under present conditions, even if they were presented.

It may be stated to the pupils that a considerable part of our national revenue comes from taxes on tobacco. This, together with the income tax and certain other taxes, constitutes the *internal revenue* of the government. In addition to this a large amount comes from the sale of postage stamps, but this is all used to pay the expenses of carrying the mails and maintaining the post offices.

INTERNAL REVENUE

1. If a man has an income of \$8475.50, of which \$1000 is exempt from taxation, find his normal income tax at 2%.

2. In a certain year a special government tax dropped from \$89,000,000 to \$62,000,000. Find the per cent of decrease.

3. When the rates were raised during the war, the income-tax receipts increased in one year from about \$360,000,000 to about \$4,000,000,000. Find the per cent of increase.

4. If a corporation had a net income of \$1,374,285 in a certain year, and if the normal rate of income tax was 2%, how much was the normal tax paid by the corporation?

5. A man had a net income of \$23,000 in a year in which the law required him to pay 4% as a normal income tax on the first \$4000 of his income above \$2000; 8% on the rest of his income above this amount; 1% on the first \$1000 above \$6000; 2% on the next \$2000; 3% on the next \$2000, and so on. Compute his total income tax.

Owing to the rapid changes in the tax laws no such problem should be taken as a standard. This is merely a type of problem that is likely to arise in the computing of income taxes.

6. A certain quality of cigar is taxed \$3 per 1000, and an extra tax of \$7 per 1000 is levied. Find the total tax on an output of 275,000 cigars.

7. A certain quality of cigarette is taxed \$3.60 per 1000, and an extra tax of \$1.20 per 1000 is levied. Find the total tax on an output of 3,250,000 cigarettes.

8. When tobacco is taxed at the rate of 8¢ per pound, with an extra tax of 5¢ per pound, what is the total tax on an output of 2,758,500 lb. of tobacco?

The Post Office. We come into close relation with our United States government more frequently through the post office than in any other way, and there is certain information that everyone needs to know about the ordinary matters relating to the mailing of letters, papers, and parcels, and to money orders. Some of the most important details are as follows:

In general, the rate is 2¢ per ounce on letters for the United States, Canada, Great Britain, Ireland, and certain other countries about which information should be asked at the post office. All fractions of a stated weight count as the full weight.

The rate on letters to all countries to which it is not 2¢ per ounce is 5¢ for the first ounce and 3¢ for each additional ounce.

On postal cards the rate to places in this country is 1¢; to most foreign countries it is 2¢.

On newspapers and periodicals the rate to places in this country is 1¢ for each 4 oz.; to most foreign countries it is 1¢ for each 2 oz.

On other printed matter the rate to places in this country or abroad is 1¢ for each 2 oz.

A special delivery stamp, for use in this country, costs 10¢.

A letter may be registered for 10¢, insuring greater security.

Parcel post allows for the sending of parcels weighing as high as 50 lb., and within a distance of 300 mi. as high as 70 lb. The cost of sending by parcel post varies from time to time, and inquiry should be made at the post office. Parcels may be insured up to $100 each on payment of a small fee.

The subject of money orders was discussed on page 181 and that of postal savings on page 156.

The above facts are of such value as pieces of general information that pupils should be familiar with them. The parcel post rates and the statistics given on page 205 should not be memorized.

Parcel Post. For the purpose of the parcel post (fourth-class mail) the country is divided into zones or rings, the center being the place at which the parcel is mailed. Roughly speaking, the first zone includes distances up to 50 mi.; the second, from 50 mi. to 150 mi.; the third, from 150 mi. to 300 mi.; the fourth, from 300 mi. to 600 mi.; the fifth, from 600 mi. to 1000 mi.; the sixth, from 1000 mi. to 1400 mi.; the seventh, from 1400 mi. to 1800 mi.; the eighth, over 1800 mi. The rates for small parcels in the first few zones were recently as follows, but are subject to change:

Pounds	Local	Zone 1, 2	Zone 3	Zone 4	Zone 5	Zone 6
1	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09
2	.06	.06	.08	.11	.14	.17
3	.06	.07	.10	.15	.20	.25
4	.07	.08	.12	.19	.26	.33
5	.07	.09	.14	.23	.32	.41
6	.08	.10	.16	.27	.38	.49
7	.08	.11	.18	.31	.44	.57
8	.09	.12	.20	.35	.50	.65
9	.09	.13	.22	.39	.56	.73
10	.10	.14	.24	.43	.62	.81

The pupils should consult this table in the work on page 206.

Expenses of the Post Office. In a recent year there were 53,084 post offices in the United States. The total length of all the postal routes was 455,439 mi. The postmasters received a total of about $33,600,000. The entire income of the Post Office Department was about $436,000,000, and the expenditures were about $363,000,000. The number of ordinary postage stamps issued was more than 15,000,000,000.

THE POST OFFICE

Numbers 1 to 15, oral

State the postage on each of the following:

1. A letter to Chicago, weight $\frac{3}{4}$ oz.; weight $1\frac{3}{4}$ oz.

A letter weighing $\frac{3}{4}$ oz. costs the same as a letter weighing 1 oz.

2. A letter to San Francisco, weight $1\frac{1}{4}$ oz., special delivery.

This costs the same as a special-delivery letter weighing 2 oz.

3. A letter to Boston, weight $2\frac{1}{4}$ oz., registered.

4. A letter to New Orleans, weight $3\frac{1}{2}$ oz., registered and special delivery.

5. A letter to France, weight 1 oz., registered.

6. A newspaper to Galveston, weight 3 oz.

7. A printed pamphlet to Sacramento, weight 2 oz.

8. A parcel from Baltimore to Washington, weight 8 lb.

Estimate the distance by consulting a map. Consult page 205 for the rate.

9. A parcel weighing 6 lb., sent to a place in the fifth zone.

10. A parcel weighing 9 lb., sent to a place in the sixth zone.

11. A letter weighing $1\frac{3}{4}$ oz., registered, sent to a place in Italy.

12. A pamphlet weighing 3 oz., sent to a place in South Africa.

13. A letter weighing $2\frac{1}{2}$ oz., sent to a place in Holland.

14. A letter weighing $\frac{3}{4}$ oz., sent to a place in England.

15. A parcel weighing 8 lb., sent to a place in the local zone.

16. In a year in which our population was 105,000,000 we used 15,400,000,000 ordinary postage stamps. Find the average number of stamps used by each person living in this country.

Such figures are, of course, only approximate, but the problem gives some idea of the enormous business done by the Post Office Department.

Government Borrowing Money. In addition to using the taxes levied by cities, villages, counties, states, and the general government, as already described, these governing bodies often borrow money, sometimes on notes but more often on printed promises to pay, known as bonds.

The precise nature of a bond will be considered later.

The United States could borrow money before the war at from 2% to 3%, and states could borrow at about 4%, while cities, counties, and villages paid a little more. The enormous increase in the expenses of the government due to our participation in the World War has raised these rates of interest considerably.

Cities often raise money by special assessments; that is, if a street is to be paved with asphalt the city will require owners of property fronting on the street to pay a special assessment in addition to their ordinary taxes because of the increase in the value of the property affected by the improvement of the street.

GOVERNMENT BORROWING MONEY

1. In a certain suburb of Atlanta, with an assessed valuation of $650,000, it was decided to expend $12,500 for a school building, levying a tax of 5 mills on a dollar to pay part of the expense and borrowing the balance at 5%. How much is the annual interest charge on the money borrowed?

2. When our government borrowed $2,000,000,000 at $3\frac{1}{2}$%, how much was the annual interest charge?

3. When our government borrowed $4,000,000,000 at $4\frac{1}{4}$%, how much was the annual interest charge?

4. When our government borrowed $6,000,000,000 at $4\frac{1}{4}$%, how much was the annual interest charge?

LIBERTY AND VICTORY BONDS

1. In June, 1917, our government issued \$2,000,000,000 in $3\frac{1}{2}\%$ bonds, this being the First Liberty Loan. These bonds were oversubscribed by a little more than 50%. What does this mean? How many dollars of bonds were oversubscribed?

2. The government issued \$3,808,766,150 in bonds, this being the Second Liberty Loan of 1917. These bonds bear 4% interest. How much is the annual interest on these bonds?

Some of the bonds in each of the first two Liberty Loans were exchanged for later bonds, but these details need not concern the schools.

3. The Third Liberty Loan, issued in May, 1918, amounted to \$4,176,516,850 and bears $4\frac{1}{4}\%$ interest. Find the amount of interest paid annually by the government.

4. The Fourth Liberty Loan was issued in October, 1918, and amounted to \$6,989,047,000, bearing interest at $4\frac{1}{4}\%$. Find the amount of interest paid annually by the government.

5. The Victory Loan, issued in May, 1919, amounted to \$4,500,000,000. Part of the bonds bear interest at $4\frac{3}{4}\%$ and part bear interest at $3\frac{3}{4}\%$. If the average rate is $4\frac{1}{4}\%$, how much is the annual interest charge?

6. If a man owns \$5000 in Victory Loan bonds of the $4\frac{3}{4}\%$ kind, how much interest does he receive semiannually?

7. In Ex. 6 how much more interest does the man receive semiannually than he would have received if he had invested the \$5000 in the First Liberty Loan?

8. If a man owns \$8500 in bonds of the Fourth Liberty Loan, how much interest does he receive semiannually?

9. If a man owns \$12,500 in bonds of the First Liberty Loan, how much interest does he receive annually?

REVIEW OF GOVERNMENT FINANCE

1. If the tax rate in a certain city is 19 mills on $1, what is the tax of a man whose property is assessed at $12,500?

2. The assessed valuation of a certain city is $37,500,000 and the total tax levied is $862,500. Find the tax rate, expressing it as so many dollars on each $1000 of assessed valuation.

3. If the tax rate in a certain city is $1.67 on $100, what is the tax on property assessed at $17,500?

4. A man left an estate worth $37,500 and on this the state levied an inheritance tax of 3%. How much was left of the estate after the tax was paid?

5. If the tax rate is 17.8 mills on $1, find by the table on page 195 the tax on property assessed at $24,000.

6. A Wisconsin farmer owns 120 acres of land assessed at $120 an acre. The buildings are assessed at $3500. He pays a highway tax of $37.59. What is the rate of highway tax per dollar of assessed valuation?

7. If the tax rate in a certain city is 14.7 mills on $1, what is the rate per cent of tax?

8. If the tax rate in a city was 1.4% last year and is 1.8% this year, what is the per cent of increase upon last year's tax rate?

9. During the last year of the World War our government appropriated $10,225,478,312 for the army, $1,573,468,415 for the navy, $2,814,348,267 for forts and fortifications, $2,515,005 for the Military Academy, $4,315,272,038 for war expenses, and $3,000,000,000 to help countries associated with us in war. In that year our population was estimated as 104,000,000. Find to the nearest dollar the amount of burden that the total of these items imposed, on the average, upon each person.

III. LITTLE EXAMINATIONS

i. 1. Disregarding interest, \$1.25 saved a day amounts to how much in 5 yr. if one of those years is a leap year?

At simple interest, at interest compounded annually, and also at interest compounded semiannually, find the amount of:

2. \$3850, 4 yr., 5%.
3. \$4850, 5 yr., 5%.
4. \$12,250, 4 yr., 4%.
5. \$23,750, 5 yr., 4%.

ii. 1. Write a 30-day note for \$1250, with interest at 6%.

Find the discounts and the proceeds on the following:

2. \$1275, 2 mo., 6%.
3. \$7550, 3 mo., 6%.
4. \$175.50, 30 da., 6%.
5. \$1275.80, 30 da., 5%.

iii. *At the rate of $5\frac{1}{2}$ mills find the taxes on the following:*

1. \$3000.
2. \$5000.
3. \$7500.
4. \$15,000.

Find the taxes on \$14,000, at the following rates:

5. 6.2 mills.
6. $17\frac{1}{2}$ mills.
7. 8.1 mills.
8. 18.3 mills.

Find the duties on goods valued as follows:

9. \$17,500, 15%.
10. \$25,750, 25%.

iv. 1. Find the cost of a draft for \$1250 at 0.1% premium.

Find the cost of each of the following drafts:

2. \$1280, at par.
3. \$1280, 0.2% premium.
4. \$13,750, 0.1% discount.
5. \$26,750, 0.2% discount.

v. 1. Write a 60-day note for \$1500, with interest at 5%.

Find the interest on:

2. \$675, for 90 da., at 6%.
3. \$540, for 60 da., at 5%.
4. \$824, for 30 da., at $4\frac{1}{2}$%.
5. \$1860, for 60 da., at 4%.

IV. REVIEW AND DRILL

MINIMUM ESSENTIALS

1. Find the cost of a draft for each amount in columns I and III below at each rate of exchange, in turn, in columns II and IV.

	I	II	III	IV
a	\$275	0.1% premium	\$217.50	0.3% premium
b	\$450	0.2% discount	\$316.75	$\frac{1}{20}$% premium
c	\$1250	0.1% discount	\$425.25	$\frac{1}{20}$% discount
d	\$1500	0.2% premium	\$675.75	0.4% discount
e	\$1875	0.3% discount	\$842.36	0.4% premium

This gives 100 examples. The teacher should select as many as necessary.

2. Find the income tax on \$7286.90 at 4%.

3. Find the tax on \$12,750 at 2.31 mills on a dollar.

4. Find the annual interest paid by the government on a loan of \$6,000,000,000 at $4\frac{1}{4}$%.

In each of the six columns given below perform the operations of Exs. 5–20, carrying each decimal to the nearest hundredth:

	I	II	III	IV	V	VI
a	$\frac{1}{32}$	0.23	41.3	1.276	$6\frac{1}{4}$	12,250
b	$\frac{1}{16}$	0.32	62.1	2.007	$7\frac{3}{5}$	13,175
c	$\frac{1}{8}$	0.41	67.9	3.982	$8\frac{2}{3}$	16,022
d	$\frac{5}{32}$	0.68	71.8	4.871	$9\frac{1}{6}$	19,685

5. $a + b$.
6. $b - a$.
7. $a + d$.
8. $a + c$.
9. $d - a$.
10. $c \div a$.
11. $c \times a$.
12. $b \times d$.
13. $b + c$.
14. $c - b$.
15. $c + d$.
16. $d - c$.
17. $a \times b$.
18. $b \div a$.
19. $a \times d$.
20. $d \div a$.

PROBLEMS WITHOUT NUMBERS

All work oral

1. How do you ascertain the interest due on the first day of the year on an account in a savings bank?

2. Which pays the better interest at the same rate if the money is left undisturbed for five years, a promissory note or deposit in a savings bank? Why?

3. How do you fill out a deposit slip? After entering the items, what operation do you perform? How do you make sure that the result is correct?

4. How do you find the discount on a promissory note?

5. If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

6. How can a manufacturer discount a claim against a dealer if the claim is not yet due? How is the discount found?

7. If you know the proceeds of a note and the rate of discount, how do you find the face of the note?

8. If you know the face of a note and the proceeds, how do you find the discount?

9. If you know the face of a note and the proceeds, how do you find the rate of discount?

10. If you change money from a postal savings bank to a government bond, how much higher rate of interest do you get?

11. If a dealer buys some goods and agrees to pay for them in 90 da., with interest at 6%, and the seller takes a note and at once discounts it at 5%, will he receive more than the selling price, or the same amount, or less? Why?

PROBLEMS FOR COMPLETION

1. Suppose that it costs \$1.92 per square yard to build a macadam road. A road is 16 ft. wide and $4\frac{3}{4}$ mi. long.

2. A Michigan farmer has 3100 bu. of wheat to haul to the railroad. Before they had good roads on the route it cost 7.1¢ a bushel to haul it. Over the good roads that they now have it costs only 3.3¢ a bushel.

3. The duty on paint of a certain kind is 20% ad valorem. A man imports a shipment of paint and pays \$1750 duty.

4. The duty on butter is $2\frac{1}{2}$¢ per pound. A dealer imports 3700 lb. of butter.

5. In a certain year, according to the government reports, the wholesale and retail trade of this country yielded personal incomes amounting to \$6,026,670,809, and the expenses of the wholesale and retail trade amounted to \$5,262,829,124.

6. A man has an income of \$9575. He is allowed \$2000 exemption from taxation. The normal income tax is 2%.

7. John wishes to send his camp kit by parcel post to his cousin, who lives in the fifth zone (600 mi. to 1000 mi.). The kit, when packed, weighs 47 lb. The rate for 40 lb. is \$2.42, and it increases 6¢ for each pound.

8. Fred's father bought \$4500 worth of bonds of the Victory Loan, taking those that paid $4\frac{3}{4}$%.

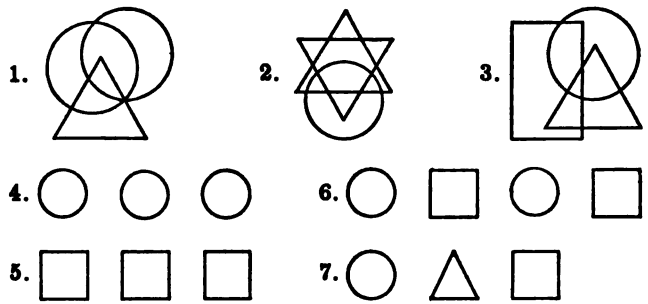
9. Mr. James owns a farm assessed at \$13,500. The tax rate is 7 mills on a dollar.

10. A wholesale dealer pays \$2750 for some fruit and sells it for \$3300.

11. In Ex. 10 the dealer's expenses are \$180 for shipping, selling, and other overhead.

ALERTNESS TEST

Copy the following figures on paper, and at a given signal by the teacher work rapidly and do precisely what is directed:



The teacher should give once and only once all directions for each test, using directions different from the following specimens, so that the pupils cannot study them in advance. Score as directed on pages 82 and 144.

In No. 1 write 3 in the space which is in the right-hand circle but not in the triangle or the other circle; also write 4 in the space which is in the triangle and one circle but not in both circles. Allow 5 sec.

In No. 2 write 4 in any space which is in one and only one of the two large triangles and also in the circle; also write 6 in the space which is in the circle but not in either triangle. Allow 5 sec.

In No. 3 write 7 in a space which is in the triangle and rectangle but not in the circle; also write 9 in the space which is in the circle and triangle but not in the rectangle. Allow 5 sec.

In No. 4 write the second letter of the word *number* in the first circle, the first letter of the same word in the second circle, and the third letter from the last in the third circle. Allow 5 sec.

In No. 5 write 1 in the third square, 3 in the first square, and half their sum in the second square. Allow 5 sec.

In No. 6 write 8 in every circle that comes after a square and write 7 over every circle that comes before a square. Allow 5 sec.

In No. 7 write 7 in the triangle, 6 in the circle, and twice their product in the square. Allow 5 sec.

Industrial Problems

MONEY IN FERTILIZERS

1. George's big brother came home from the agricultural college and decided that on their farm they ought to try more modern methods of feeding the plants. He told George that the plants needed nitrogen, phosphoric acid, and potash, and that if these are put into the soil by means of fertilizers, the plants grow better. George's brother tried applying \$5.16 worth of fertilizer to one acre of a field of corn, and the increase in the yield was 8.6 bu. Corn being worth 90¢ a bushel at the farm, what was the value of the increase? The value of the increase was how much more than the fertilizer cost?

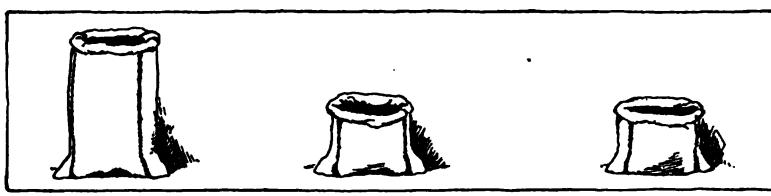
2. George's brother tried another kind of fertilizer on another acre of corn, paying \$7.64 for this kind. He increased the yield 7.2 bu. If corn was then worth 90¢ a bushel at the farm, was this experiment profitable or not? Find the gain or loss.

3. George's brother knew of another fertilizer, said to be excellent for wheat, and put \$14.26 worth on an acre. The increase in yield was 8.4 bu., and wheat was then selling at the farm for \$2.12 a bushel. What was the gain or loss in this case?

4. George's brother explained to him that a complete fertilizer is one containing all three of the foods that a plant needs. These are nitrogen, phosphoric acid, and potash. A formula written 2–8–2 means that this fertilizer contains 2% nitrogen, 8% phosphoric acid, and 2% potash. How many pounds are there of each of these foods in a ton of 2–8–2 fertilizer?

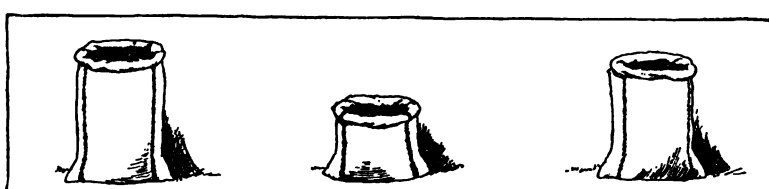
In trade, the order of the formula given in Ex. 4 is often stated thus: nitrogen, potash, phosphoric acid, instead of nitrogen, phosphoric acid, potash. In this book the order recommended by experiment stations and given in Ex. 4 is followed. Pupils should inquire as to local customs.

5. George's brother showed him the picture below, representing different yields of wheat, and asked George to find how many pounds of nitrogen, phosphoric acid, and potash there are in 300 lb. (the amount used per acre) of the fertilizer that gave the 35-bushel yield. How many pounds are there of each?



4–8–4 Formula	Unfertilized	4–0–4 Formula
35 bu. per acre	15 bu. per acre	17 bu. per acre

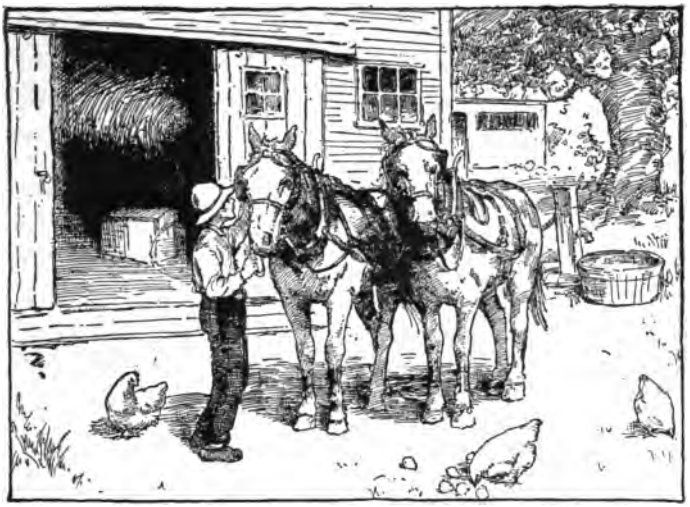
6. In the same picture what ingredient is omitted in the 17-bushel formula? How many pounds of each of the other plant foods are there in 300 lb. of this fertilizer? How many in a ton?



4–8–0 Formula	Unfertilized	0–8–4 Formula
33 bu. per acre	15 bu. per acre	30 bu. per acre

7. In the above picture, which also represents different yields of wheat, how many pounds of each of the three plant foods are there in 300 lb. of each fertilizer?

Teachers in rural schools should talk with practical farmers on this subject and get the pupils to do the same. Different localities have different soils, and hence the question of fertilizers is to some extent a local one.



GETTING HORSES READY FOR MARKET

1. To one lot of horses a dealer fed corn. In this case the average weight increased from 1436 lb. to 1674 lb. in 112 da. What was the average increase per day?

2. To another lot he fed three parts corn to one of oats, and the average weight increased from 1468 lb. to 1734 lb. in 112 da. What was the average increase per day?

3. To a third lot he fed half corn and half oats, the average increase being from 1463 lb. to 1736 lb. in 112 da. What was the average increase per day in this case?

4. Calling the result found in Ex. 1 the standard, say 1, what numbers represent the results in Exs. 2 and 3?

Answer to two decimal places, that is, to the nearest hundredth.

Examples like these, representing scientific experiments, not only give the pupils exercise in computation but also give them some idea of the way in which the government is trying to solve the problem of how to improve stock.

FARM PROBLEMS

1. In 1 T. of clover hay there are 39 lb. of nitrogen, 13 lb. of phosphoric acid, and 44 lb. of potash. Find the per cent of each.

This is practically the same as finding the number of pounds of each in 100 lb. of clover hay. State each result to the nearest tenth of 1%.

As in all special vocations of this kind, the problems may be omitted if the teacher feels that they are not such as relate to the pupils' experiences.

2. In 1 T. of corn stover there are 12 lb. of nitrogen, 9 lb. of phosphoric acid, and 39 lb. of potash. How many pounds of potash are there in 27 T. of corn stover?

3. In Ex. 2 what is the potash worth at $7\frac{1}{4}¢$ a pound?

4. A man sprayed the vines in his vineyard with Bordeaux mixture. This mixture contains 4 lb. of blue vitriol (copper sulphate) and 4 lb. of stone lime to 50 gal. of water. If he used 200 gal. of Bordeaux mixture, how much did the blue vitriol cost at 7¢ a pound and how much did the stone lime cost at $\frac{3}{4}¢$ a pound?

5. A farmer has a potato field of 6 A. which he sprays with Paris green solution. This solution is made by mixing 4 oz. of Paris green with 50 gal. of water. If he uses 1500 gal. of the solution in a season, how much does the Paris green cost at $37\frac{1}{2}¢$ a pound?

6. In Ex. 5 the farmer raised 180 bu. of potatoes per acre, worth \$1.60 a bushel, but experience shows that if he had not sprayed he would have raised only 148 bu., worth \$1.50 a bushel. How much were his receipts increased by spraying?

7. If one man raises 160 bu. of potatoes per acre and another raises 176 bu., the second man raises what per cent more than the first? The first man raises what per cent less than the second?

CHAPTER IV

I. BUSINESS RELATIONS

Nature of Insurance. If your home should burn, the loss would be very great to you and your family. A long time might be needed to save the money required to buy or build a new home. A sudden calamity like this means a great loss to those who bear it. For this reason people sometimes join together and contribute each a small sum annually. Then if the home of any one of them burns, they pay him enough to fit up a new home from the money they have collected.

This shows the general nature of insurance. Usually, however, instead of a number of persons joining together, they pay annually each a small sum, or *premium*, to an insurance company which agrees to pay for some specified loss.

Insurance. A written agreement to compensate anyone for some specified loss is called *insurance*. The written agreement of an insurance company to pay a certain amount in case of loss is called a *policy*. The sum of money specified to be paid in case of loss is called the *face of the policy*.

The teacher should encourage the pupils to make inquiry as to the insurance of houses and of various forms of property in the vicinity and to report to the class. In particular, the value of life, accident, and health insurance, of fire insurance, and of plate-glass insurance should be mentioned. The workmen's compensation act affords a kind of insurance, and the leading features should be ascertained, particularly with reference to the industries of the city or village in which the pupils live.

Fire Insurance. The money paid for insurance is called the *premium.* The *rate of premium* in fire insurance is usually stated as a certain sum for each \$100 of the face for a certain period of years, generally from 1 yr. to 3 yr.

For example, what is the premium for insuring a store for \$4000 against loss by fire, at \$1.20 a year?

The premium is $40 \times \$1.20$, or \$48.

FIRE INSURANCE

All work oral

1. What is the premium at 80¢ on a policy for \$1000?
2. What is the premium at 60¢ on a policy for \$3000?
3. A man paid \$50 for insuring his house, the rate being \$2.50. What was the face of the policy?

State the premiums on the following policies at the rates specified:

4. \$1000, 50¢.	8. \$1750, \$1.	12. \$3500, \$1.20.
5. \$2000, 80¢.	9. \$3750, \$2.	13. \$6000, \$2.25.
6. \$1250, \$1.	10. \$725, \$2.	14. \$10,000, \$0.80.
7. \$4250, \$2.	11. \$825, \$2.	15. \$50,000, \$1.50.

State the faces of the policies, given these premiums and rates:

16. \$25, \$2.50.	18. \$40, \$2.	20. \$13.75, \$1.
17. \$17.50, \$1.	19. \$80, \$1.	21. \$18.50, 50¢.

State the rates, given these faces of policies and premiums:

22. \$5000, \$25.	24. \$5000, \$50.	26. \$7500, \$150.
23. \$6000, \$60.	25. \$4000, \$80.	27. \$6500, \$130.

Teachers will recognize the fact that Exs. 16–27 are of little practical value except as exercises in oral computation.

FIRE INSURANCE

1. At \$0.95, what is the premium on a \$2500 policy?

2. At \$1.10, what is the premium on a \$2800 policy?

3. At \$1.15, what is the premium on a \$3750 policy?

4. A building worth \$12,000 is insured for $\frac{2}{3}$ of its value at \$1.10. What is the premium?

5. If you insure the contents of your store for \$6000, what is the premium at \$1.25?

6. The premium at \$1.50 for insuring a stock of goods is \$52.50. What is the face of the policy?

7. A man insured his factory, valued at \$135,000, for $\frac{4}{5}$ of its value, at \$1.90. What was the premium?

8. A dealer insured his stock of goods, valued at \$14,000, for $\frac{7}{8}$ of its value, at \$1.65. What was the premium?

9. If a three-year policy for \$1800 costs \$29.70, what is the rate of premium for the three years? What is the rate per year?

10. If a three-year policy for \$3000 costs \$36, what is the rate of premium for the three years? What is the rate per year?

11. A merchant insures his stock for $\frac{3}{4}$ of its value, at \$1.75. The premium is \$131.25. What is the value of the stock?

12. A manufacturer insures his factory for $\frac{5}{8}$ of its value, at \$2. The premium is \$210. What is the face of the policy? What is the value of the factory?

13. A factory worth \$33,000 is insured for $\frac{5}{6}$ of its value, at \$2. If a fire caused a total loss, how much would the owner lose, including the premium paid?

14. A man insured his library for its full value, \$2500, at \$1.25. What was the premium? If a fire destroyed $\frac{3}{5}$ of the value of his library, how much could he recover?

Life Insurance. If a man is careful as to the future, when there will probably be persons dependent upon him, he will wish to insure his life. Then if he should die while the policy is in force, the insurance company will pay the face of the policy to the *beneficiary*, the person to whom he has directed the money to be paid. The younger a man is when he takes out his insurance, the lower will be the premium, and the sooner he will cease to make payments if he takes out a policy on which he pays premiums for a definite number of years.

Premiums in life insurance are stated at so much on $1000. They vary with the age of the person insured, reckoned at the time he takes out his policy.

LIFE INSURANCE

All work oral

State the premium on each of the following policies, the premium on $1000 being given:

1. $2000, $26.
2. $3500, $25.
3. $2000, $21.
4. $4500, $30.
5. $3000, $22.
6. $10,000, $29.
7. $30,000, $32.
8. $20,000, $27.
9. $5000, $28.40.
10. $2500, $22.20.
11. $1000, $27.65.
12. $12,000, $25.

State the face of each of these policies, given the premium and the premium on $1000 as follows:

13. $131, $26.20.
14. $55, $27.50.
15. $96.60, $32.20.

State the premiums on $1000, given the policies and premiums:

16. $2000, $30.
17. $4000, $96.
18. $5000, $105.50.

Teachers will recognize the fact that Exs. 13–18 are of little practical value except as exercises in oral computation.

Kinds of Policies. The following are four of the leading kinds of life-insurance policies:

Ordinary Life, the insured agreeing to pay during his life a certain premium, usually annually or semiannually.

The rate is always given as the cost of insurance for $1000; that is, the rate $25.50 means that the annual premium on a policy for $1000 is $25.50. Some insurance companies allow *dividends* each year, thus reducing the premiums.

Limited Life, the premiums being payable for some fixed number of years, as twenty, after which the policy is called *paid up*, but the face is not paid until the death of the insured.

Endowment, the premiums being paid for some fixed number of years, as twenty, ten, or fifteen. At the end of this time many companies give the insured the choice of receiving the face of the policy in cash or a paid-up policy payable at his death.

In limited life and endowment insurance, since there is a limited period during which the premiums must be paid, and since the insurance may be paid before the last premium is due, the premiums are higher than on an ordinary life policy.

Term Insurance, the premiums being paid for a specified length of time and the face of the policy being payable if the insured dies within the term of insurance.

Thus, a man may insure his life or his health for thirty days, as in certain forms of accident insurance. He may also insure his life for a certain number of years only. The principle of accident and health insurance is so nearly that of ordinary life insurance that an extended discussion is not necessary.

There are various other forms of policies, but most of them are modifications of the above types. Some are arranged so that a certain amount will be paid annually (an *annuity*) during the lifetime of the one for whose benefit they are written.

LIFE INSURANCE

1. How much is the annual premium at $102.60 that a man must pay on a 10-year endowment policy for $4000? How much would he pay in the 10 yr., not considering the loss in interest on the several payments?

2. If a young man takes out at $27.39 a 20-payment policy for $5000, how much, not considering interest, will he have paid when the policy matures; that is, after 20 payments?

3. How much is the premium on a 20-payment policy for $5000, taken out by a man at such an age that the rate is $34.20? How much are the total premiums for the 20 yr., not considering the loss in interest on the several payments?

4. If a man takes out a $2500 policy in one company at $26.40 and a $3500 policy in another at $23.50, how much does he pay each year for insurance?

5. A young man takes out a $10,000 policy on the 25-payment plan at $26.36. How much is the annual premium? How much are the total premiums, together with simple interest at 6% on each premium to the end of the 25 yr.?

6. A man took out a $5000 policy at $26.40. He died just before the eighth annual payment was due. How much did the beneficiary receive above what the man had paid in premiums? How much would the man have saved in the 7 yr. if he had put the amount of the premium each year at simple interest at 5%?

7. Twenty years ago a young man took out a 20-payment policy for $5000, paying $136.95 a year. How much has he paid in the 20 yr.? Estimating that the insurance company has had the use of all this amount for the equivalent of 10 yr. at 5%, what is the total amount received by the company?

LIFE INSURANCE

1. If life insurance is taken by a man 21 yr. old, the annual premium on an ordinary life policy is \$19.12 per thousand. What would be the total amount of premiums paid on \$5000 in 20 yr.? What would be the net amount if the company allowed \$77.80 per thousand in dividends during this period?

2. How much more would the policy of Ex. 1 cost if taken out when the man is 32 yr. old, the premium then being \$25.09, and the dividends for 20 yr. being \$99.65 per thousand?

3. If life insurance is taken by a man 21 yr. old, the annual premium on a 20-payment limited life policy is \$28.98, and the dividends allowed by a certain company amount to \$88.23 per thousand in 20 yr. Not considering interest, what does the net cost of a \$3000 policy amount to by the time it is paid up?

4. How much more would the policy of Ex. 3 cost if taken out when the man is 50 yr. old, the premium being \$54.65, and the amount of the dividends for 20 yr. being \$198.60 per thousand?

5. If life insurance is taken by a man 21 yr. old, the annual premium on a 20-year endowment policy is \$48.48, and the dividends amount to \$189 per thousand in 20 yr. What is the difference between the amount paid the company (less dividends) and the amount received in return on a \$5000 policy?

6. A man pays \$50 a year, in advance, for a \$10,000 accident policy. How much does he pay in 10 yr., adding to each payment simple interest at 6% to the end of the 10 yr.?

7. A man pays \$75 a year, in advance, for health insurance. Owing to sickness he draws \$175 the second year. Allowing 6% interest on his premiums to the end of the second year, how much does he draw in excess of his premiums?

Workmen's Compensation Laws. In many states there are laws requiring that employers pay their workmen a certain compensation in case of injuries arising out of employment and in course of employment. The laws of various states specify the industries covered, such as construction or operation of railroads, electrical construction or operation, work in machine shops, in subways, on buildings, and the like. Usually the law does not include, in the list of persons compensated, farm laborers, domestic servants, or public employees.

The law varies in different states, but in one state the compensation includes, in case of death, \$100 for funeral expenses, 30% of the wages of the deceased for the widow during her life, to cease at her death or remarriage, 10% additional for each child until the child reaches the age of 18 yr., and 15% for each orphan until the orphan reaches the age of 18 yr. Wages are considered up to \$150 a month, but not beyond that amount.

This page is merely suggestive to the teacher. In states where there is such a law it is desirable for the teacher to ascertain its provisions. The law relating to disability should, in that case, also be considered.

Usually an employer insures his employees in some company which agrees to pay the compensation required by this law.

Pupils should not be required to memorize the facts mentioned above, but should refer to them in solving the following typical problems.

WORKMEN'S COMPENSATION LAWS

1. A workman's wages are \$28 a week. In case of his death how much does his widow receive per week?

2. In Ex. 1 what is the total amount that will be paid to a child who is exactly 17 yr. old when the father dies?

3. A workman's wages are \$30.60 a week. If he has a disability allowance of $66\frac{2}{3}\%$ for 20 wk., how much will he receive?

Nature of a Corporation. Some boys in the eighth grade have organized a baseball club. There are fourteen boys, and they pay \$100 for uniforms and \$12 for balls and bats. If each of the boys contributes $\frac{1}{14}$ of \$112, he will contribute \$8, and if they make some money from selling tickets to the games, each boy will have $\frac{1}{14}$ of the profits after the expenses are paid.

But some of the boys cannot afford to contribute as much as others, so they divide the \$112 into 224 shares of 50¢ each and sell to each member as many shares as he cares to buy.

James is one of the chief promoters of the club, and he takes 40 shares, thus making his payment \$20. Fred takes 20 shares, and the others take various amounts.

At the first three games the gate receipts are heavy. The boys divide the profits according to the number of shares they hold. Fred tries to buy some of the shares that James owns, so as to get more of the profits, but James will not sell for less than 60¢ a share. He says his stock is now above par.

The boys really formed a *corporation.* The *capital* was \$112. There were 224 *shares* of *stock*, the *par value* of each being 50¢. The profits they divided were *dividends*, and these dividends were so high that the stock went *above par.*

In practical life men form corporations in this way, only they play the game of business instead of the game of baseball.

Corporation. The laws of our various states permit a number of persons who wish to go into business together to organize as one body, called a *corporation.*

Capital. The total amount contributed by the members of a corporation to carry on the business is called the *capital.*

The capital varies with different corporations. Some corporations have very large capital.

CORPORATIONS

All work oral

1. Some boys organize a tennis club. They pay in $150 and issue shares at 50¢ each. How many shares are issued?

2. If William buys 30 shares, how much does he pay?

3. If Frank pays in $10, how many shares does he take?

4. If there are enough profits from renting the courts and from tickets bought by the spectators at the match games so that the boys who bought the stock can sell each share for 55¢, what per cent has the stock increased in value?

5. If the stock goes still higher and becomes 20% above par, how much is each share worth?

6. If a heavy storm injures the courts, and the boys have to raise more money, probably no one will want to buy the stock, although Frank offers some of his for 45¢ a share. This price is what per cent below par?

7. The boys appoint a few of their number to direct or manage the club. These are called directors, and they decide to order a 10% assessment on all stockholders. How much has to be paid on each share of stock?

8. Why is stock sometimes above par and sometimes below?

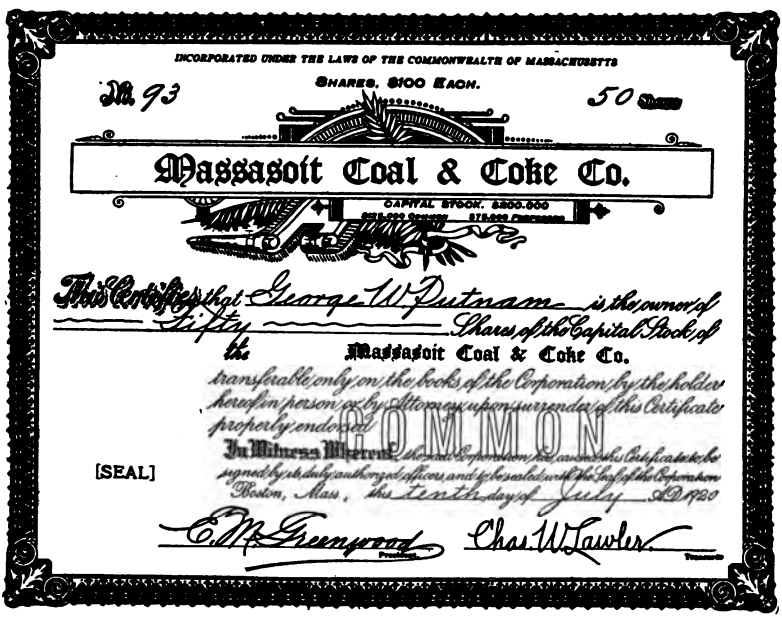
9. Why is it necessary in business to have stock companies, or corporations, instead of having one man furnish the capital?

10. Why is it necessary for the stockholders to select a few men to direct the business instead of managing it themselves?

11. If you own $5 worth of stock in a baseball club and the stock goes 20% above par, how much is it then worth?

12. If you own $5 worth of stock in a tennis club and the stock falls 10% below par, how much is it then worth?

Shares and Dividends. The capital is divided into *shares*, and a person who owns any shares is a *stockholder* and receives a *certificate of stock*. Railway shares are usually $100 each, although sometimes $50. Mining and other shares are often $25, $5, or


INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS

No. 93 SHARES, $100 EACH. 50 Shares

Massasoit Coal & Coke Co.

CAPITAL STOCK, $200,000

This Certifies that George W Putnam is the owner of Fifty Shares of the Capital Stock of the Massasoit Coal & Coke Co.

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

SPECIMEN

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers and its seal to be hereunto affixed.

[SEAL]

Boston, Mass., this tenth day of July A.D. 1920

E. M. Greenwood, President. Chas. W. Lawler, Treasurer.

A Certificate of Stock

$1 each, but the amount varies. In our work the par value of each share is to be taken as $100 unless otherwise stated.

The net earnings of a company are divided among the stockholders in the form of *dividends*. There are often two kinds of stock: *preferred stock*, which pays a fixed rate of dividend if that amount is earned; and *common stock*, which takes whatever dividends may be paid after other claims have been satisfied.

Directors and Officers. The stockholders elect a few of their number to have general direction of the company. These are called *directors*, and they elect the *officers*. These officers include a president, a vice president, a secretary, and a treasurer.

For example, if the capital stock of a corporation is $100,000 and the shares are $100 each, there are 1000 shares. If a man has 50 shares, he owns $5000 worth of stock, and at the election of directors he is entitled to one vote for each share. If this company earns $6000 over all expenses, the directors may declare a dividend of 6%. Then a man who owns $5000 worth of stock will receive a check for 6% of $5000, or $300.

The custom as to preferred and common stock and the par value of stock varies with different companies. The pupils should study these different forms of securities as issued by some local corporation, such as a traction company or a railroad.

CORPORATION PROBLEMS

1. A company divides $50,000 in dividends, and the stockholders receive $4 per share. What is the capital?

2. A company with a capital of $3,000,000 declares a 5% dividend. How much does the holder of 100 shares receive?

3. A company with a capital of $500,000 divides $45,000 in dividends. How much does the holder of 30 shares receive?

4. A company with $50,000,000 capital declares a dividend of $2\frac{3}{4}\%$ every six months. How much money does it distribute annually among the stockholders?

5. If a company with $500,000 capital has earned $11,725.40 above expenses in the last quarter (3 mo.), what rate of dividend can it declare and still carry about $1700 to the next quarter?

6. A company with a capital of $250,000 has earned $15,000 this year above all expenses. It decides to divide in dividends all but $2500. What is the rate of dividend?

Buying and Selling Stock. Stocks are usually bought and sold through a *broker*, who buys and sells the stock in the *stock exchange*, a kind of auction room for such business.

Stock Quotations. A quotation of $118\frac{3}{4}$ means that the *market price* per share in the stock exchange is \$118.75. The fractions used in stock quotations are halves, fourths, eighths, and sometimes sixteenths. If a stock is paying a high rate of dividend, people will pay more than \$100 for a \$100 share, and the stock is said to be *above par*. If a \$100 stock is quoted at 100, it is said to be *at par*, and if below 100 it is *below par*.

Brokerage. The broker charges *brokerage*, or commission, for his services. A charge of \$15 per 100 shares on stocks selling between \$10 and \$125 per share, and \$20 per 100 shares on stocks selling at \$125 per share or over, is added to the cost of the stock at the market price when the broker buys for a customer. When the broker sells stock he deducts the brokerage from the amount received and pays the balance to the customer. There is a minimum brokerage charge of \$1 in all cases.

Different rates of brokerage are in use, but in this book we shall use only the two rates given above. The war tax on the sale of stocks and bonds may be mentioned, but need not be computed.

For example, how much does a man pay for 300 shares of stock quoted at $117\frac{1}{8}$, bought through a broker?

The cost of the stock at the market price is $300 \times \$117.12\frac{1}{2}$, or \$35,137.50. The brokerage is $3 \times \$15$, or \$45, so that the man pays for the stock \$35,137.50 + \$45, or \$35,182.50.

The man who sold the stock received \$35,137.50 − \$45, or \$35,092.50 from his broker. The brokerage on only 6 shares or less of this stock would be the minimum charge of \$1; on 10 shares it would be $\frac{1}{10}$ of \$15, or \$1.50.

BUYING AND SELLING STOCK

Numbers 1 to 6, oral

State the brokerage on the following sales of stock:

1. 100 shares @ $97\frac{7}{8}$.
2. 200 shares @ 137.
3. 500 shares @ $69\frac{3}{4}$.
4. 50 shares @ $87\frac{3}{8}$.
5. 75 shares @ $172\frac{1}{2}$.
6. 3 shares @ 149.

Find the amounts paid on the following purchases of stock bought through a broker:

7. 100 shares @ $87\frac{1}{8}$.
8. 300 shares @ $136\frac{1}{2}$.
9. 400 shares @ $54\frac{3}{8}$.
10. 1000 shares @ $47\frac{7}{8}$.
11. 50 shares @ $142\frac{5}{8}$.
12. 75 shares @ $172\frac{5}{8}$.
13. 125 shares @ $97\frac{3}{4}$.
14. 10 shares @ $119\frac{7}{8}$.
15. 5 shares @ $63\frac{3}{8}$.
16. 60 shares @ 147.

Find the amounts received from the following sales of stock made through a broker:

17. 200 shares @ $127\frac{1}{4}$.
18. 500 shares @ $37\frac{1}{8}$.
19. 600 shares @ $96\frac{3}{8}$.
20. 50 shares @ $99\frac{7}{8}$.
21. 78 shares @ 67.
22. 4 shares @ $158\frac{3}{8}$.
23. 15 shares @ $56\frac{1}{4}$.
24. 2400 shares @ $80\frac{1}{2}$.
25. 40 shares @ $129\frac{5}{8}$.
26. 3 shares @ $88\frac{3}{4}$.

27. A man bought 30 shares of American Locomotive @ $98\frac{3}{4}$, 20 shares of Bethlehem Steel @ $88\frac{1}{2}$, and 5 shares of Union Pacific @ 126. How much did he pay, including brokerage?

28. A man sold 40 shares of General Motors @ $27\frac{5}{8}$, 75 shares of American Sugar @ $127\frac{1}{2}$, and 5 shares of Lackawanna Steel @ $71\frac{1}{4}$. How much did he receive from his broker?

Bond. A printed or written promise to pay a certain sum at a specified time, signed by the maker and, unlike a note, bearing his seal, is called a *bond*. When corporations wish to borrow any considerable amount of money, they usually issue bonds. These bonds generally have small notes called *coupons* annexed, agreeing to pay the interest as due. Bonds, unlike most notes, are usually secured by a *mortgage*, an agreement by which the holders of the bonds may, if the bonds and the interest are not paid when due, sell the property mortgaged.

Difference between Bonds and Shares of Stock. Bonds differ from shares of stock in this way: the stockholders of a corporation are the owners of the property, while the bondholders are owed money by the corporation and so are creditors. Bonds bear a fixed rate of interest, but the income from shares of stock depends upon the earnings of the company after all the expenses of conducting the business have been paid.

Computing Dividends. Stock dividends and bond incomes are always computed on the *face value*, that is, on the par value.

1. How much is the income from bonds of par value \$5000 which pay 4% interest?

The income is 4% of \$5000, or \$200.

2. What is the rate of income from the investment if 10 shares of stock, par value \$100, paying 5% are bought at $112\frac{1}{4}$?

The stock costs \$1122.50 + \$1.50 (brokerage), or \$1124.

The income from the stock is 5% of \$1000, or \$50.

Since we have the product, \$50, and one factor, \$1124, we divide and have

$$\$50 \div \$1124 = 0.044.$$

Hence the rate of income from the stock is 4.4% *on the investment*, but 5% *on the par value.*

Newspaper Quotations. The daily newspapers give the market prices of the leading stocks and bonds. The examples in the next exercise may be solved by using the current stock quotations found in the newspapers or by using the following short list:

At., Top., & S. F.	$84\frac{7}{8}$	United Fruit	$201\frac{1}{8}$
N.Y. Central	$76\frac{1}{2}$	U.S. Rubber	$82\frac{1}{4}$
Union Pacific	$123\frac{1}{2}$	General Electric	$142\frac{5}{8}$

In these examples use the brokerage rates given on page 231 whenever stocks are bought or sold.

The par value of a share of some stocks, like the Pennsylvania R. R., is \$50, and quotations on the New York Stock Exchange are now made on this basis, but such matters are technicalities of the stock market which do not concern the school. In our work, as before stated, consider the par value of a stock as \$100 per share, unless otherwise instructed.

Illustrative Problems. 1. If a man buys 100 shares of Western Union Telegraph at $83\frac{1}{4}$ and sells it at $82\frac{1}{2}$, having received no dividend, how much does he lose?

The man loses \$83.25 − \$82.50, or \$0.75, on each share.

On 100 shares he loses 100 × \$0.75, or \$75.

But he has paid two brokerage charges, one when he bought the stock and one when he sold it.

He pays 2 × \$15, or \$30, in brokerage charges.

His total loss on the transaction is \$75 + \$30, or \$105.

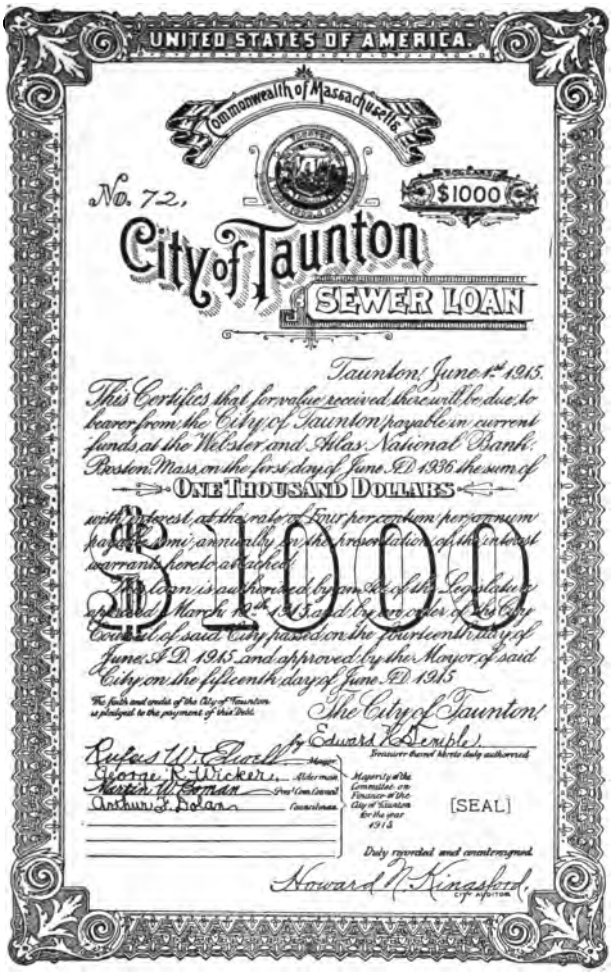
2. If a man buys 1000 shares of American Locomotive at $94\frac{1}{8}$ and sells it later at $97\frac{5}{8}$, having received no dividend, how much does he gain or lose?

The man gains on each share $\$97.62\frac{1}{2} - \$94.12\frac{1}{2}$, or \$3.50.

On 1000 shares he gains 1000 × \$3.50, or \$3500.

The brokerage is 2 × 10 × \$15, or \$300.

His net gain is \$3500 − \$300, or \$3200.


UNITED STATES OF AMERICA.

Commonwealth of Massachusetts

No. 72. $1000

City of Taunton

SEWER LOAN

Taunton, June 1st 1915.

This Certifies that for value received there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston, Mass. on the first day of June A.D. 1936 the sum of

ONE THOUSAND DOLLARS

with interest at the rate of Four per centum per annum payable semi-annually on the presentation of the interest warrants hereto attached.

This loan is authorized by an Act of the Legislature approved March 10th 1915 and by vote of the City Council of said City passed on the fourteenth day of June A.D. 1915 and approved by the Mayor of said City on the fifteenth day of June A.D. 1915

The faith and credit of the City of Taunton is pledged to the payment of this Debt.

The City of Taunton,
by Edward K. Semple,
Treasurer thereof duly authorized.

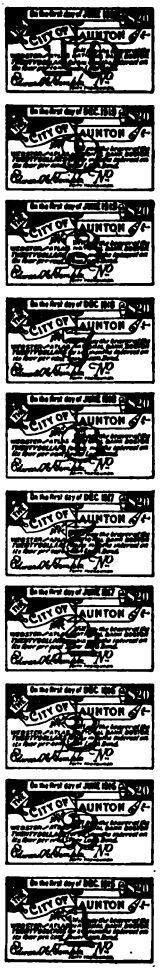
Rufus W. Hurll, Mayor
George R. Dickers, Alderman
Martin W. Carman, Vice Chairman
Arthur L. Dolan, Councilman

Majority of the Committee on Finance of the City of Taunton for the year 1915

[SEAL]

Duly recorded and countersigned.

Howard P. Kingsford, City Auditor.



A Coupon Bond

STOCKS AND BONDS

1. What is the par value of 50 shares of stock, the par value of a share being $100? What is the annual income if the stock pays a dividend of $1\frac{3}{4}\%$ quarterly?

2. If a $100 bond paying 6% is bought for $104, including brokerage, what is the rate of income from the investment?

In actual practice bond yields depend also upon the date of maturity of the bond, but since the computations involved are beyond the scope of arithmetic, this feature should be disregarded in all problems dealing with incomes from bonds. In actual life this feature must not be disregarded.

Find the gain or loss on the following sales of stocks, bought at the quotations given on page 234, no dividends having been received:

3. 100 shares of At., Top., & S. F. @ $87\frac{1}{8}$.

4. 50 shares of N.Y. Central @ $72\frac{3}{8}$.

5. 15 shares of Union Pacific @ $127\frac{3}{4}$.

6. 3 shares of United Fruit @ $197\frac{7}{8}$.

7. 400 shares of U.S. Rubber @ $85\frac{5}{8}$.

8. 150 shares of General Electric @ $119\frac{1}{4}$.

9. If a First Liberty Loan bond, par value $100, paying $3\frac{1}{2}\%$ is bought for $90.25, including brokerage, what is the rate of income from the investment?

10. If a $4\frac{1}{4}\%$ Fourth Liberty Loan bond, par value $100, is bought for $86.80, including brokerage, what is the rate of income from the investment?

11. A man buys 100 shares of stock at $129\frac{3}{4}$. If the stock pays $6\frac{1}{2}\%$ annually, find the rate of income from the investment.

12. At the time the man in Ex. 11 bought his stock, he could have invested $13,000 in a mortgage paying $5\frac{1}{2}\%$, but he preferred the stock. What is the difference in the incomes?

Thrift and Investments. After a man has saved money, either he uses it to pay for something he buys or he invests it. A man feels much more independent if he has saved money for use in case of need or for the purchase of a home.

Formerly people hoarded their savings in hidden places about their homes. It is customary now to invest money that is not needed for immediate use, so that it will not only be safer than when kept at home but will also bring some income. Unfortunately some of the investments offered to the public are offered by dishonest men and are not desirable, but many others are perfectly safe. It is well to remember that the larger the income offered by any form of investment, the greater is likely to be the risk of losing the money put into it. Would it be better to put money into an investment which promises a large income, but which might not be safe, or into something which promises a smaller income, but which is perfectly safe, like Liberty Bonds and Victory Bonds? We should learn how to distinguish between good investments and bad investments so that we may invest wisely whatever money we may be able to save.

THRIFT AND INVESTMENTS

1. If you have \$75 to invest and deposit it on Jan. 1 in a savings bank that pays 2% interest semiannually, how much interest is due you at the end of 6 mo.?

2. If a man has \$1250 to invest and can so invest it as to receive 4% interest, how much will it bring him each year?

3. A man invests his savings in $5\frac{1}{2}\%$ bonds. If the par value of the bonds is \$4000, what is the income from them?

4. A woman has saved \$1750. She can deposit it in a savings bank which pays 4% per annum, compounded semiannually, or can invest it in an oil company which advertises that it will probably pay 10% on the investment, but which her banker advises her is a risky speculation. What is the difference in income for the first year? Which investment should she make?

5. A man has saved \$3500. He can invest it in a well-managed manufacturing business about which he is thoroughly informed and can receive 7% on his investment. Is this a good investment? How much income will he receive a year?

6. A man comes to town and offers investments in a fruit plantation in Mexico. Nobody has heard of the man or of his plantation. He says the business will pay 15%, and he asks Mr. Doe to invest \$7500. How much income does he promise to Mr. Doe? Mr. Doe can invest the money near home and get 6%. How much will this bring him per year? Which investment would you advise him to make? Write the reason.

7. How much more income is received from an investment of \$6500 at 6% than from one of the same amount at $4\frac{1}{2}$%?

Find the income from each of the following:

8. \$400, 5%.	**14.** \$925, $5\frac{1}{2}$%.	**20.** \$8750, 6%.
9. \$475, 5%.	**15.** \$875, $4\frac{1}{2}$%.	**21.** \$8225, 6%.
10. \$575, 4%.	**16.** \$1250, 4%.	**22.** \$6500, 3%.
11. \$725, 4%.	**17.** \$1575, 4%.	**23.** \$7500, $4\frac{1}{2}$%.
12. \$650, 6%.	**18.** \$6250, 5%.	**24.** \$8500, 7%.
13. \$875, 6%.	**19.** \$7500, 5%.	**25.** \$2500, 7%.

26. Mr. Sinclair has \$7500 invested at 6% and \$9000 at 5%. Which investment, if either, brings him the greater income?

27. How does the income from \$9000 at 8% compare with that from \$12,000 at 6%?

28. Mrs. Radcliff has inherited \$2500 and has a chance to invest it at 5% in a first-class security or at 12% in a banana plantation that her banker tells her is very risky. How much would be the income from each? Which should she take?

29. One of the best ways to save money is to save each week a fixed sum, even if small. Many banks conduct Thrift Clubs in which each member deposits each week for 50 wk. a fixed amount, such as 50¢, \$1, \$2, or \$5. The bank pays interest that is equivalent to $1\frac{1}{2}\%$ on the total amount deposited in the 50 wk., and at the end of that time sends to each member a check for his deposits and interest. How large is the check which a member receives if he deposits 50¢ per week? \$1? \$2? \$5?

30. Every year for 5 yr. a young man takes out a \$5 membership in the Thrift Club mentioned in Ex. 29. At the end of each year he deposits his check in a savings bank which pays interest equivalent to $4\frac{1}{4}\%$ simple interest. What is his balance in the savings bank at the end of the 5 yr.?

The purpose of the examples on this page is to show that small regular savings soon amount to a considerable sum. Of course the savings bank pays compound interest, which is an important consideration if money is kept on deposit for several years, but this is approximated here by allowing a slightly higher rate of simple interest. If the bank in Ex. 30 pays 4% compounded semiannually, the balance is \$1375.41 instead of \$1376.55.

31. The young man in Ex. 30 finds that with the money which he has in the bank he can buy $4\frac{1}{4}\%$ Liberty Loan bonds of par value \$1500. If he withdraws his savings and buys the bonds, how much more interest will he receive in a year than if he had left the money in the bank?

32. If the young man in Ex. 31 keeps the bonds for 5 yr., and if then the price has risen so that when he sells them he receives their par value after paying the brokerage, how much more is his total income from the bonds than the interest which the bank would have paid him during the 5 yr.?

33. Considering his total income for 5 yr. from the bonds in Ex. 32, find the rate of return per year from his investment.

Investing in Real Estate. Besides depositing money in banks or purchasing stocks and bonds, people who have saved some money often buy real estate; that is, land and buildings.

The following suggestions should be read and understood:

Real estate often makes a desirable form of investment because of its comparative safety, of the tendency of wisely selected property to increase in value, and of the possibility of using such property for a home for the owner and thus saving some of the expense of renting.

The disadvantages of this form of investment are, first, the expense of maintaining the property, including insurance, taxes, and repairs, together with the loss of interest on the investment; second, the liability to depreciation in value, which is generally greater with houses than with land; and third, the difficulty of selling the property quickly at its full value.

Let each pupil select some piece of property and ascertain the price at which it was held five years ago and its present price. Discuss any changes in the value of the property. Obtain if possible figures on the income from rents, and the cost of insurance, taxes, repairs, etc. If the property is occupied by the owner a fair rental figure should be assumed. Compare the return from the investment with returns from other forms of investment.

INVESTING IN REAL ESTATE

1. A man paid \$4000 for a house and rents it for \$30 a month. He pays \$44 a year for taxes, \$4 for insurance, and \$32 for repairs. What per cent on his investment does he receive?

2. Mr. Davids rents a house in St. Louis for \$45 a month. He pays \$65 a year for taxes, \$30 for insurance and repairs, and \$130 which the city assesses him for paving the street. How much is his net income from the property?

3. A man owns a house which he rents for \$30 a month, but in 3 yr. the house is vacant for 2 mo. The house cost him \$4500, and his annual expenses for taxes, repairs, and insurance are \$87.50. What rate of income does he receive?

4. Mr. St. John bought a house for $4500 and rented it for $28 a month. He paid annually $50 for taxes, $4.50 for insurance, and $45 for repairs. Three years later he sold the property for $5200. How much more did he gain than he would have gained by investing the $4500 for 3 yr. at 6% ?

5. A man bought a house for $4800 and placed on it a mortgage of $2000 on which he has to pay 6%. Allowing $50 a year for taxes, insurance, and repairs, and 5% interest on his investment of $2800, how much does the house cost him each year? If he had not bought the house he would have to pay $30 a month rent for a home. How much does he save per year by owning his home instead of renting?

6. Mr. Young paid $6000 for a house and lot. He rents it for $45 a month. In a certain year he pays $66 for taxes, $6 for insurance, and $48 for repairs. What net per cent on his investment does he receive that year?

7. Mr. Jewett buys a house for $4500, mortgaging it for $2000 at 6%. His annual expenses for taxes, insurance, and repairs are $75, and he rents the property for $28 a month, paying a collector 5% for collecting the rent. After 3 yr. he sells the property for $5200 and pays the mortgage. Is this a better investment than that of putting the same amount of money into a security which would have paid him 6% ? Find the difference in the two incomes.

8. Mr. Carson buys a house and lot for $4200, mortgaging the property for $1800 at 6% and paying the balance in cash. His annual expenses for taxes, repairs, and insurance are $68, and he rents the property for $25 a month. After 2 yr. he sells it for $3000, the buyer agreeing to pay the mortgage. Find the per cent of return from his investment per year.

Budget. A systematic plan for future expenditures, based upon income, is called a *budget.*

One advantage of a budget is that a person, a family, or a corporation has a check upon any sudden desire to be extravagant, thus tending to spend beyond the income for the year. One cause of financial difficulty in the home or in business is that money is often spent freely as it comes in, with little thought of future needs or of emergencies that are likely to arise.

In the case of families experience has shown about what per cent of income may safely be assigned to various lines of expenditures. In a general way the house may take about 30% of a small income, food 30%, clothing, 10%, higher life 5%, health 5%, miscellaneous 10%, and investments 10%, but these figures will vary greatly according to circumstances.

The student should gather data from his own family or other sources and prepare for his own use, but not for discussion in class, a budget for a month. At the end of the month he should see how closely the expenses came to his estimates and should then try to improve upon his budget. After a little experience the budget can be made for a year in advance.

General Divisions of a Budget. There are many ways of making up a budget, but the following divisions are typical:

1. *House*, including rent, heat, light, telephone, taxes, insurance.
2. *Food*, including groceries, meat, milk, fruit, meals bought away from home.
3. *Clothing*, including suits, underwear, shoes, hats, repairs.
4. *Higher Life*, including education, books, papers, magazines, travel, religion, charity.
5. *Health*, including exercise, recreation, bills of physicians and dentists, medicine.
6. *Miscellaneous*, including laundry, presents, small expenses.
7. *Investments*, including savings, household property.

Form for a Budget. The following is one of various convenient forms that may be used in preparing a budget:

I shall allow for my expenses during the next year the following amounts, and I will endeavor to keep within each allowance.

1. *House*

a. Rent			*e.* Taxes		
b. Heat			*f.* Insurance		
c. Light			*g.* Miscellaneous		
d. Telephone			*Total*		

2. *Food*

a. Groceries			*d.* Fruit		
b. Meat			*e.* Meals bought		
c. Milk			*Total*		

3. *Clothing*

a. Suits			*d.* Hats		
b. Underwear			*e.* Small items		
c. Shoes			*Total*		

4. *Higher Life*

a. Education			*e.* Travel		
b. Books			*f.* Religion		
c. Magazines			*g.* Charity		
d. Papers			*Total*		

5. *Health*

a. Exercise			*d.* Dentist		
b. Recreation			*e.* Medicine		
c. Physician			*Total*		

6. *Miscellaneous*

a. Laundry			*c.* Small items		
b. Presents			*Total*		

7. *Investments*

a. Bank			*c.* Other investments		
b. Household			*Total*		

BUDGETS

1. A family with an income of $1500 a year allowed 20% of the income for house, 25% for food, 20% for clothing, 20% for higher life, 10% for other expenses, and the rest for investments. How much money was allowed for each purpose?

2. A family with an income of $2500 a year allowed 20% for house, 25% for food, 15% for clothing, 25% for higher life, 10% for other expenses, and the rest for investments. How much money was allowed for each purpose?

The per cents stated in Exs. 1 and 2 are the results of careful studies of family expenses where the incomes range from $1500 to $4000. The changes in the cost of living do not materially affect the relative per cents of the income expended for various purposes.

3. A study of family expenditures in a manufacturing community showed that 19% of the income was paid for house, 43% for food, 11% for clothing, 22% for all other expenses, and the rest for investments. If a family had an income of $1200 a year, how much money was spent for each purpose?

4. Using the form given on page 243, make up a budget for an income of $3000 a year, allotting the specified per cent to each item. 1. House: rent, 15%; heat, 4%; light, 2%; telephone, $\frac{1}{2}$%; taxes, $\frac{1}{2}$%; insurance, $\frac{1}{2}$%; miscellaneous, $2\frac{1}{2}$%. 2. Food: groceries, 13%; meat, 6%; milk, 2%; fruit, $1\frac{1}{2}$%; meals bought, $2\frac{1}{2}$%. 3. Clothing: suits, 8%; underwear, 2%; shoes, 2%; hats, 1%; small items, 2%. 4. Higher Life: education, 1%; books, 1%; magazines, 1%; papers, $\frac{1}{2}$%; travel, $2\frac{1}{2}$%; religion, 2%; charity, 2%. 5. Health: exercise, 1%; recreation, 1%; physician, $1\frac{1}{2}$%; dentist, $\frac{1}{2}$%; medicine, 1%. 6. Miscellaneous: laundry, 3%; presents, 2%; small items, 5%. 7. Investments: bank, 4%; household, 3%; other investments, 3%.

USING WHAT YOU HAVE LEARNED

1. Mr. Upton is left a legacy of $3000 by his uncle and wishes to invest it wisely. He can invest the money in a mortgage which runs for 3 yr. and pays $5\frac{1}{2}\%$ per annum. How much income would this bring him in 3 yr.?

2. If he should put the $3000 into a certain savings bank, he would receive 4% compounded semiannually. How much interest would the bank pay him in 3 yr.? This is equivalent to what per cent simple interest per year?

3. Mr. Upton also investigates the returns on an industrial stock in which he has been advised to invest. For several years the company has paid $1\frac{3}{4}\%$ quarterly dividends, and its stock is quoted at $124\frac{3}{4}$. Allowing for brokerage at $15 per 100 shares, how many shares could Mr. Upton buy with his $3000? What per cent of return from his investment would he receive?

4. Mr. Upton's broker tells him that $4\frac{3}{4}\%$ Victory Loan bonds are selling below par and that with his $3000 he can buy bonds of this issue of par value $3200. What per cent on the investment would these bonds yield?

5. His broker suggests that in 3 yr. he probably could sell the bonds for more than he paid, and that probably after the brokerage is paid there would be a profit of $2 on each $100 par value. Including this gain, what would be the total income in the 3 yr. if Mr. Upton bought the bonds described in Ex. 4? This is equivalent to what per cent of return per year?

6. Write a statement of the plan which you advise Mr. Upton to adopt in investing his legacy, and give your reasons.

At the discretion of the teacher, the pupils, in considering Ex. 3, may discuss the effect a decrease or an increase in the dividend rate or in the market price would have on the desirability of the investment.

USING WHAT YOU HAVE LEARNED

1. A man hires a cottage for $26 a month. How much rent does he pay in 1 yr.? The owner pays $42.80 a year in taxes, $5.60 for insurance, $60 for repairs, and 5% of the rental for collecting. How much does the owner have left from the rental?

2. The owner of the cottage in Ex. 1 sells it to the tenant for $3150 and pays $150 to an agent for making the sale. If the former owner invests the proceeds in a 6% mortgage, is his income increased or is it decreased, and how much?

3. The new owner in Ex. 2 spends $500 in improvements to the property. What is now his total investment in the cottage? What is the annual interest on the investment at 6%?

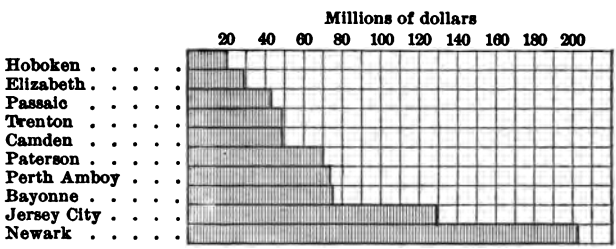
4. Owing to the improvements made in Ex. 3 the taxes are increased to $51.30 a year, and the insurance to $6.40. Since he has just had the house improved, he has no repair bill the first year. What is the monthly cost of his owning the cottage the first year, including interest on the investment? How does this compare with the rent which he formerly paid (Ex. 1)?

5. The second year that he owns the cottage in Ex. 4 he pays $75 for repairs as well as the other expenses given. At the end of the year he sells it for $4000, and figures out whether he has lost or gained by owning it. He adds the expenses, including interest on the investment, and compares them with the amount received from the sale. What is the result?

6. On studying the result in Ex. 5 the man realizes that he has made no allowance for the fact that he has lived in the house for 2 yr. and has paid no rent. Owing to the improvements made he could have rented the cottage for $30 a month. Taking this figure as a fair rental, at what amount should the man in Ex. 5 calculate his gain or loss?

II. GRAPHS

Graph. When we are considering statistics of any kind it is often very helpful to represent them graphically. For example, the values of the products of the leading manufacturing cities in the state of New Jersey in a certain year were as follows: Newark, $202,000,000; Jersey City, $129,000,000; Bayonne, $74,000,000; Perth Amboy, $73,000,000; Paterson, $70,000,000; Camden, $49,000,000; Trenton, $49,000,000; Passaic, $42,000,000; Elizabeth, $29,000,000; and Hoboken, $20,000,000. The figures do not strike us as strongly as they would if expressed graphically, as in the following diagram, in which each space represents $10,000,000.

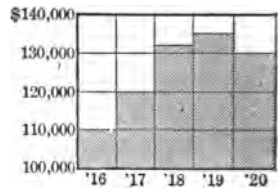


Any graphical representation of statistics of any kind is called a *graph*. The kind of graph shown above is called a *bar graph* or a *bar pictogram*.

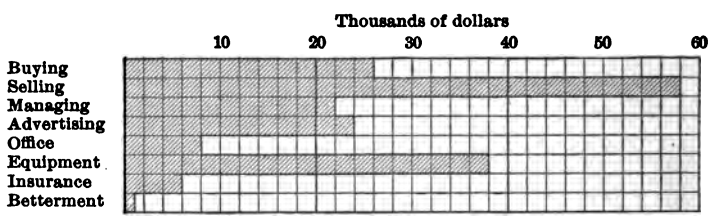
Sometimes the bars in such a graph stand vertically, as in the upper graph shown on page 248. Pupils should be familiar with both kinds of bar pictograms.

The word "graph" is applied to all such pictorial representations. Graphs have come to be used in all kinds of modern business, and the pupil should become familiar with a few of the more important types. Such graphs are now found in newspapers and magazines and are also used to record the temperature in the schoolroom, the standing of pupils, and the like.

Bar Pictograms. The value of bar pictograms in business may easily be seen by examining two practical cases. The first case shows the sales of goods in a business house from 1916 to 1920. The owners of the business could see at a glance that the growth of the business from 1916 to 1918 was not maintained in 1919, and that in 1920 there was an actual decrease in the amount of the sales. It will be noticed that the graph begins with $100,000. To draw a graph showing the total sales to the scale used here would require 3 in. of space, and since all the figures are over $100,000 and the purpose of the graph is to compare the annual sales of this company alone, the smaller form is better.



To take another illustration, when the statistics setting forth the expenses of a certain company were presented to the directors



by the above graph, they saw much more clearly than ever before that the selling expenses were excessive, that the amount paid for advertising the goods was relatively too small, that the expenditure for equipment needed investigating, and that the amount assigned to the social betterment of their employees was altogether too small to produce good results.

Squared paper like the above is very convenient for drawing graphs. It may be purchased at a stationer's shop.

BAR PICTOGRAMS

Represent the following statistics by bar pictograms:

1. In a recent year the number of clothing, millinery, and laundry establishments in Manhattan was 12,000, and in Brooklyn, 3600.

2. Recently the number of printing and paper-goods establishments in New York City was as follows: Manhattan, 2150; The Bronx, 47; Brooklyn, 311; Queens, 36; Richmond, 17.

3. In a recent year the number of textile factories in New York City was as follows: Manhattan, 752; The Bronx, 71; Brooklyn, 255; Queens, 32; Richmond, 5.

4. The net income of a certain railway for a recent year was applied as follows:

Sinking-fund requirements	$1,500,000
Dividends on preferred stock	6,500,000
Dividends on common stock	25,000,000
Additions and betterments	1,500,000
Surplus to profit-and-loss account	6,500,000

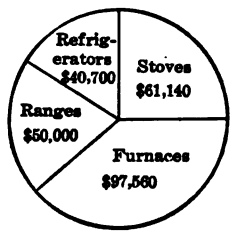
Draw a vertical bar pictogram, using $\frac{1}{8}$ in. to represent $1,000,000.

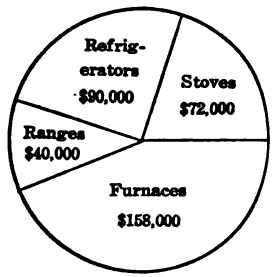
5. The annual business of a company was as follows: 1915, $48,000; 1916, $72,000; 1917, $88,000; 1918, $124,000; 1919, $130,000; 1920, $142,000; 1921, $144,000.

6. The total annual business of a company for the last ten years was as follows: $275,000, $285,000, $295,000, $325,000, $310,000, $330,000, $360,000, $340,000, $375,000, $400,000.

In Exs. 5 and 6 the pupils should be asked to interpret the graph; that is, to tell what it signifies in the business. In Ex. 5 they may naturally consider the effect of the war upon the business, and in each case they should consider the prospects as shown by the graph.

Circular Pictogram. It has become common in connection with the financial statistics of large business houses to make use of the *circular pictogram.* For example, these circles show at a glance the following comparative amounts of business done by a certain house in two consecutive years:

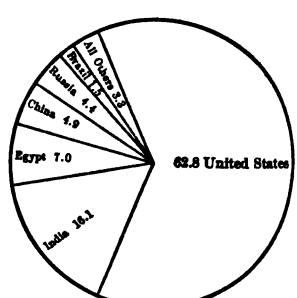




	First Year	Second Year
Stoves	$61,140	$72,000
Furnaces	97,560	158,000
Ranges	50,000	40,000
Refrigerators	40,700	90,000

The graphs show that the total business has increased, the second circle being considerably larger than the first. More stoves were sold in the second year, yet there was a relative shrinkage. The sale of ranges has fallen off both actually and relatively.

A circular pictogram is particularly convenient when we are dealing with per cents. For example, this pictogram shows the per cents of the world's mill supply of cotton contributed by each country in a recent year. It tells us at a glance that the United States contributes considerably more than half of the supply of the whole world.



CIRCULAR PICTOGRAMS

1. Of the pupils who attend school in a certain city 10% are in the first grade, 16% in the second, 17% in the third, 15% in the fourth, 13% in the fifth, 12% in the sixth, 10% in the seventh, and 7% in the eighth. Represent these facts by a circular pictogram.

2. Of the six most important textile fibers, cotton is 54% in quantity; wool, 14½%; jute, 14%; flax, 10%; hemp, 7%; and silk ½%. Represent these facts by a circular pictogram.

Represent by circular pictograms the following sets of facts:

3. A drug store finds that 37% of its income is derived from the sale of candies, cigars, and soda water; 42% from patent medicines; 12% from sundries; and the balance of the income from prescriptions.

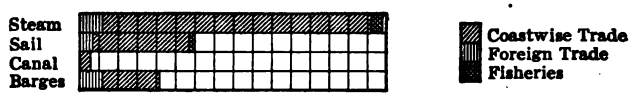
4. A class in school kept a record of the weather in the first 25 da. of April of a certain year. They found that the rainy days were 20% of the period; cloudy days, 40%; foggy days, 8%; clear weather, 32%.

5. A man's various investments are as follows: 12% of his capital is in land, 45% in bonds, 27% in stocks, 6% in banks, and the balance in notes.

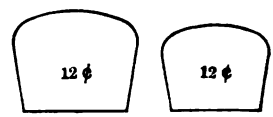
6. A manufacturing company devotes 64% of its gross income to wages, 9% to office expenses, 11% to advertising, 8% to selling expenses, 5% to dividends, 2% to interest, and the balance to incidental expenses.

7. A company finds that the buying expenses are 17% of the gross income; office expenses, 4%; management, 10%; other overhead, 24%; selling expenses, 29%; interest, 10%; dividends, 4%; incidentals, 2%.

Multiple Pictogram. The bar pictogram is often used to compare several features of statistics at once. For example, this *multiple pictogram*, in which each square represents 1000, not only classifies the American ships afloat in a certain year but also shows the distribution of them in the various trades.

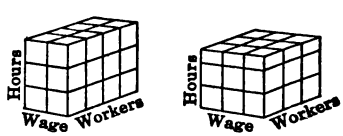


Value in Advertising. Advertisers are very quick to see the value of the appeal to the eye. A baker's advertisement that he sold a 16-ounce loaf for the same price that his rival charged for a 12-ounce loaf had little effect until he resorted to the graph here shown.



This topic and the one which follows are introduced for the purpose of showing the various uses of graphs. They are not used in the exercises.

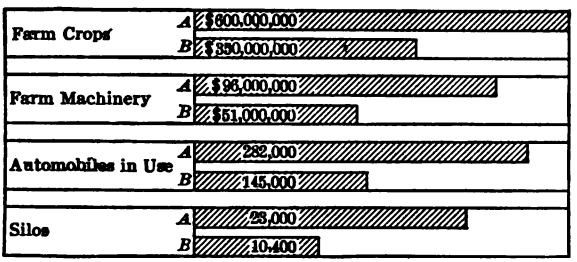
Solid Pictogram. A *solid pictogram* is often used in showing related sets of statistics. For example, these rectangular solids compare the number of workers, the wage per hour, the hours per week, and the total weekly wages paid in the clothing industry of two cities. The number of workers is shown in one dimension, the wage per hour in a second, and the number of hours per week in the third. We see that city A has $\frac{4}{5}$ as many workers as city B, that the wage per hour in A is only $\frac{2}{3}$ as much as in B, and that the number of hours per week in A is $\frac{6}{5}$ as many as in B. The total weekly wages paid are shown by the volumes of the two solids, which are respectively 24 and $22\frac{1}{2}$.



City A City B

PICTOGRAMS

1. Certain recent statistics from two neighboring agricultural states *A* and *B*, each with a rural population of approximately 2,250,000, are compared in the following bar pictogram:



Find to the nearest cent the average value in each state of the farm crops per person.

2. Of the total number of automobiles in use in the two states, what per cent is in use in each state?

3. Of the total number of silos in the two states, what per cent is in use in each state?

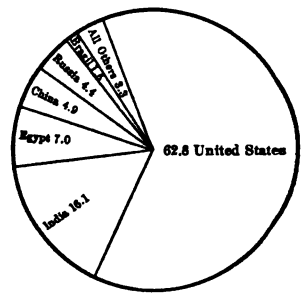
4. Of the total value of the farm machinery in the two states, what per cent is the value in each state?

5. Of the total value of the crops in the two states, what per cent is the value in each state?

6. Compare the results in Exs. 2–5 and write a statement giving your impression of the value of up-to-date methods as shown by these results.

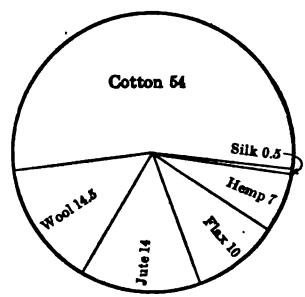
In composite graphs like the one shown above the same scale is often not used throughout. For each pair of figures compared the bars are drawn to scale, but it would be manifestly impossible within reasonable dimensions to represent $600,000,000 (value of farm crops) and 10,400 (number of silos) by the same scale.

7. The first pictogram shows the percentage of the world's mill supply of cotton contributed by each country in a recent year. Write in statistical form a statement of what the graph tells.



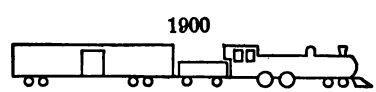
8. In Ex. 7 write a statement as to four important facts which the graph tells you even if the per cents are omitted. For example, how would you rank the United States in comparison with India or with Egypt?

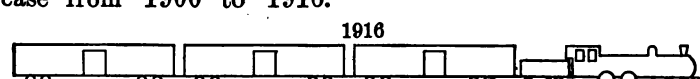
9. The second pictogram shows the relative quantities of the six most important textile fibers. Write a statement as to four important facts which the graph tells you.



10. From Ex. 9 find the per cent which the quantity of flax is of the quantity of jute; of the quantity of hemp; of the quantity of silk.

11. This pictogram shows the comparative sizes of a trainload on a prominent Eastern railroad in 1900 and in 1916. Estimate the relative sizes and find the per cent of increase from 1900 to 1916.

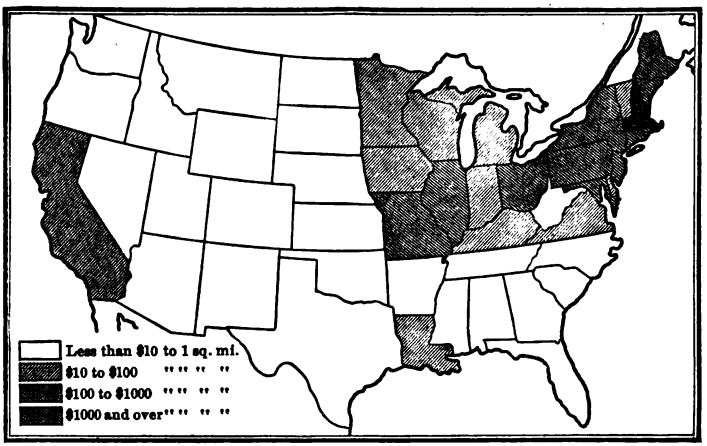




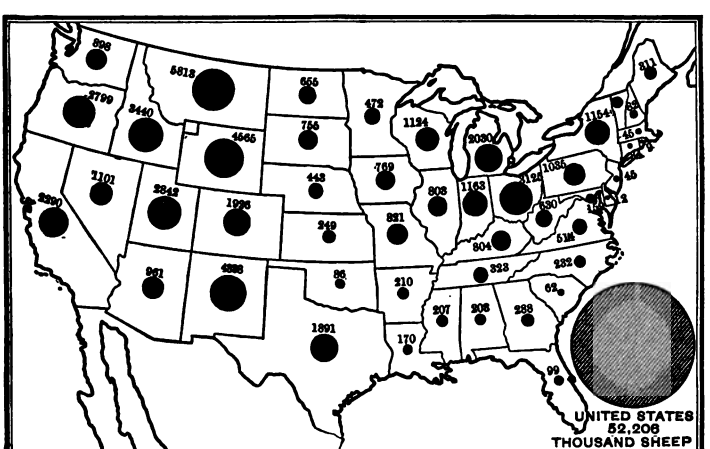
Write several reasons that tend to explain why the size of the trainload has increased in the period mentioned.

12. Draw a pictogram in any form showing the attendance in your class for a week.

Cartogram. The following map, or *cartogram*, shows the distribution of the manufacture of boots and shoes in a certain year:



The following cartogram shows the number of thousands of sheep in the United States in a certain year:



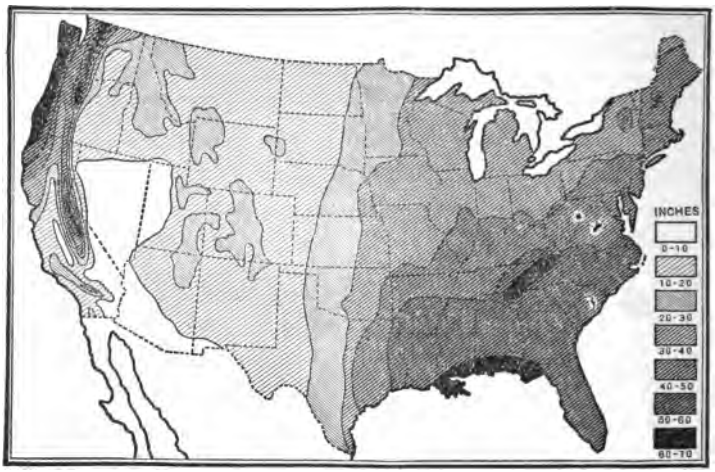
CARTOGRAMS

1. Using the upper map on page 255, write the names of the states in which the manufacture of boots and shoes was $1000 and over per square mile in the year considered.

2. Give a reason for the concentration of boot and shoe manufacture in the states indicated in Ex. 1.

3. Using the lower map on page 255 write in order of number of sheep the names of the ten states having the largest numbers.

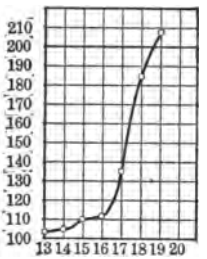
4. Write a statement in the way of a report to a shipping house as to the mean annual rainfall in inches, containing four important facts and based on the following cartogram:



Such information is of great commercial importance, particularly to houses that sell goods which deteriorate quickly in a damp climate.

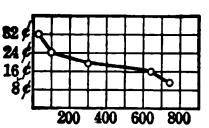
5. Sketch a map of your city or village and shade it to show approximately the areas occupied chiefly by stores, chiefly by manufacturing plants, and chiefly by residences.

Curve-Line Graph. In many kinds of statistics the most convenient kind of graph is a *curve-line graph.* Such a line shows in a convenient form the changes from day to day or from year to year.



For example, a company that did a business of $103,000 in 1913, $105,000 in 1914, $110,000 in 1915, and $112,000 in 1916 found its business increased to $135,000 in 1917, $185,000 in 1918, and $208,000 in 1919. While these figures show a decided increase, we appreciate the sudden change more easily if we look at the graph. Such graphs are familiar to all readers of the newspapers and magazines of the present day, having become very common of late years.

Demand Curve. Curve-line graphs may be applied to the prices of goods, in which we may have a series of short straight lines connected to give the effect of a curve.



For example, it was found in a certain market that when strawberries sold at 32¢ a quart, 25 qt. were sold daily; when the price dropped to 24¢, 100 qt. were sold daily; at 20¢, 300 qt. were sold; and so on, the average number of quarts sold per day being related to the price. The relation of price to sales may be indicated graphically as here shown, and the story is told much more vividly than by a mere recital of figures. We see at once that it is not always more profitable to sell at the highest prices, because the sales may be so small as to give but a small total profit.

Such a graph is known as a *demand curve*, and commercial houses often find that a study of these curves is profitable in making their plans for purchasing stock.

CURVE-LINE GRAPHS

1. Represent by a line graph the change in the average price of lake copper for six consecutive years, as follows: year A, 12.7¢; year B, 16.7¢; year C, 16.7¢; year D, 13.5¢; year E, 28¢; year F, 32¢.

2. The total annual sales of a grocery for a period of ten years were as follows: $16,000, $18,000, $20,000, $25,000, $24,000, $26,000, $28,000, $36,000, $43,000, $52,000. Draw a graph to show the trend of business.

3. From the graph in Ex. 2 estimate the probable amount of business the next year. Will it be advisable for the store to enlarge its plant and stock in this neighborhood?

4. Construct a demand curve based on the following facts which have been ascertained with respect to a certain article:

Price . . .	12¢	10¢	8¢	6¢	4¢	2¢
Number sold .	400	500	1800	3000	8000	8200

5. A grocer's records of sales of peaches for a certain year showed that he sold 10 baskets at $1.75 a basket; 30 baskets at $1.50; 70 baskets at $1.20; 80 baskets at $1.10; and 90 baskets at $1. Draw the demand curve and estimate the probable demand when the price is 90¢ a basket.

6. If the business of the grocer mentioned in Ex. 5 has increased 20% since last year, when the demand facts were recorded, draw a demand curve for this year, assuming that he will sell 20% more at each price mentioned above than he did last year. Using your curve as a guide, estimate how many baskets of peaches he will probably be able to sell this year at $1.45.

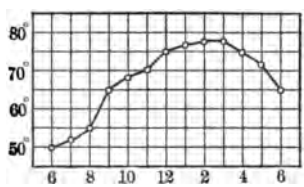
CURVE-LINE GRAPHS

1. The distances of a train from its starting point in a run of 10 min. were measured and were found to be as follows:

Minutes	1	2	3	4	5	6	7	8	9	10
Miles	0.2	0.5	0.9	1.5	2.2	3	3.8	4.6	5.1	5.5

Draw a graph and tell what it shows about the full speed.

2. The temperature at a certain school between 6 A.M. and 6 P.M. on the first day of last May was as shown by this graph. Write the approximate temperature for each hour and half hour of the day.



3. The height of a plant in the schoolroom was measured on eight consecutive days in June and was found to be as follows:

Day of the month	1	2	3	4	5	6	7	8
Height in inches	3.1	4.2	5.2	6.1	6.8	7.5	8.2	8.7

Draw a graph showing the growth, and from it determine in what part of the 7-day period it was growing the most rapidly.

4. In seven hours of a rainy day in April the amount of rain which fell was measured and was found to be as follows:

Hours	1	2	3	4	5	6	7
Number of inches	0.1	0.3	0.2	0.1	0.3	0.4	0.5

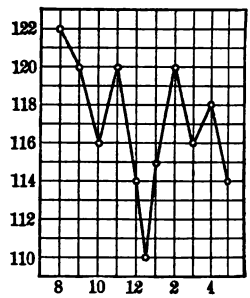
Draw a graph showing the variation in rainfall by the hour.

5. A boy's height each year between the ages of 5 yr. and 15 yr. was found in inches to be as follows:

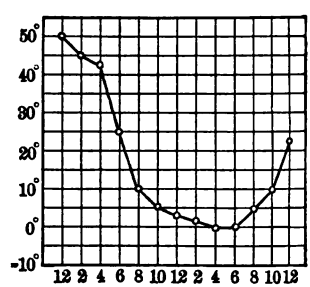
Age	5	6	7	8	9	10	11	12	13	14	15
Height	42	44	46	49	52	54	56	58	61	63	68

Draw a graph and from it determine in what part of this period the boy was growing most rapidly.

6. In a certain mill the steam gauge on the boiler showed at 8 A.M. a pressure of 122 lb. to the square inch. The pressure varied through the day as shown by this graph. Write the pressure for every hour from 8 A.M. to 5 P.M. and also for 12.30 P.M. Why was the pressure so high at 8 A.M.?



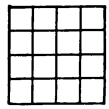
7. When the temperature is below zero this fact is indicated by a minus sign. That is, $-5°$ means $5°$ below zero. This graph shows the changes in temperature from noon on a day in January to noon on the following day. Write the approximate temperatures for every hour of this period, and a statement telling when the temperature fell the most rapidly and when it rose the most rapidly.



8. The population of the earth by continents in millions is approximately as follows: Africa, 143; North America, 140; South America, 56; Asia, 873; Australasia, 16; Europe, 465. Draw a graph, using that kind of graph which you think shows the facts most effectively.

III. ADVANCED MENSURATION

Square and Square Root. If a square has a side of 4 units, it has an area of 16 square units. Therefore 16 is called the *square* of 4, and 4 is called the *square root* of 16.



Square Root applied to Areas. Hence, considering the numbers as abstract,

The side of a square is the square root of the area.

Symbols. The square of 4 is written 4^2, and the square root of 16 is written $\sqrt{16}$.

Perfect Square. Such a number as 16 is called a *perfect square*, but 10 is not a perfect square. We may say, however, that $\sqrt{10}$ equals 3.16 +, because 3.16^2 nearly equals 10.

Square Root of a Perfect Square. Square roots of perfect squares may often be found by simply factoring the numbers.

For example,
$$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$$
$$= \sqrt{3 \times 7 \times 3 \times 7}$$
$$= \sqrt{21 \times 21} = 21.$$

$$\begin{array}{r|l} 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ & 7 \end{array}$$

That is, we separate 441 into its factors and then separate these factors into two equal groups, 3×7 and 3×7. Hence we see that 3×7, or 21, is the square root of 441.

FINDING SQUARE ROOTS

All oral work

State the square roots of the following:

1. 9.	**3.** 49.	**5.** 144.	**7.** 1.44.	**9.** 4900.
2. 64.	**4.** 121.	**6.** 900.	**8.** 0.64.	**10.** 2500.

SQUARES AND SQUARE ROOTS

Find the squares of the following:

1. 2.3.	3. $2\frac{1}{2}$.	5. 2.4.	7. 0.7.	9. 1200.
2. 4.7.	4. $3\frac{1}{3}$.	6. 3.6.	8. 0.81.	10. 3602.

By factoring, find the square roots of the following:

11. 121.	13. 625.	15. 729.	17. 1296.	19. 12.25.
12. 169.	14. 324.	16. 576.	18. 1089.	20. 40.96.

Find the sides of squares whose areas are as follows:

21. 4.84 sq. in.	23. 4.41 sq. in.	25. 5929 sq. yd.
22. 1.96 sq. ft.	24. 4356 sq. in.	26. 10.89 sq. yd.

Find the perimeters of squares whose areas are as follows:

27. 6561 sq. ft.	29. 19,600 sq. ft.	31. 16,384 sq. in.
28. 12,100 sq. in.	30. 146.41 sq. in.	32. 16,900 sq. ft.

33. From a corner of a square piece of land that contains 57,600 sq. ft. there is cut a square lot containing 6400 sq. ft. Draw a plan of the lots and find the perimeter of each lot.

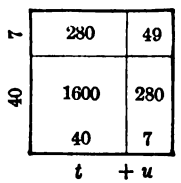
34. A square lot contains 18,225 sq. ft. What is the perimeter of the lot? What is the perimeter of a square lot of four times this area? The second perimeter is how many times the first?

35. Two square building lots front on the street. The area of the two together is 8900 sq. ft., that of the larger being 6400 sq. ft. What is the frontage of each lot?

36. A square lot has an area of 28,900 sq. ft. What is the perimeter of the lot? What is the perimeter of a square lot that contains nine times this area? The perimeter of the second lot is how many times the perimeter of the first?

Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained as follows:

$$\begin{array}{rl} & 40+7 \\ & 40+7 \\ \hline & (40\times 7)+7^2 \\ 40^2+ & (40\times 7) \\ \hline & 40^2+2\times(40\times 7)+7^2 \\ =1600+2\times 280 & +49 \\ =1600+560 & +49 \\ =2209. & \end{array}$$



This relationship is conveniently seen in the annexed figure.

Every number consisting of two or more figures may be regarded as composed of tens and units. Therefore,

The square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.

This principle, important in square root, should be clearly understood both from the multiplication and from the illustration.

Separating into Periods. Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, it is evident that the square root of any number between 1 and 100 lies between 1 and 10, and that the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number of *one* figure or *two* figures is a number of *one* figure; the square root of any integral number of *three* or *four* figures is a number of *two* figures; and so on.

Therefore, if an integral number is separated into *periods* of two figures each, from the right to the left, the number of figures in the square root is equal to the number of the periods of figures. The last period at the left may have one figure or two figures.

Finding the Square Root. Required the square root of 2209.

If the figures of the number are separated into periods of two figures each, beginning at the right, we see that there are two integral places in the square root of the number.

The first period, 22, contains the square of the tens' number of the root. Since the greatest square in 22 is 16, then 4, the square root of 16, is the tens' figure of the root required.

```
  22 09 (47
  16
 ------
 80  6 09
 87  6 09
 ------
```

Subtracting the square of the tens, the remainder contains twice the tens × the units, plus the square of the units. Dividing by twice the tens (that is, by 80, which is 2 × 4 tens), we find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

Since twice the tens × the units, plus the square of the units, is equal to (twice the tens + the units) × the units, that is, since $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47. Checking the work, we find that 47^2 is 2209.

FINDING SQUARE ROOTS

Find the square roots of the following:

1. 3249. 2. 3721. 3. 3969. 4. 5041.

Find the sides of squares that have the following areas:

5. 6724 sq. ft. 7. 9025 sq. ft. 9. 7921 sq. ft.
6. 7569 sq. ft. 8. 9409 sq. ft. 10. 6889 sq. yd.

Find the square roots of the following fractions by taking the square roots of the terms of the fractions:

11. $\frac{144}{529}$. 12. $\frac{961}{1024}$. 13. $\frac{1089}{1156}$. 14. $\frac{1936}{5041}$.

Square Root with Decimals. Find the square root of 151.29.

Separate the number into periods, beginning at the decimal point. The greatest square of the tens in 151.29 is 100, and $\sqrt{100} = 10$.

Then 51.29 contains $2 \times 10 \times$ the units' number of the root, plus the square of the units' number.

Dividing 51 by 2×10, or 20, the next figure of the root is 2.

We have now found 12, the square being $100 + 44 = 144$.

	1 51.29 (12.3
	1
20	51
22	44
240	7 29
243	7 29

As before, dividing 7.29 by 24, we find that the tenths' figure of the root is 3.

Hence the square root of 151.29 is 12.3.

If the number is not a perfect square we may annex pairs of zeros at the right of the decimal point and continue as far as we choose.

Summary of Square Root. To find a square root:

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its root as the first figure of the required root.

Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

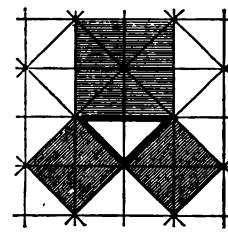
Divide this new dividend by twice the part of the root already found and write the quotient as the second figure of the required root. Annex to this divisor the figure thus found and multiply by the number representing this figure.

Subtract this result, bring down the next period, and proceed as before until all the periods have been thus annexed.

The result is the square root required.

Square on the Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse* of the triangle.

If a floor is made up of triangular tiles as in this figure, it is easy to mark out a right triangle. It is seen that the square on the hypotenuse contains eight equal triangles, while each square on a side contains four such triangles. Hence



The square on the hypotenuse equals the sum of the squares on the other two sides.

This important fact is proved in plane geometry for all right triangles.

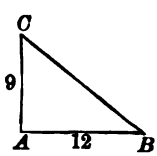
Illustrative Problem. If $AB = 12$ and $AC = 9$, find BC.

Since $\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2$,

we have $\overline{BC}^2 = 12^2 + 9^2$,

or $\overline{BC}^2 = 144 + 81 = 225$,

and $BC = \sqrt{225} = 15$.



SQUARE ON THE HYPOTENUSE

1. In the above figure, if $AB = 24$ ft. and $AC = 9$ ft., find BC.

2. In the above figure, if $BC = 105$ in. and $AB = 84$ in., find AC.

Show the class that, in the above figure, $\overline{AC}^2 = \overline{BC}^2 - \overline{AB}^2$.

3. If the side of a square is 17 in., find the diagonal to the nearest hundredth of an inch.

4. Find, to the nearest hundredth of an inch, the side of a square whose area is 3 sq. in.

5. In Ex. 4 suppose the given area is 6 sq. in.

6. Find, to the nearest hundredth of an inch, the diagonal of a square whose area is 81 sq. in.

USING WHAT YOU HAVE LEARNED

1. Find the diagonal of the floor of a hall 51 ft. by 68 ft.

2. Find, to the nearest hundredth of a foot, the diagonal of a square that contains 4 sq. ft.

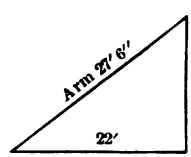
3. The two sides of a right triangle are 20 in. and 30 in. Find, to the nearest hundredth of an inch, the hypotenuse.

4. The two sides of a right triangle are 57 in. and 76 in. Find the hypotenuse.

5. What is the length of a wire drawn taut from the cornice of a 100-foot building to a spot 75 ft. from the base?

6. A telegraph pole is set perpendicular to the ground, and a wire is fastened to it 18 ft. from the ground and then is stretched to a stake 13 ft. 6 in. from the foot of the pole, so as to hold it in place. How long is the wire?

7. A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top of the derrick above the opening?



Find, to the nearest hundredth, the hypotenuse of each of the right triangles of which the other two sides are as follows:

8. 32 ft., 28 ft.
9. 40 ft., 75 ft.
10. 321 ft., 428 ft.
11. 36 in., $67\frac{1}{2}$ in.
12. 72 ft., 135 ft.
13. $56\frac{1}{2}$ in., $72\frac{1}{2}$ in.

14. The foot of a 45-foot ladder is placed 27 ft. from the wall of a building against which the top of the ladder rests. How high does the ladder reach on the wall?

15. How far from the wall of a house must the foot of a 45-foot ladder be placed in order that the top of the ladder may just touch a window sill 40 ft. from the ground?

USING WHAT YOU HAVE LEARNED

Find, to the nearest hundredth, the hypotenuse of each of the right triangles of which the sides are as follows:

1. 35 ft., 29 ft.
2. 81 ft., 35 ft.
3. 10 in., 15 in.
4. $42\frac{1}{2}$ ft., $63\frac{3}{4}$ ft.
5. 4.5 in., 7.2 in.
6. 6.25 in., 7.5 in.
7. $3\frac{3}{4}$ in., $8\frac{1}{2}$ in.
8. 23 in., 34 in.
9. 3.4 in., 2.7 in.

10. A room is 16 ft. long, 12 ft. wide, and 9 ft. high. Find, to the nearest hundredth, the distance from an upper corner through the room to the opposite lower corner.

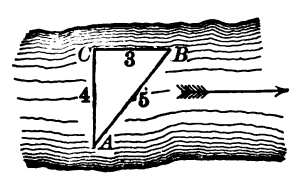
First find the hypotenuse on the floor and then the hypotenuse required. This hypotenuse is called the *diagonal* of the room.

11. Find, to the nearest hundredth, the diagonal of a cube of which the edge is 2 in.

First find the diagonal of one face. Then proceed as in Ex. 10.

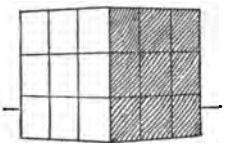
12. A school flagpole is broken by the wind 16 ft. from the ground. The two pieces hold together, and the top touches the ground 30 ft. from the foot. Find the length of the flagpole.

13. If I start to row directly across a stream, in the direction AC, at the rate of 4 mi. an hour, and if the stream carries me in the direction CB, at the rate of 3 mi. an hour, my course will really be AB, the resultant of these two motions. Suppose that I row 4.5 mi. per hour, and the stream flows 6 mi. per hour, what is my rate of progress?



14. The captain is walking at the rate of 4 mi. per hour directly across the deck of his steamer, which is moving at the rate of 8 mi. per hour. Find, to the nearest hundredth, the rate at which the captain is moving. Draw a plan to scale.

Cube and Cube Root. If a cube has an edge of 3 units, it has a volume of 27 cubic units. Therefore 27 is called the *cube* of 3, and 3 the *cube root* of 27.



Symbols. The cube of 3 is written 3^3, and the cube root of 27 is written $\sqrt[3]{27}$.

Cube Root applied to Volumes. Therefore, considering the numbers as abstract,

The edge of a cube is the cube root of the volume.

Power. Squares and cubes are called *powers*. We have also higher powers, like the fourth, fifth, and so on. Raising to powers is sometimes called *involution*. Finding roots is sometimes called *evolution*.

Cube Root of a Perfect Cube. The cube root of a perfect cube may often be found by factoring.

For example, $\sqrt[3]{216} = \sqrt[3]{2\times2\times2\times3\times3\times3}$

$= \sqrt[3]{(2\times3)\times(2\times3)\times(2\times3)}$

$= \sqrt[3]{6\times6\times6} = 6.$

$$\begin{array}{r|l} 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

The subject of cube root is now so commonly postponed until algebra is studied that an extended explanation is not felt to be necessary. A brief treatment will therefore suffice. Teachers are advised to omit the subject unless the local course of study calls for it.

FINDING CUBE ROOTS

Find the following cube roots by factoring:

1. $\sqrt[3]{729}$.
2. $\sqrt[3]{1331}$.
3. $\sqrt[3]{512}$.
4. $\sqrt[3]{1728}$.
5. $\sqrt[3]{2744}$.
6. $\sqrt[3]{15625}$.
7. $\sqrt[3]{4096}$.
8. $\sqrt[3]{5832}$.
9. What is the edge of a cube of volume 13.824 cu. in.?
10. What is the edge of a cube of volume 10,648 cu. in.?

Finding Square and Cube Roots. There are methods of finding the cube root of any number, just as we have seen that there is a method of finding the square root. These methods are seldom used in ordinary computations. When we wish to find squares, cubes, square roots, or cube roots we generally use tables. For example, on page 271 there is a convenient table showing the squares, cubes, square roots, and cube roots of all integral numbers from 1 to 100, which will be sufficient for our present needs.

1. **Illustrative Problems.** Find the square root of 39.

By the table, $\sqrt{39} = 6.245$. This means that 6.245^2 is approximately 39. By squaring 6.245 we see that the result is 39.000025. That is, the square root of 39 is 6.245 to the nearest thousandth.

2. Find the cube root of 88.

By the table, $\sqrt[3]{88} = 4.448$. That is, 88 is not an exact cube, and its cube root is between 4.4475 and 4.4485.

3. Find the square root of 841.

In the table the column of numbers (under No.) goes only to 100. We therefore look to see if 841 is in the column of squares. We find that it is, being the square of 29. Therefore $\sqrt{841} = 29$.

4. Find the cube root of 884,736.

The column of cubes in the table shows that $884{,}736 = 96^3$. Hence $\sqrt[3]{884736} = 96$.

5. Find the square root of 2000.

Since $2000 = 20 \times 100$ we may find the square roots of 20 and 100 and take their product. We see that $\sqrt{20} = 4.472$, and $\sqrt{100} = 10$. We therefore have $\sqrt{2000} = 10 \times 4.472 = 44.72$.

6. Find the square root of 1765.

The column of squares in the table shows that $1764 = 42^2$. It is then evident that $\sqrt{1765} = 42 +$ some fraction; that is, this table gives us only the two integral figures of the root.

Table of Powers and Roots. This table will be found helpful:

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	51	2,601	132,651	7.141	3.708
2	4	8	1.414	1.260	52	2,704	140,608	7.211	3.733
3	9	27	1.732	1.442	53	2,809	148,877	7.280	3.756
4	16	64	2.000	1.587	54	2,916	157,464	7.348	3.780
5	25	125	2.236	1.710	55	3,025	166,375	7.416	3.803
6	36	216	2.449	1.817	56	3,136	175,616	7.483	3.826
7	49	343	2.646	1.913	57	3,249	185,193	7.550	3.849
8	64	512	2.828	2.000	58	3,364	195,112	7.616	3.871
9	81	729	3.000	2.080	59	3,481	205,379	7.681	3.893
10	100	1,000	3.162	2.154	60	3,600	216,000	7.746	3.915
11	121	1,331	3.317	2.224	61	3,721	226,981	7.810	3.936
12	144	1,728	3.464	2.289	62	3,844	238,328	7.874	3.958
13	169	2,197	3.606	2.351	63	3,969	250,047	7.937	3.979
14	196	2,744	3.742	2.410	64	4,096	262,144	8.000	4.000
15	225	3,375	3.873	2.466	65	4,225	274,625	8.062	4.021
16	256	4,096	4.000	2.520	66	4,356	287,496	8.124	4.041
17	289	4,913	4.123	2.571	67	4,489	300,763	8.185	4.062
18	324	5,832	4.243	2.621	68	4,624	314,432	8.246	4.082
19	361	6,859	4.359	2.668	69	4,761	328,509	8.307	4.102
20	400	8,000	4.472	2.714	70	4,900	343,000	8.367	4.121
21	441	9,261	4.583	2.759	71	5,041	357,911	8.426	4.141
22	484	10,648	4.690	2.802	72	5,184	373,248	8.485	4.160
23	529	12,167	4.796	2.844	73	5,329	389,017	8.544	4.179
24	576	13,824	4.899	2.884	74	5,476	405,224	8.602	4.198
25	625	15,625	5.000	2.924	75	5,625	421,875	8.660	4.217
26	676	17,576	5.099	2.962	76	5,776	438,976	8.718	4.236
27	729	19,683	5.196	3.000	77	5,929	456,533	8.775	4.254
28	784	21,952	5.292	3.037	78	6,084	474,552	8.832	4.273
29	841	24,389	5.385	3.072	79	6,241	493,039	8.888	4.291
30	900	27,000	5.477	3.107	80	6,400	512,000	8.944	4.309
31	961	29,791	5.568	3.141	81	6,561	531,441	9.000	4.327
32	1,024	32,768	5.657	3.175	82	6,724	551,368	9.055	4.344
33	1,089	35,937	5.745	3.208	83	6,889	571,787	9.110	4.362
34	1,156	39,304	5.831	3.240	84	7,056	592,704	9.165	4.380
35	1,225	42,875	5.916	3.271	85	7,225	614,125	9.220	4.397
36	1,296	46,656	6.000	3.302	86	7,396	636,056	9.274	4.414
37	1,369	50,653	6.083	3.332	87	7,569	658,503	9.327	4.431
38	1,444	54,872	6.164	3.362	88	7,744	681,472	9.381	4.448
39	1,521	59,319	6.245	3.391	89	7,921	704,969	9.434	4.465
40	1,600	64,000	6.325	3.420	90	8,100	729,000	9.487	4.481
41	1,681	68,921	6.403	3.448	91	8,281	753,571	9.539	4.498
42	1,764	74,088	6.481	3.476	92	8,464	778,688	9.592	4.514
43	1,849	79,507	6.557	3.503	93	8,649	804,357	9.644	4.531
44	1,936	85,184	6.633	3.530	94	8,836	830,584	9.695	4.547
45	2,025	91,125	6.708	3.557	95	9,025	857,375	9.747	4.563
46	2,116	97,336	6.782	3.583	96	9,216	884,736	9.798	4.579
47	2,209	103,823	6.856	3.609	97	9,409	912,673	9.849	4.595
48	2,304	110,592	6.928	3.634	98	9,604	941,192	9.899	4.610
49	2,401	117,649	7.000	3.659	99	9,801	970,299	9.950	4.626
50	2,500	125,000	7.071	3.684	100	10,000	1,000,000	10.000	4.642

USE OF THE TABLE OF ROOTS

Using the table on page 271, find the square roots of the following:

1. 25.	3. 64.	5. 7.	7. 29.	9. 67.
2. 49.	4. 81.	6. 13.	8. 47.	10. 91.

Using the table on page 271, find the cube roots of the following:

11. 8.	13. 64.	15. 17.	17. 28.	19. 51.
12. 27.	14. 13.	16. 19.	18. 46.	20. 99.

Using the table on page 271, find the square roots of the following:

21. 225.	23. 961.	25. 5184.	27. 7744.	29. 9025.
22. 529.	24. 2116.	26. 6889.	28. 8649.	30. 9604.

Using the table on page 271, find the cube roots of the following:

31. 5832.	33. 19,683.	35. 85,184.	37. 531,441.
32. 9261.	34. 42,875.	36. 97,336.	38. 804,357.

Find the integral parts of the square roots of the following:

39. 230.	41. 509.	43. 1475.	45. 3850.	47. 6730.
40. 375.	42. 850.	44. 1695.	46. 4775.	48. 8675.

Find the integral parts of the cube roots of the following:

49. 2775.	50. 22,325.	51. 75,750.	52. 675,750.

53. The volume of a cube is 2197 cu. in. Find the edge.

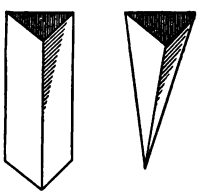
54. The volume of a cube is 39 cu. ft. Find the edge.

55. The area of one face of a cube is 78 sq. in. Find the volume of the cube.

56. The total area of all six faces of a cube is 36 sq. in. Find the volume of the cube.

Pyramid. A solid of this shape, in which the base is any polygon and the other faces are triangles meeting at a point, is called a *pyramid.* The point at which the triangular faces meet is called the *vertex* of the pyramid. The distance from the vertex to the base is called the *height*, or *altitude*, of the pyramid.

Volume of a Pyramid. A contractor or builder occasionally needs to find the volume of a pyramid. The rule for finding the volume is easily seen by taking a hollow prism and a hollow pyramid of the same base and the same height, as here shown, filling the pyramid with water, and pouring the water into the prism. It will be found that the prism can be exactly filled with three times the amount of water that fills the pyramid. That is,



The volume of a pyramid is equal to one third the product of the base and height.

VOLUME OF A PYRAMID

Find the volumes of pyramids with these bases and heights:

1. 129.3 sq. in., 3.72 in.
2. 202.11 sq. ft., 8.92 ft.
3. 702.39 sq. in., 16.8 in.
4. 187.02 sq. ft., 10.7 ft.

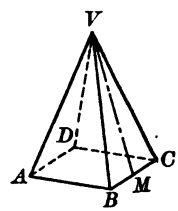
Find the bases of pyramids with these volumes and heights:

5. 89.6 cu. in., 12.8 in.
6. 178.2 cu. in., 19.8 in.
7. 123.2 cu. in., 17.6 in.
8. 140.4 cu. in., 23.4 in.

9. The Great Pyramid in Egypt was originally 482 ft. high and had a square base 768 ft. on a side. What was the volume? If it weighs 168 lb. per cubic foot, what was the weight in tons?

Lateral Area of a Pyramid. We have already learned how to find the volume of a pyramid, a problem more often found in practical life than the one of finding the surface.

The teacher will observe that the only kind of pyramid considered in arithmetic is the kind in which the vertex is exactly over the center of a regular base, the so-called *regular pyramid* here illustrated.



The altitude of any triangular face of a pyramid is called the *slant height* of the pyramid.

For example, VM is the slant height of the pyramid here shown.

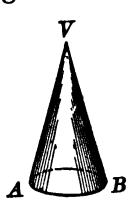
The sum of the areas of the triangular faces of a pyramid is called the *lateral area* of the pyramid.

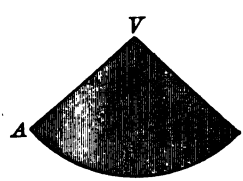
Since the area of each triangular face of a pyramid is equal to half the product of the base and altitude,

The lateral area of a pyramid is equal to half the product of the perimeter of the base and the slant height.

Lateral Area of a Cone. If we slit the surface of a cone and flatten it out as shown in the right-hand figure below, we have part of a circle.

The teacher will observe that the only kind of cone considered in arithmetic is the so-called *right circular cone* here shown, in which the vertex is directly over the center of the base. The terms "lateral area" and "slant height" will be understood from the study of the pyramid.



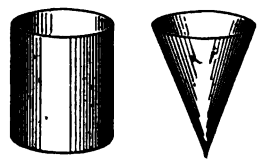


In the way that we found the area of a circle on page 115, we find that

The lateral area of a cone is equal to half the product of the circumference of the base and the slant height.

Volume of a Cone. In the way that we found the volume of a pyramid we may find the volume of a cone. That is,

The volume of a cone is equal to one third the product of the base and height.



What is the volume of a cone 5 in. high, the radius of the base being 2 in.?

Area of base $= \frac{22}{7} \times 4$ sq. in.

Volume $= \frac{1}{3} \times 5 \times \frac{22}{7} \times 4$ cu. in. $= 20.95$ cu. in.

That is, the volume is a little greater than 20.95 cu. in. Common sense should be used as to rejecting small fractions in cases like this.

MEASUREMENTS OF PYRAMIDS AND CONES

Find the lateral area of the pyramid which in each case has its perimeter of base and slant height as follows:

1. 24 in., 16 in. 2. 4 ft. 3 in., 9 in. 3. 4 ft. 9 in., 12 in.

Find the lateral area of the cone which in each case has its circumference of base and slant height as follows:

4. $15\frac{3}{4}$ in., $4\frac{3}{4}$ in. 5. 7 ft. 6 in., 4 ft. 6. 3 ft. 6 in., 15 in.

Find the volume of the cone which in each case has its radius of base and height as follows:

7. 15.4 in., 6 in. 8. 6.3 in., 15 in. 9. 42 in., 18 in.

10. The Great Pyramid originally had a slant height of 616 ft. and a square base 768 ft. on a side. Find the lateral area.

11. How many square feet of slating are used on a pyramidal steeple of slant height 40 ft. and perimeter of base 48 ft.?

12. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in., the radius of the base being 3 ft. 6 in. How many square feet must be allowed for gilding?

Surface of a Sphere. If we wind half of the surface of a sphere with a cord as here shown, and then wind with the same

length of cord the surface of a cylinder whose radius is equal to the radius of the sphere and whose height is equal to the diameter, the cord will cover half the curved surface of the cylinder.

Therefore *the surface of a sphere is equal to the curved surface of a cylinder of the same radius and height.*

Since, as we already know, the curved surface of the cylinder is $\frac{2}{7}\frac{2}{}$ × diameter × height, and the height is 2 × radius, the

$$\text{area of the sphere} = \tfrac{2}{7}\tfrac{2}{} \times 2 \times \text{radius} \times 2 \times \text{radius}$$
$$= 4 \times \tfrac{2}{7}\tfrac{2}{} \times \text{square of radius.}$$

Volume of a Sphere. It is sometimes desirable to be able to find the volume of a sphere, although this is not one of the essentials of arithmetic. It is shown in geometry that

The volume of a sphere is equal to $\frac{4}{3}$ *times* $\frac{22}{7}$ *times the cube of the radius.*

1. If the earth is a sphere of 4000 mi. radius, find the surface.

$4 \times \frac{2}{7}\frac{2}{} \times 4000^2 = 4 \times \frac{2}{7}\frac{2}{} \times 16{,}000{,}000 = 201{,}142{,}857\frac{1}{7}$.

Therefore the surface is about 201,143,000 sq. mi.

2. If the diameter of a ball is 4 ft., find the volume.

The radius is $\frac{1}{2}$ of 4 ft., or 2 ft.

The volume is $\frac{4}{3} \times \frac{2}{7}\frac{2}{} \times 2 \times 2 \times 2$ cu. ft., or 33.52 cu. ft.

MEASUREMENTS OF SPHERES

1. If a ball has a radius of $2\frac{1}{4}$ in., what is the surface?

2. Find the surface of a hemispherical dome of diameter 40 ft.

3. The sun's diameter is 866,500 mi. Find the surface to the nearest 1000 sq. mi.

4. If a tennis ball is $2\frac{3}{8}$ in. in diameter, what is the surface?

5. If a cubic foot of granite weighs 165 lb., find the weight of a sphere of granite 1 ft. in diameter.

6. A bowl is in the form of a hemisphere 4.2 in. in diameter. How many cubic inches does it contain?

7. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water and 1 cu. ft. of water weighing 1000 oz.?

8. The diameter of a lead ball is $1\frac{1}{2}$ in., lead is 11.35 times as heavy as water, and 1 cu. ft. of water weighs 1000 oz. Find, to the nearest ounce, the weight of the ball.

9. Taking the radius of the earth as 4000 mi. and the earth as an exact sphere, find the volume to the nearest 1000 cu. mi.

10. Considering the moon as a sphere of diameter 2160 mi., compute the surface and also the volume to the nearest thousand of the unit required.

11. A ball with a diameter 6 in. is placed in a cubic box that is 6 in. on an edge, inside measure. Find the number of cubic inches of the box not occupied by the ball.

12. Find the volume of a sphere 1 ft. in diameter, of a cylinder of the same diameter and altitude, and of a cone of the same diameter and altitude. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone?

USING WHAT YOU HAVE LEARNED

1. A pyramid has a lateral area of 200 sq. in. The slant height is 8 in. Find the perimeter of the base.

2. A conic spire has a slant height of 68 ft. The perimeter of the base is 60 ft. What is the lateral surface?

3. What is the entire surface of a cone if the slant height is 7 ft. and the diameter of the base is 7 ft.?

4. A cornucopia has a circular top $3\frac{1}{8}$ in. across. If the cornucopia is $8\frac{3}{4}$ in. deep, how many cubic inches does it contain?

5. Compute the surface of a dome in the shape of a hemisphere having a diameter of 25 ft.

6. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindric marble column that is 10 ft. high and 16 in. in diameter?

7. Find to the nearest 100 cu. yd. the amount of earth which must be removed in digging a canal 2 mi. 1740 ft. long, 200 ft. wide, and 16 ft. deep.

8. A prism has a base of 22 sq. in. and an altitude of 28 in. How does its volume compare with that of a cone whose base has a radius 7 in. and whose altitude is 12 in.?

9. A marble 1 in. in diameter is dropped into a cylindric jar 4 in. high and 3 in. in diameter, half full of water. How much does the marble cause the water to rise?

10. A steel rod 6 ft. 10 in. long has a triangular cross section 3 sq. in. in area. If 1 cu. ft. of water weighs 62.5 lb., and steel is 7.83 times as heavy as water, how much does the rod weigh?

11. A funnel in the shape of a cone is closed at the bottom. If the circular top is 5 in. across, and the funnel is 6 in. deep (to the vertex), how many cubic inches of water will it hold?

USING WHAT YOU HAVE LEARNED

1. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. a second. Find the average number of gallons carried over the falls in a minute.

2. If $\frac{7}{8}$ in. on a map represents an actual distance of $35\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

3. How many loads (cubic yards) of gravel, to the nearest 100 loads, will be required to grade $2\frac{3}{4}$ mi. of road, the gravel to be laid 14 ft. wide and 6 in. deep?

4. A dealer pays \$9737.50 for some wool, including the purchasing agent's commission of $2\frac{1}{2}\%$. Find the commission.

5. The sugar contained in the sugar beet is $14\frac{1}{2}\%$ of the weight of the beet. How many pounds of beets will be required to produce 1740 lb. of sugar?

6. How many boards each 12 ft. long will be required to build a straight fence 4 boards high about a rectangular field which is 40 rd. wide and contains 20 acres?

7. Find the circumference of a circle whose diameter is 22.6 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

8. Find the diameter of a circle whose circumference is 471.24 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

9. If 24 burners, each consuming $7\frac{1}{2}$ cu. ft. of gas per hour, are used on an average 5 hr. per day for 365 da., what will be the gas bill for the year at \$1.05 per thousand?

10. A bell weighing 1110 lb. is composed of 76.5% copper, 21.5% tin, and the rest zinc. How many pounds of each metal does the bell contain?

IV. LITTLE EXAMINATIONS

i. 1. Find the premium on a fire-insurance policy for \$12,500 at \$1.24.

2. Find the premium on a life-insurance policy for \$15,000 at \$32.75.

3. Find the cost of 250 shares of stock at $96\frac{7}{8}$, including the brokerage of \$15 per 100 shares.

4. Find the income from $4\frac{1}{4}\%$ bonds of par value \$9500.

5. Draw a graph of the changes of temperature during a day when the thermometer indicated the following: 8 A.M., 60°; 9 A.M., 65°; 10 A.M., 68°; 11 A.M., 70°; noon, 75°; 1 P.M., 76°; 2 P.M., 78°; 3 P.M., 75°; 4 P.M., 70°; 5 P.M., 70°; 6 P.M., 68°; 7 P.M., 65°; 8 P.M., 58°.

ii. *Find the square roots of the following:*

1. $\frac{49}{144}$. 2. $\frac{225}{361}$. 3. $\frac{529}{729}$. 4. $\frac{1369}{1681}$. 5. $\frac{2209}{2601}$.

Find, to the nearest hundredth, the square roots of:

6. 13. 7. 17. 8. 30. 9. 125. 10. $\frac{5}{9}$.

iii. *Find the circumferences, given the following diameters:*

1. 34.1 in. 2. 32.2 ft. 3. 3.5 in. 4. $4\frac{1}{2}$ in. 5. 9 in.

Find, to the nearest hundredth, the square roots of:

6. 19. 7. 15. 8. 50. 9. 675. 10. $\frac{7}{12}$.

iv. *Find the discounts and the proceeds on the following notes:*

1. \$500, for 60 da., at 6%.
2. \$625, for 60 da., at 5%.
3. \$375, for 30 da., at $4\frac{1}{2}\%$.
4. \$825, for 90 da., at 4%.
5. \$675, for 90 da., at $4\frac{1}{2}\%$.
6. \$575, for 30 da., at 5%.

V. REVIEW AND DRILL

MINIMUM ESSENTIALS

Multiply, and also divide these numbers:

1. 172.8 by 1.44. 2. 473.48 by 6.23. 3. 47.595 by 5.01.

Add, and also subtract these numbers:

4.	5.	6.	7.	8.	9.
$3\frac{9}{10}$	$4\frac{7}{8}$	$7\frac{5}{8}$	$9\frac{1}{16}$	$8\frac{1}{32}$	$6\frac{4}{5}$
$2\frac{2}{3}$	$3\frac{1}{5}$	$2\frac{15}{16}$	$3\frac{5}{8}$	$2\frac{3}{4}$	$3\frac{3}{8}$

Multiply, and also divide these numbers:

10. $\frac{2}{3}$ by $\frac{4}{5}$. 11. $\frac{7}{32}$ by $\frac{1}{5}$. 12. $\frac{9}{10}$ by $\frac{2}{3}$. 13. 37 ft. 8 in. by 2.

Find, to the nearest hundredth, the values of the following:

14. 35% of \$23,468.25. 15. $7.231 \div 0.643$. 16. $2 \div 0.6234$.

Add these numbers without writing them in columns:

17. \$4.48 + \$5.92 + \$1.75 + \$5.75 + \$9.50 + \$19 + \$8.75.

18. \$24.17 + \$25.29 + \$41.50 + \$65.70 + \$79.85 + \$82.50.

19. \$42.96 + \$58.37 + \$87.62 + \$54.63 + \$42.85 + \$53.75.

20. \$58.17 + \$65.42 + \$96.54 + \$84.90 + \$68.79 + \$48.70.

21. \$63.44 + \$88.96 + \$17.28 + \$62.95 + \$78.69 + \$42.55.

22. Add 125% of 5280 ft. to $\frac{3}{4}$ mi., using a short method.

23. Find the value of $\sqrt{138}$ to the nearest hundredth.

24. Which will bring the better income, a 5% note or 6% stock, par value \$100, which costs, including brokerage, \$120?

25. A manufacturer pays \$5642.40 for materials, labor, and overhead charges on a certain contract, and makes a profit of $12\frac{1}{2}$% on these costs. Find the amount of the contract.

PROBLEMS WITHOUT NUMBERS

All work oral

1. If you know the face of a fire-insurance policy and the premium per $100, how do you find the premium?

2. There are companies that insure employers against accidents to the workmen. If you know the annual pay roll of an employer and the per cent of the pay roll that the insurance company charges, how do you find the total premium?

3. If you know the newspaper quotation on some railroad stock, how do you find the cost of a share?

4. If you know the newspaper quotation on a certain stock, how do you find the amount you would receive for a share?

5. How do you find the square of a number?

6. If you know the side of a square, how do you find the area? Illustrate by an example.

7. If you know the area of a square, how do you find the side? Illustrate by an example.

8. How do you find, by factoring, the square root of a number which is a perfect square? Illustrate by an example.

9. How do you find the hypotenuse of a right triangle when you know the other two sides?

10. If you know the base and height of a pyramid, what else can you find, and how do you find it?

11. If you know the height of a cylinder and the diameter of the base, what other measurements can you find relating to the cylinder, and how do you find them?

12. How do you find the area of a circle? the volume of a cube? the volume of a sphere?

PROBLEMS FOR COMPLETION

1. A man takes out a 20-payment limited life insurance policy for $15,000. The premium at his age is $26.40 annually.

2. A man buys 50 shares of stock when it is quoted at $108\frac{3}{4}$. The brokerage is $15 per 100 shares.

3. A man owns 25 shares of stock. A dividend of $2.75 a share is declared.

4. A man buys 30 shares of Western Union Telegraph stock at $87\frac{1}{2}$ and sells it at 90. The brokerage is as usual on each transaction.

5. Suppose that you have $3000 to invest and that with it you can buy $4\frac{1}{4}\%$ Liberty Bonds of par value $3200, including brokerage.

6. Mr. Huntington invests $12,500 in some property that pays him $5\frac{3}{4}\%$ a year.

7. Mr. Driscoll buys a house for $9800 and rents it at $68 a month. In a certain year he pays $56 for taxes, $32 for insurance, and $75 for repairs.

8. A square has an area of 22 sq. in.

9. Another square has a diagonal of 10 in.

10. A third square has a perimeter of 17 in. and it is to be covered with gold leaf at a cost of 3¢ per square inch.

11. A ladder 28 ft. long leans against a building. The foot of the ladder is 8 ft. from the building.

12. The edge of a cube is 16 in.

13. The area of one side of a cube is 289 sq. in.

14. The area of a circle is 44 sq. in.

15. The area of the surface of a sphere is 154 sq. in.

ALERTNESS TEST

Beginning at a signal from the teacher, copy the following on paper, insert in each set of parentheses the number which the teacher announces, then place a cross (+) before each statement that is correct, and a circle (○) before each one that is incorrect:

1. $3+(\ \ )=20.$
2. $(\ \ )-4=12.$
3. $2\times(\ \ )=30.$
4. $10\times(\ \ )\div 2=80.$
5. $5+(\ \ )=4\times 5.$
6. $9\times(\ \ )=153.$
7. $(\ \ )+(\ \ )=40.$
8. $10\times(\ \ )=120.$

The teacher should see that the pupils have copied all eight exercises, and should then say, "Take the number 15 and begin." The number given may be 15, 16, or 17, and should be changed as the test is repeated. The teacher should score the results as directed on pages 82, 144, and 214. Allow 10 sec.

Proceed as in the exercises above, using the following statements and inserting the numbers which the teacher directs:

1. $(\ \ )\div 2=24+1.$
2. $(\ \ )\div 2=51-1.$
3. $(\ \ )\div 7=10-3.$
4. $(\ \ )-3=47+1.$
5. $3\times(\ \ )=153.$
6. $2+(\ \ )=50.$
7. $(\ \ )-9=40.$
8. $(\ \ )\div 5=10.2.$

As before, the teacher should give a number when the class is ready. This number may be 48, 49, 50, or 51. Allow 15 sec.

By the aid of such tests a teacher's records will soon show the degree of alertness of each member of the class, and the work can be adjusted according to the individual powers of the pupils.

Such psychological-arithmetical tests are extensively used in the army and at present are becoming common in the placing of men and women in industrial positions that require alertness.

As already stated in connection with alertness tests, the work may be omitted at the discretion of the teacher.

Industrial Problems

PROBLEMS IN IRRIGATION

1. An acre-foot of water is the amount of water 1 ft. deep that it will take to cover a perfectly flat acre. What is an acre-inch of water, how many acre-inches are there in an acre-foot, and how many cubic feet are there in an acre-foot of water?

These terms are used in irrigation, particularly in the mountain states. The exercise may be omitted in those parts of the country in which irrigation is not common. The technical terms on this page need not be learned unless needed in the locality, and they are sufficiently explained in the problems to make formal definitions unnecessary.

2. A second-foot of water is a cubic foot passing a particular point in 1 sec. If 7 cu. ft. of water pass in 1 sec., the irrigation ditch is delivering 7 second-feet of water. In a flume 4 ft. wide, the water being 6 in. deep, it was found that the water was flowing at the rate of 10 ft. per second. How many second-feet of water was the flume carrying?

Allowance has to be made for friction of flowing water, but neither this nor the method of measuring the rate need be considered in the school.

3. A farmer has 80 A. of beets which he wishes to irrigate. How many cubic feet should he store in a reservoir if each acre requires 2 acre-feet of water for the season?

There will have to be an allowance for evaporation and other waste, but this problem is supposed to require the net amount.

4. A company is organized to furnish 25 sections (25 sq. mi.) of land with $1\frac{1}{2}$ acre-feet of water per acre annually. If 25% is allowed for loss by seepage and evaporation, so that only 75% of the water is available, and if the reservoir has an average depth of 40 ft. and must store at one time all the water necessary for a year, how many acres will the reservoir cover?

PROBLEMS RELATING TO WESTERN FARMS

1. A farmer has 200 A. on which he rotates his crops, with 3 yr. given entirely to alfalfa and 2 yr. entirely to sugar beets. The beets require 2 acre-feet of water per acre each season, and the alfalfa requires four fifths as much. How many acre-feet of water will the farmer need during the five-year period?

2. In a certain reclamation project it cost \$86,500,000 to reclaim 2,500,000 acres of land. Find the average cost per acre.

3. A farmer rented 50 A. of irrigated land for 5 yr. at \$5 per acre per year. It costs \$2 per acre to seed the alfalfa, or an average of 40¢ per acre per year; the preparation of the land costs \$1 per acre per year; and the fertilization costs 80¢ per acre per year. It cost \$5 per acre every time the alfalfa was cut, and it was cut twelve times in the 5 yr. The yield was 2.8 T. per acre per year. Find the total cost for 5 yr., the amount of alfalfa grown, and the cost per ton.

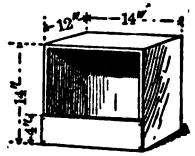
The crops as well as the industries vary in different parts of the country. The teacher may omit pages which relate to industries with which the pupils are not familiar.

4. Some experiments in Utah show that alfalfa grows best with 1.6 acre-feet of water per season, wheat and oats with 1.2 acre-feet, barley with 1.7 acre-feet, and sugar beets with 1.9 acre-feet. A Utah farmer had 100 A. of alfalfa, 50 A. of wheat, 75 A. of oats, 40 A. of barley, and 50 A. of sugar beets. He used uniformly 2 acre-feet of water per acre on all this land. Disregarding any damage to the crops, how much water did he waste?

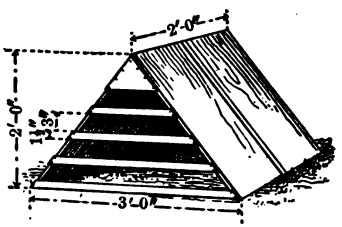
5. Some farmers use this rule for finding the number of bushels of corn in a bin: Take eight times the product of the three dimensions of the bin and divide by ten. Using this rule, find the number of bushels in a bin 6 ft. by 10 ft. by 4 ft.

HOUSING POULTRY

1. Charles helped his father make some nests and coops. This is a picture of one of the nests for a sitting hen. Charles figured out the number of feet of lumber $\frac{7}{8}''$ thick needed for twelve nests. How many feet were required? How much would the lumber cost at \$80 per M?

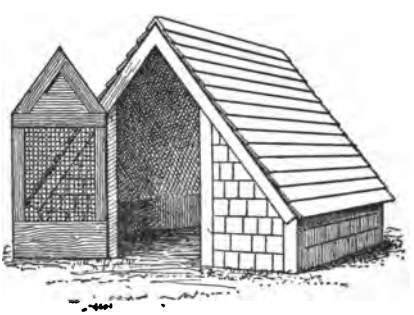


2. For the hens and chickens Charles and his father made twelve coops like the one pictured below. Charles drew the figure to the scale of 1 in. to 1 ft. and then measured the size of each part of the coop. Do this, and find the number of feet of lumber $\frac{7}{8}''$ thick required and also find its value at \$80 per M.

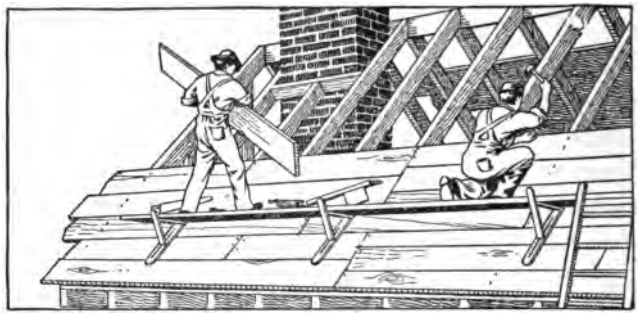


Some approximation is allowed here, but this is exactly what happens in practical work, and is very desirable.

3. Charles's father built a sufficient number of poultry houses like this for his hens. The length of each was 9 ft., the width 7 ft., the height in the center 6 ft., and the height of the eaves 18 in. Allowing 20 cu. ft. for each hen, how many hens will this house accommodate?



If Charles gets an answer like 11.8 hens he will say that the house will accommodate a dozen hens. In other words, he will use his common sense with such an answer, just as you should always do.



PROBLEMS OF THE BUILDER

1. The distance from the eaves to the ridge of this roof is 13 ft., and the span is 24 ft. Find the rise of the roof, that is, the height of the peak above the level of the eaves.

2. The span of a common A-shaped roof is 26 ft. and the rise of the roof is 6 ft. What is the distance, to the nearest hundredth, from the eaves to the ridge?

3. A semicircular arch over a door has a diameter of 4′ 1″. Find the length of the semicircumference.

4. A builder is putting in a steam pipe 168 ft. long and 2 in. in diameter. Find the radiating surface, that is, the curved surface that gives out heat.

5. A contractor places on the top of a city apartment house a cylindric water tank 20 ft. high and 10 ft. in diameter. How many gallons of water will the tank hold?

6. The excavation needed for the cellar of a city apartment house is to be 92 ft. long, 60 ft. wide, and 7 ft. deep. In addition to this the excavation is to be extended 8 in. beyond each of the four sides, so as to allow space for working. How many loads (cubic yards) of earth must be removed?

HAULING BY MOTOR TRUCK

1. In a recent year the cost of a one-ton motor truck was \$1850 for the chassis and \$125 for the body. The freight and the war tax amounted to \$60. Find the total cost.

2. Find the amount of the yearly upkeep of the motor truck in Ex. 1, the items being as follows: repairs, \$75; tires, \$420; insurance, \$61.05; depreciation due to wear, 10% of the original cost of the truck.

3. The truck ran 14,400 mi. in the year, averaging 9 mi. to a gallon of gasoline. Find the total cost of the gasoline for the year at 28¢ a gallon.

4. The motor truck used 1 qt. of lubricating oil for each 90 mi. that it ran. Find the total cost of the oil for the year, the distance run being 14,400 mi. and the oil costing 95¢ a gallon.

5. The total cost for the year in which the motor truck ran 14,400 mi. was found to be \$1457, not including wages. Find to the nearest cent the average cost per mile.

6. In a recent year the cost of a two-ton motor truck was \$2600 for the chassis and \$150 for the body. The freight and the war tax amounted to \$70. Find the total cost.

7. In Ex. 6 the yearly operating expenses were \$1977.95. Find to the nearest tenth the per cent of original cost to be charged to operating cost.

8. During a certain season 158,400 hogs were delivered to the Omaha market by motor truck. Allowing 80 hogs to a car, how many freight cars would it take to transport these hogs?

9. In a recent year 138,000 cattle were delivered to the Cincinnati stockyards by motor truck. Allowing 30 cattle to a car, how many freight cars would it take to transport these cattle?

COMPARATIVE COST OF HAULING

1. A farmer who lives 9 mi. from the railroad station finds that he can haul one wagonload of corn daily, each load containing 39 bu., and each bushel containing 56 lb. Find the weight of his load.

2. If the farmer in Ex. 1 hauls corn 5 da. each week for 7 wk., what is the total distance traveled by his horses? How many bushels does he haul? How many pounds? How many tons?

3. Before the war it cost on the average 19¢ to haul 1 T. a mile in a wagon, counting the return trip as part of the haul. At this rate, what was the cost of hauling the corn in Ex. 2?

4. At the close of the war the cost of hauling by wagon had increased to 33¢ per ton per mile. What did it then cost the farmer to haul the corn as in Ex. 2?

5. A year later the farmer bought a motor truck which made four round trips a day and hauled 58 bu. of corn per trip. Taking the same amount as in Ex. 2, how many trips did it then require? How many days? How many days of labor were saved?

If you have a little less than a load for the last trip, common sense tells you to call it a whole load.

6. If it cost 15¢ per ton per mile to haul corn in a motor truck, counting each return trip as part of the haul, what was the cost of hauling the corn of Ex. 5 in the truck? How much was saved by the use of the truck over the cost of the haul as found in Ex. 4?

7. Considering the above problems on a percentage basis, how did the amount of corn hauled daily by wagon compare with that by truck? The cost of hauling by truck in Ex. 6 was what per cent of that of hauling by wagon as found in Ex. 4?

SOME FIGURES FOR THE FARMER

1. If it takes a farmer $1\frac{1}{2}$ hr. longer to make a trip over an ordinary dirt road than it does over a concrete one, how many hours are lost in a year by 1000 farmers who average 12 trips each per month over a dirt road? How much is the financial loss at \$1.10 per hour for each man and his team?

2. In a concrete road 18 ft. wide transverse joints are placed every 50 ft. These transverse joints are cross seams in the concrete, dividing it into large blocks. Each joint is filled with tarred paper, three strips to each joint. How many feet of strips will be needed for 1 mi. of road?

3. Many states and counties sell bonds to obtain the money for building concrete roads. A certain county has a valuation of \$75,000,000 and is allowed to issue bonds up to 5% of the valuation. If a concrete road 18 ft. wide costs \$30,000 a mile, how many miles can be built from the proceeds of these bonds?

4. A farmer wishes to build a garage 10 ft. wide and 16 ft. long, the sides being 8 ft. high to the eaves. Making no allowance for doors or windows and allowing $\frac{1}{6}$ extra for matching, how much will the siding, $\frac{7}{8}$ in. thick, cost at \$85 per M?

5. Find the cost of cementing the floor of the garage in Ex. 4 at \$1.80 per square yard. Also find the cost of painting the outside of the garage two coats, the first costing 18¢ per square yard, and the second costing $33\frac{1}{3}\%$ more than the first.

6. It is estimated that a herd of 70 dairy cows will consume 336 T. of silage in 240 da. This allows an average of how many pounds per cow per day?

7. If it requires 240 T. of silage to fill a silo 18 ft. in diameter and 42 ft. high, what is the average weight of silage per cubic foot?

FARM PROBLEMS

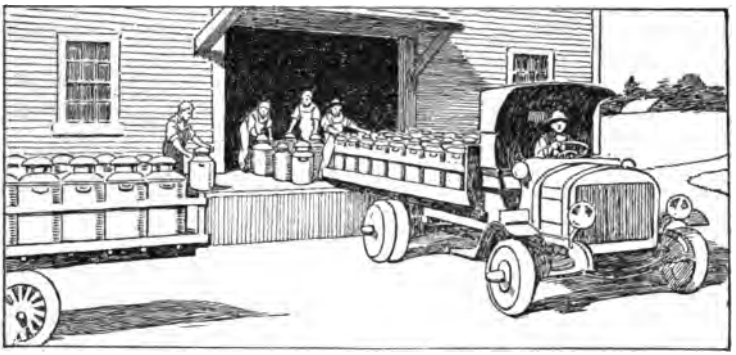
1. A farmer who owns a square piece of land containing 40 A. decided to improve the property by tile draining, constructing four drains of 5-inch tile across the land and parallel to one of the sides. The tile was laid at an average depth of 38 in. and cost 70¢ a rod. The man who dug the ditches charged 50¢ a rod for a depth of 29 in. and 3¢ a rod for each additional inch of depth. Find the cost of putting in the drains.

2. Mr. James laid 128 rd. of 8-inch tile, 16 tiles to a rod, in a ditch 5 ft. deep. He paid $85 per thousand for the tile and $16\frac{1}{2}$¢ a foot for digging the ditch, laying the tile, and covering it. Find the total expense.

Count any part of 100 tiles as a hundred.

3. A farmer wishing to seed his three wheat fields to clover asked his son to find the cost of the seed. The son found that he could buy the seed at $26 per bushel, and that he must use 6 qt. to an acre. He measured the fields and found them to be as follows: 40 rd. by 60 rd., 165 ft. by 330 ft., and 14 rd. 9 ft. by 18 rd. 8 ft. Find the cost of the seed.

4. A farmer is building a barn 120 ft. long and 40 ft. wide. The main posts are $20\frac{1}{2}$ ft. high, and the peak of the A-shaped roof is 15 ft. higher than the top of the main post. The eaves extend 1 ft. beyond the side walls, and the roof extends 9 in. beyond the end walls of the barn. An allowance of 10% of the area of the walls should be made for the openings for doors and windows. Estimating 1000 shingles to 120 sq. ft., how many thousand shingles will the farmer need to shingle the roof and sides of the barn? If in ordering the shingles he orders a whole thousand for any fraction of a thousand needed, how much do the shingles cost at $11.50 per M?



GENERAL REVIEW

DAIRY PROBLEMS

1. How many pounds of butter fat are there in 26,250 lb. of milk which tests 4.2% butter fat?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value of the butter produced from 1236 lb. of butter fat, the butter being worth 44¢ a pound?

3. A farmer shipped a quantity of milk to a creamery, the milk averaging 3.75% butter fat. If the butter fat weighs 630 lb., how many pounds of milk were there in the shipment?

4. A farmer has two cows, one supplying 986 lb. of milk testing 3.1% butter fat in a certain month, and the other 812 lb. testing 4.2%. If the creamery allows him 38¢ a pound for butter fat, which cow pays him the more for butter fat in that month, and how much more, the feed costing the same?

5. A creamery uses 7500 lb. of milk in a week. The skim milk, containing 0.6% butter fat, amounts to 80% of the whole milk. How much butter fat is lost in the skim milk?

PROBLEMS ON SPRAYING

1. In order to lessen potato scab some seed potatoes are soaked in a solution of $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture?

2. In Ex. 1 what would be the per cent of formalin in the mixture if $\frac{5}{8}$ pt. had been used to 15 gal. of water?

3. Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$ by weight of copper sulphate, $1\frac{1}{4}\%$ of unslaked lime, and the rest water. How many pounds of each substance are there in a barrel containing 250 lb. of the mixture?

4. A cubic foot of Bordeaux mixture weighs 63 lb. From the statements made in Ex. 3 find the number of pounds of copper sulphate and unslaked lime in the Bordeaux mixture that fills a tank 6 ft. long, 4 ft. wide, and 1 ft. 6 in. deep.

5. Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 284.8 lb. of fruit were taken, and from the other half 95% less. How many pounds were taken from the tree?

6. In preparing a spray for use on his trees, a man uses 3 lb. of arsenate of lead and 4 lb. each of lime and copper sulphate to 50 gal. of water. If the arsenate of lead costs 40¢ a pound, the lime 1¢ a pound, and the copper sulphate 15¢ a pound, and the man uses $2\frac{1}{2}$ gal. of the spray on each tree, what is the cost of the material required for spraying 125 trees?

7. For spraying potato plants a man prepares a solution of Paris green, using $\frac{1}{2}$ lb. of Paris green costing 60¢ a pound to 100 gal. of water. He uses 300 gal. to an acre and sprays a field of $3\frac{1}{2}$ acres. What is the cost of the material? If the labor costs \$14.50, what is the total cost? If the crop raised brings him \$56 more than if he had not sprayed, how much is his gain?

PROBLEMS ON TRANSPORTATION

1. If a freight car is 36 ft. long, 8 ft. 3 in. wide, and 7 ft. 6 in. high, inside measure, how many cubic feet does it contain?

2. If the width of the car in Ex. 1 were increased 1 in., and the length 1 ft. 6 in., what would be the increase in capacity?

3. If a wooden coal car can carry 80,000 lb., and a steel car 100,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

4. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Reckoning a bushel as containing $1\frac{1}{4}$ cu. ft., and a bushel of wheat as weighing 60 lb., what is the weight of the wheat in the car?

5. If the car of Ex. 4 has a carrying capacity of 80,000 lb., to what depth must it be filled with wheat to contain exactly this amount?

6. If a locomotive weighing $115\frac{1}{2}$ tons can exert a pull of $22\frac{1}{4}\%$ of its weight in starting a train, how many pounds of pull does it exert?

7. If a locomotive and coal tender together weigh 95 tons, and there are seven cars to the train, averaging 21 tons each, and a horizontal pull equal to 0.5% of the total weight of the train is required to maintain the necessary speed on a level track, what is this pull in pounds?

8. A 20,000-ton ship carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses per year are only four times as much. In a year such a modern boat will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

Problem Data. The following price list may be used in solving the problems on page 297 and similar problems:

Coffee: Maracaibo, 40¢ per pound; 5 lb. for $1.70.
Mocha and Java, 62¢ per pound; 5 lb. for $2.90.
Old Government Java, green, 54¢ per pound; 5 lb. for $2.60.
Tea: Black India, 80¢ per pound.
English Breakfast, 70¢ per pound.
Ceylon, finest, $1.40 per pound.
Cocoa: half-pound cans, 45¢; $4.50 per dozen cans.
Sugar sirup: half-gallon cans, 90¢; 5-gallon cans, $8.
Maple sirup: pints, 50¢; gallon cans, $2.90; $33 per dozen cans.
Honey: 16-ounce bottles, 65¢; $7 per dozen bottles.
Flour: buckwheat, 10¢ per pound; a bag of 24½ lb., $2.20.
Self-raising, 3 lb. for 35¢; 6 lb. for 65¢.
Wheat, 9¢ per pound; $15 per barrel of 196 lb.; $2.05 per bag of 24½ lb.
Breakfast cereal: 24¢ per package; $2.45 per dozen packages.
Macaroni: 24¢ per package; 25 packages for $5.50.
Crackers: Alberts, 32¢ per tin; $3.25 per dozen tins.
Soda, 10¢ per carton; $1.05 per dozen cartons.
Soups: half-pint cans, 12¢; pint cans, 22¢; quart cans, 38¢; $4.35 per dozen quart cans.
Bacon: American, 45¢ per pound.
Sliced in jars, 48¢ per pound; $4.85 per dozen jars.
Herrings: 25¢ per can; $2.55 per dozen cans.
Asparagus: 45¢ per can; $4.80 per dozen cans.
Loaf sugar: 14¢ per pound.
Granulated sugar: 12¢ per pound.
Olives: 54¢ per bottle; $5.65 per dozen bottles.
Olive oil: $1.30 per bottle; 75¢ per half bottle; $12 per dozen bottles.

Allspice: 25¢ per can.	**Cloves**: 80¢ per pound.
Cinnamon: 90¢ per pound.	**Nutmegs**: 95¢ per pound.
Pepper: 48¢ per pound.	**Ginger**: 65¢ per pound.

The data may also be secured by the pupils by inquiry at home or at some grocery. The prices secured may be made the basis of practical problems in simple domestic bookkeeping.

INDUSTRY OF THE KITCHEN

1. If a family needs a dozen cans of cocoa, how much is saved by purchasing at the dozen rate?

2. If a family needs 5 gal. of sugar sirup, how much is saved by purchasing a 5-gallon can instead of buying ten half-gallon cans? What per cent is saved?

3. How much would a hotel save by purchasing 120 gal. of maple sirup by the dozen gallon cans instead of buying single cans? What per cent would the hotel save?

4. In which is the per cent of saving greater, in buying maple sirup by the dozen cans instead of by the can, or in buying honey by the dozen bottles instead of by the bottle?

5. At the rate charged for a pound of wheat flour how much will $24\frac{1}{2}$ lb. cost? How much is saved in buying a bag?

6. What is the per cent of saving in buying self-raising flour by the 6-pound package instead of by the 3-pound package?

7. After inquiry at home, make out a list of the groceries needed for a week. Using the prices on page 296, make and balance two pages of a home account, the left-hand page showing the amount allowed for housekeeping expenses for the week, and the right-hand page the amounts spent for groceries.

8. Make out a bill for six items of groceries, making the proper extensions and footing. Receipt the bill.

9. Make out a bill for twelve items of groceries, as in Ex. 8.

10. If gas costs \$1 per 1000 cu. ft. and your family uses 2300 cu. ft. in May, how much is the gas bill for that month?

11. At the beginning of the month the gas meter registers 14,200, and at the end of the month 17,100. How much is the gas bill for the month, at \$1.10 per 1000 cu. ft.?

PROBLEMS IN MANUFACTURING

1. Find the cost of 8 rockers at \$14.50, 9 dressers at \$29, and 16 bedsteads at \$37.50, a discount of 8% being allowed.

2. A manufacturer sells the following: 8 doz. plates @ \$2.84; 9 doz. cups @ \$4.70; $\frac{3}{4}$ doz. covered dishes @ \$19.90; 16 doz. plates @ \$3.30; $\frac{5}{6}$ doz. pitchers @ \$13.50. The bill is discounted at 3%. If the net receipts represent a profit of 20% on the cost, find the cost of manufacturing the lot.

3. A manufacturing company has a capital of \$350,000, its annual business is $3\frac{1}{4}$ times its capital, and its profits are $7\frac{1}{2}$% of its business. How much are its annual profits?

4. A dealer sells a lamp for \$15 at a profit of 20% on the cost, and the manufacturer who sold it to the dealer made a profit of 25% on the cost. Find the cost of manufacture.

5. On which will a manufacturer make the greater per cent of profit on the cost: an article which costs \$2.90 to make and sells for \$3.50 less 2%, or an article which costs \$8.70 to make and sells for \$12 less $\frac{1}{6}$? How much greater is this per cent?

6. A manufacturer makes a bid on a contract to turn out certain articles. He wishes to make 15% profit on the cost, and finds that the cost of labor, materials, and overhead is \$3782.40. What bid to the nearest dollar should he make?

7. A company manufactures circular water tanks 20 ft. in diameter and 28 ft. high. If a cubic foot contains $7\frac{1}{2}$ gal., what should the company advertise as the capacity of the tanks?

8. A factory pays a salesman a commission of $4\frac{1}{2}$% on all sales. If the salesman sells \$245,000 worth of goods in a certain year, and his expenses, which he himself pays, are \$4487.60, how much is his net income for that year?

PROBLEMS OF THE MERCHANT

1. A dealer received \$31.50 for a suit of clothes which he sold at 10% off the marked price. What was the marked price?

2. How much is paid for a bill of goods amounting to \$82.60 which are bought at a sale where a discount of 15% is given?

3. A merchant sold for \$308 goods which cost him \$274.60. What is the per cent of profit on the cost? on the selling price?

4. A furniture dealer buys tables at \$16.75 each and sells them at \$25 each. His buying and selling expenses and overhead are 24% of the cost. What per cent does he make on the total cost?

5. If the dealer in Ex. 4 had calculated his cost of doing business as 24% of the selling price, what would have been his per cent of profit on the selling price?

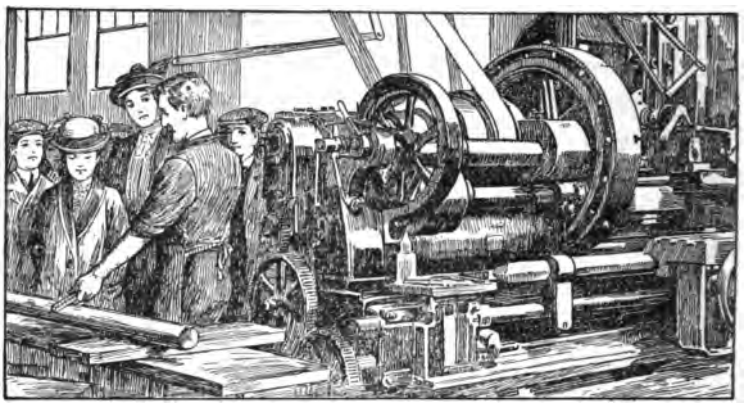
In the following problems on this page the profit and the cost of doing business are to be calculated on the selling price.

6. A hardware dealer buys stoves at \$28.50 each. His cost of doing business is 28% and he wishes to make a profit of 15%. Find the selling price which the dealer must put on the stoves.

7. A merchant buys a line of carpeting at \$1.84 a yard. His cost of doing business is 14% and his profit is to be 10%. Find to the next larger multiple of 10¢ the price per yard at which he must sell the carpeting.

8. A dealer buys some kitchen chairs at \$2.24 each with discounts of 25% and 10%. If the cost of doing business is 16% and the profit is to be 12%, find to the next larger multiple of 25¢ the price at which he must sell the chairs.

9. A dealer pays \$972.40 for a lot of goods. The cost of doing business is 20% and the profit is 12%. Find the profit on the sale of the entire lot.



A VISIT TO THE MACHINE SHOP

1. Miss Adams took her class to visit a machine shop, and the foreman told them that he and his men used arithmetic a great deal in the work of the shop. He showed them a casting that was 3 ft. 7 in. long and explained that iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat. How long would this casting be when red hot?

2. The foreman told Miss Adams that all metals did not expand at the same rate, and that in casting brass hinges they had to allow for a shrinkage of $\frac{1}{64}$ in length when the brass cooled. The boys were told to figure out the length of the mold for a hinge that was to be 5.6 in. long when cooled. How long was it?

3. The foreman showed them a piece of steel shafting that the men were just setting up, which he said was 3 in. in diameter, 20 ft. long, and weighed 490 lb. to the cubic foot. He said they had to know the weight of the shafting in order to be sure to give it ample support. The girls were told to figure it out. How much should they find that the shafting weighed?

PROBLEMS OF THE MACHINIST

1. If the diameter of a wheel is 7 in., find the circumference.

2. Machinists often use decimals in measuring. Find the length in feet and inches, to the nearest hundredth of an inch, of the circumference of a flywheel 7 ft. 6 in. in diameter.

3. The diameter of the piston of an engine is 8 in. Find the area of the cross section of the piston.

4. If the pressure of steam in a boiler is 140 lb. to the square inch, find the pressure on a valve 2.5 in. in diameter.

5. A boiler contains 480 flues, each having an internal diameter of $2\frac{5}{8}$ in. Find the total area for draft through them.

6. If a pressure of 130 lb. to the square inch is applied to a circular plunger 2 ft. in diameter, what is the total pressure?

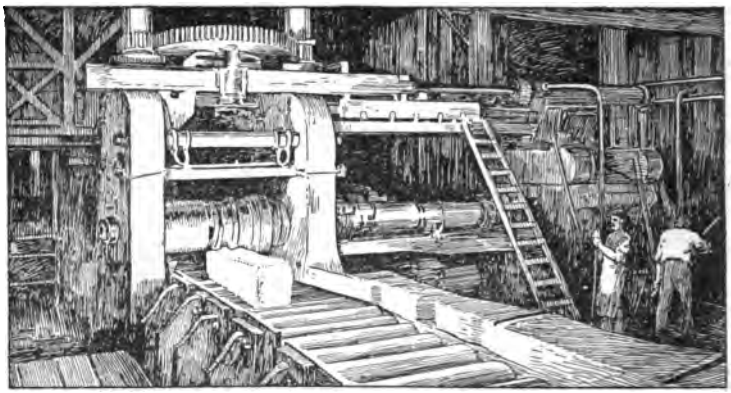
7. The grinding speed of an emery wheel, usually given in feet per minute, is the product of the circumference and the number of revolutions per minute. Find the grinding speed of an emery wheel $10\frac{1}{2}$ in. in diameter which makes 2000 revolutions per minute.

8. A flywheel 6 ft. in diameter makes 100 revolutions per minute. At what rate does a point on the rim travel?

9. How many revolutions per minute must be made by a circular saw the diameter of which is 28 in., in order that the saw may have a cutting speed of 8800 ft. per minute?

10. Two cogwheels are so geared that for every revolution of the larger wheel the smaller revolves 12 times. The larger wheel has 204 cogs. How many cogs has the smaller?

11. A flywheel 8 ft. in diameter which makes 250 revolutions per minute is connected by a belt with a wheel 2 ft. in diameter. How many revolutions per minute does the smaller wheel make?



THE IRON INDUSTRY

1. What is the weight of a steel girder rolled out at this mill, the girder being 26′ 6″ long and weighing $42\frac{1}{2}$ lb. to the foot?

2. The mill also rolls out iron rods. What is the cost of 2240 ft. of iron rod, $2\frac{1}{8}$ lb. to the foot, at $2\frac{3}{4}$¢ a pound?

3. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}\%$ as much as the iron. The pattern weighs $45\frac{5}{8}$ lb. How much does the casting weigh?

4. If steel rails weighing 120 lb. to the yard are used between New York and Chicago, a distance of 980 mi., how many tons will be required for a double-track road between these cities?

5. An iron tire expands $1\frac{3}{16}\%$ on being heated for shrinking on a wheel. A wooden wheel needs a tire 4′ 8″ in diameter. How much longer will the tire be when thus heated?

6. In a certain blast furnace the casting machine turns out 20 pigs of iron per minute, averaging in weight 110 lb. each. If this machine runs at this rate for 308 da., 16 hr. a day, how many long tons, 2240 lb. each, of pig iron will it turn out?

THE POTTERY INDUSTRY

1. In a shipment of a carload of earthenware to Chicago there were 15 washstands weighing 150 lb. each, 10 bathtubs weighing 900 lb. each, and 50 laundry tubs weighing 250 lb. each. What was the freight on the shipment at 90¢ per hundred pounds?

2. A manufacturer ships a carload of bathtubs. As packed, each tub is 5 ft. 6 in. long, 30 in. wide, and 21 in. deep. They are stood on end, one deep, in a car that is 36 ft. long and 8 ft. wide. How many tubs can be put in the car?

The problem requires practical judgment. In the first place, one dimension is not used. In the second place, more tubs can be put in the car if placed one way than can be put in if they are placed another way.

3. Each of the tubs mentioned in Ex. 2 weighs 980 lb., and the weight of the crate and packing is 15% of the weight of the tub. Find the total weight of the shipment.

4. The cost of material used in making each of the tubs mentioned in Exs. 2 and 3 is 4¢ per pound. Find the cost of the raw material used in manufacturing the lot.

5. The cost of a 1000-pound charge of glaze needed for 150 washstands is 14¢ per pound. Find the cost of the glaze needed for 75 washstands.

6. It requires $18\frac{1}{2}$ T. of egg coal to fire a kiln. How many kilns will a carload of coal weighing 106,720 lb. fire, and how much will it be worth at $9.50 per ton?

7. A press of clay weighing 3200 lb. costs $42 per ton. How many washstands, each weighing 200 lb., will this press of clay turn out, and what is the cost of the clay in each?

When possible the class should visit industrial plants and obtain problems for themselves. Technical expressions should be avoided as far as possible. Such problems should be discussed as informational exercises.

PROBLEMS OF THE CONTRACTOR

1. If a contractor makes $\frac{1}{4}$ of the amount he receives for a \$4500 job, how much profit does he make?

2. If a contractor makes \$675 on a job for which he receives \$3375, how much profit does he make on each dollar received? How much profit on each dollar which the job cost him?

3. A contractor builds a brick wall 68′ 8″ long, 5′ 8″ high, and 2 bricks thick. Allowing 14 bricks per square foot of outside surface for a 2-brick wall, how many bricks does he use? How much will the bricks cost at \$17 per M if in buying them he has to order to the next higher hundred required?

4. A man contracted to paper a house. The rooms are $14' \times 16'$, $8' \times 12'$, $15' \times 18'$, $10' \times 12'$, $16' \times 16'$, and $12' \times 12'$. For openings he deducts 6 ft. from the perimeter of each room and allows one roll of paper for every $3\frac{1}{2}$ ft. of perimeter. How many rolls does he need in all? If the paper for the first two rooms costs $35\cent$ a roll; for the third and fourth, $40\cent$; and for the last two, $55\cent$; what is the total cost of the paper?

5. A man has a contract to put up the four walls of a building which is to be 80′ long, 35′ wide, and 78′ high. The walls are to be 18″ thick, and 750 cu. ft. is to be allowed for openings. Allowing 22 bricks to a cubic foot, how much will the bricks cost at \$18.50 per M if he orders to the next $\frac{1}{2}$ M required?

6. A contractor wishes to build a concrete wall 66 ft. long, 8 ft. high, and 18 in. thick. Concrete is $\frac{1}{9}$ Portland cement, which costs \$3.80 a barrel of 400 lb., and the rest is gravel costing \$1.75 a load (cubic yard), delivered. A cubic foot of dry cement weighs 150 lb. In placing his order the contractor cannot order a fraction of a barrel or of a load. What will be the cost of all the materials for the wall?

Review Examinations. Pages 305–314 are devoted to oral and written review examinations, ten examples to a set.

WRITTEN EXAMINATION

1. Find the interest on \$428 for 3 mo. 6 da. at 6%. Solve in two different ways, stating the operations in steps.

2. From a lot 12 rd. by 16 rd. a path 4 ft. wide is cut on all four sides. How many square yards are left?

3. A man sold a lot for \$600, gaining 20%. He sold another lot for \$600, losing 20%. Did he gain or lose on the two transactions together, and how much?

4. Which is the better investment of \$1500 for 2 yr.: a note bearing $4\frac{1}{2}\%$ interest, or a savings-bank account at 4%, the interest being compounded semiannually? How much better?

5. If $x : 17\frac{1}{2} = 9\frac{1}{2} : 35$, what is the value of x?

6. A woman living in a city earned \$1638 last year by shopping for her friends. On half of her purchases she received 4% commission and on half she received 5%. How much was the cost of the goods that she bought?

7. Multiply 7 ft. 6 in. by 4 and divide the product by 5.

8. A wholesale jeweler advertised his stock to be sold at 50% and 25% off. I bought a watch for \$30. What was the list price?

9. A man paid \$12,800 for a house. He rents it for \$840, paying taxes at the rate of 12 mills on \$1, \$24.20 for insurance, and \$144 for other expenses. Find his net rate of income.

10. How many yards of carpet $\frac{1}{2}$ yd. pattern and $\frac{3}{4}$ yd. wide will it take to cover a room 16 ft. 8 in. long and 12 ft. wide, cutting so as to match patterns and running the strips in the more economical way?

WRITTEN EXAMINATION

1. Find to the nearest hundredth of a foot the side of a square whose area is 50 sq. ft.

2. The lunar month, the time from one new moon to the next, is 29 da. 12 hr. 44 min. 3 sec. Express 6 lunar months in days and the decimal of a day.

3. Find the difference in the number of acres in two fields, one of which contains 120 sq. rd., and the other of which is 120 rd. square.

4. If a certain gun metal is composed of 84% copper and the remainder tin, how many pounds of tin must be used with 638.4 lb. of copper to make the gun metal?

5. The net earnings of a company with a capital of $1,250,000 were $103,750.75 last year. It paid a dividend of 6% and carried the balance to surplus. How much did the company pay in dividends and how much did it carry to surplus?

6. A farmer has a bin 18 ft. long, 5 ft. 3 in. wide, and 5 ft. deep which holds 378 bu. How many bushels in a bin that is 4 ft. longer, 1 ft. 9 in. narrower, and has the same depth?

7. Water is flowing into a tank whose base is 9 ft. 4 in. long and 6 ft. 9 in. wide, at the rate of 3 cu. ft. in 2 min. How long will it take to increase the depth of the water 4 ft. 6 in.?

8. One face of a cube has an area of 3 sq. in. What is the volume of the cube?

9. A woman pays taxes at the rate of $1.09 per $100 on property assessed at $7500. Find the amount of her taxes.

10. How high is a tree which casts a shadow 85 ft. long at the same time that an upright post 3 ft. high casts a shadow 4 ft. 3 in. long?

ORAL EXAMINATION

1. How much is the interest at 6% on \$250 for 30 da.? on \$675 for 60 da.? on \$840 for 90 da.?

2. What is the amount of a 90-day note for \$1200 if the note bears 6% interest? if the note bears 5% interest?

3. During the year a merchant borrows at various times \$100, \$200, \$1200, \$1500, each time for 60 da. at 6%. How much interest does he pay in all?

4. A man having \$10,000 invested at $4\frac{1}{2}$% reinvests half of it at 5% and leaves the rest as before. What is the increase in his income?

5. A man borrowed \$8000 at 5% and invested it in a distant state at 8%. How much did he make per year by the transaction? Why does not everyone who has money to invest follow this plan?

6. On a bill of goods amounting to \$1500 a dealer is allowed a 2% discount for cash. If his bank will discount a 30-day note for \$1500 at 6%, how much does the dealer gain by borrowing the money from the bank and taking the cash discount?

7. At a bargain sale a dealer sells a suit of clothes marked \$33 at $16\frac{2}{3}$% off the marked price. Find the selling price.

8. A dealer places an order for goods amounting to \$1000 at list prices, and is allowed a discount of 20%, 10%. What is the net amount of the bill for these goods?

9. If a man needs \$740 to pay for goods and his bank will discount a 60-day note for \$750 at 6%, will the proceeds of the note be more or less than the man needs? How much more or less?

10. What single discount is equivalent to a discount of 10%, 10%? to a discount of 20%, 10%?

WRITTEN EXAMINATION

1. On a certain run of 161.88 mi. the rates of the express and a local train are 45.6 mi. an hour and 28.4 mi. an hour respectively. How much time is saved by taking the express train?

2. From a tank full of water $\frac{3}{5}$ of the contents was drawn off. The tank then lacked 70 gal. of being half full. What is the capacity of the tank?

3. A tank of capacity $166\frac{1}{2}$ gal. is $\frac{2}{5}$ full of water. How long will it take a pipe supplying water at the rate of 3.7 gal. a minute to fill the rest of the tank?

4. A man who owned some shares of stock in a mill sold $\frac{1}{3}$ of his shares to A, $\frac{1}{5}$ to B, and $\frac{1}{6}$ to C. He then had 90 shares left. How many shares had he at first?

5. A boy lost $\frac{1}{6}$ of his kite string in a tree and 30 ft. of it in some wires. If he had $\frac{3}{4}$ of it left, how long was it at first?

6. How many bricks will be required for the four walls of a house 42 ft. long, 28 ft. wide, and 18 ft. high, allowing for 3 doors 4 ft. by 7 ft. 6 in. and for 9 windows 3 ft. by 5 ft., the walls to be 1 ft. thick, and using 22 bricks to 1 cu. ft.?

7. The sugar beet contains by weight $14\frac{1}{2}\%$ of sugar. In a recent year the United States produced 7,265,000 T. of sugar beets. How much sugar did this represent?

8. A man has a salary of \$3500 a year. He pays 15% of his salary for rent, 18% for clothing, 20% for groceries, 8% for meats, \$945 for other expenses, and saves the rest. What per cent of his salary does he save?

9. Find the square root of 7 to the nearest thousandth.

10. How many tons of coal can be placed in a bin 16 ft. 6 in. long, 7 ft. wide, and 6 ft. deep, allowing 33 cu. ft. to the ton?

WRITTEN EXAMINATION

1. Find the difference in cross section of two shafts whose diameters are 2 in. and 4 in. respectively.

2. At $2.90 a yard what is the cost of a stair carpet for a straight flight of 16 steps 12 in. wide and 8 in. rise, allowing 20 in. extra at the bottom of the stairs and the same amount, besides the 12 in. for the top step, at the top?

3. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 650 lb. of flint glass.

4. A bell weighing 875 lb. is composed by weight of 77% copper, 2% zinc, and the rest tin. How many pounds of each metal does the bell contain?

5. If gun metal is composed by weight of 85½% copper and the rest tin, how many pounds of copper must be melted with 159½ lb. of tin to make gun metal?

6. If bronze is composed by weight of 18% tin and the rest copper, how many pounds of tin must be melted with 1066 lb. of copper to make bronze?

7. The driving wheels of a locomotive have a diameter of 6 ft. 5 in. How many revolutions a minute will each wheel make when the locomotive is traveling at the rate of 50 mi. an hour?

8. The front wheels of a wagon are 3 ft. 6 in. in diameter and the rear wheels are 4 ft. 1 in. How many more revolutions will a front wheel make than a rear wheel in going a mile?

9. Multiply 17 ft. 9 in. by 6; by 12; by 24.

10. It is required to make a rectangular tin box that shall contain exactly 1 gal. Find the three dimensions of the box such that they shall each be in exact inches.

WRITTEN EXAMINATION

1. A boy computing the cost of 56 articles made the mistake of multiplying by 65 instead of by 56, and obtained $487.50. What was the cost of the articles?

2. Two masts of a ship are 52 ft. apart, one being 80 ft. high and the other 70 ft. high. Supposing them perpendicular to the deck, how far is it from the top of one to the top of the other?

3. A merchant asked 25% less than the cost of a certain article, but at an auction he received $33\frac{1}{3}\%$ more than the asked price. Did he gain or lose on the cost, and what per cent?

4. Water is flowing at the rate of 1 cu. ft. every 90 sec. into a rectangular tank whose base measure is 3′ 6″ by 4′ 2″. How long will it take to increase the depth of the water by 1′ 6″?

5. A 12-inch gun can fire a shell weighing 850 lb. every 30 sec. At this rate how many tons of shell could a battleship fire from four 12-inch guns in 2 min. 30 sec.?

6. A train leaves a city at 10 45 A.M. and reaches another city $127\frac{1}{2}$ mi. distant at 5 min. past 2 P.M. Allowing 20 min. for stops, what is the rate per hour?

7. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in the capacity of a bin that is half as long, 6 in. wider, and 1 ft. less in depth?

8. If 12 men in 16 da. can lay the asphalt blocks in a certain length of a city street, how many men must be added to the force to complete the work 4 da. sooner?

9. Divide 124 ft. 3 in. by 7 and check the result.

10. A village has a water tank 42 ft. in diameter. In 3 hr. 45 min., when no water is being pumped in, the water is lowered 3 ft. How many gallons per hour are drawn out on an average?

ORAL EXAMINATION

1. Last year a gardener had 3 A. of potatoes and raised 200 bu. to the acre. How many bushels did he raise in all? If he sold these potatoes at $1.40 a bushel, how much did he get for them all? Potatoes weigh 60 lb. to the bushel. What was the weight of his potato crop?

2. For spraying his potato plants a gardener uses a mixture of 1 lb. of Paris green to 100 gal. of water. Since 1 gal. of water weighs 8.3 lb., the weight of the Paris green is what part of the weight of water?

3. A gardener soaked his seed potatoes in a solution of 1 pt. of formalin to 25 gal. of water to keep off the potato scab. What part of the solution is formalin?

4. If $\frac{3}{4}$ of the weight of potatoes is water, and if a bushel of potatoes weighs 60 lb., what is the weight of water in a bushel of potatoes?

5. A farmer had 2 A. of celery, and the yield was 1550 doz. to the acre. How many dozen did he have in all?

6. A farmer bought seed for 1000 celery plants at 48¢ an ounce. Allowing an ounce of seed to 2000 plants, how much did he pay for the seed?

7. When wheat costs $2.12\frac{1}{2}$ a bushel, how much will 100 bu. of wheat cost? 1000 bu.? 2000 bu.?

8. If wheat is worth $2 a bushel, how much will the yield of 100 A. be worth if the average yield is 25 bu. per acre?

9. If wheat is worth $2.20 a bushel, how much will the yield of 50 A. be worth if the average yield is 20 bu. per acre?

10. If wheat is worth $2.12\frac{1}{2}$ a bushel, how much will the yield of 80 A. be worth if the average yield is 25 bu. per acre?

WRITTEN EXAMINATION

1. The following numbers are the given per cents more than what numbers and the same per cents less than what other numbers? That is, 390 is $62\frac{1}{2}\%$ more than what number and $62\frac{1}{2}\%$ less than what other number?

390, $62\frac{1}{2}\%$	175, $66\frac{2}{3}\%$	2.4, 20%	9.1, 30%
150, $87\frac{1}{2}\%$	350, $16\frac{2}{3}\%$	42.3, 50%	17.6, 60%
640, $33\frac{1}{3}\%$	143, $8\frac{1}{3}\%$	210, 40%	56.1, 70%

2. If a man spends 5% of his salary for clothes and then has \$1710 left, how much is his salary? How much does he spend for clothes?

3. If you have read all but 40% of the pages of a book and have read 234 pages, how many pages has the book?

4. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

5. If you lose $\frac{1}{10}$ of a kite string and have 585 ft. left, how long was the string? How many feet did you lose? What per cent of the string is left?

6. A man sold some damaged goods for \$1547, at a loss of 9%. How much did the goods cost him?

7. A village has a population of 3384, having lost 6% in the last ten years. What was the population ten years ago? What is the number lost?

8. A man sold a car for \$1000, at a loss of 20%. How much did the car cost him?

9. A man sold some property and gained \$360, which was 12% on the money he invested. How much did he invest?

10. A man sold some property for \$3348, at a profit of 24%. How much did the property cost him?

WRITTEN EXAMINATION

1. Find the side of a square whose area is 529 sq. in.

2. What will it cost for rails for 5 mi. of double track if the rails cost \$56 a ton and weigh 150 lb. per yard?

3. A and B together have \$153. If $\frac{3}{4}$ of A's money is equal to $\frac{2}{3}$ of B's, how much has each?

4. A horse tied to a stake can reach the grass 30 ft. in any direction from the stake. Over how many square feet of land can the horse graze?

5. Allowing 1000 shingles to 120 sq. ft. of surface, find how many thousand shingles will cover a pitched roof each side of which is 64 ft. long and 28 ft. 6 in. wide.

6. A corner lot has 72 ft. 6 in. front and is 144 ft. 6 in. deep. Find the cost of laying a 5-foot stone walk along the front and side of the lot, at 42¢ per square foot.

7. Find the depth of a cylindric tank 5 ft. in diameter that has the same capacity as a rectangular cistern which is 8 ft. square and 6 ft. deep.

8. A boat travels 15 mi. downstream in $2\frac{1}{2}$ hr. The boat's rate of travel in still water is $4\frac{1}{2}$ mi. an hour. How long will it take the boat to return?

9. A man has $\frac{5}{8}$ of his property invested in a farm and $\frac{1}{6}$ in railway stock; the remainder, \$1500, is deposited in a bank. Find the value of the farm and of the railway stock. Find the value of the entire property.

10. Two men start from points 33 mi. apart and walk toward each other, the first at the rate of $4\frac{1}{2}$ mi. per hour, and the second at the rate of $3\frac{3}{4}$ mi. per hour. How far from the point where the second man started will they meet?

WRITTEN EXAMINATION

1. How much will 275 cu. in. of steel weigh if 1 cu. ft. weighs 490 lb.?

2. What is the weight of a circular piece of steel shafting 5 in. in diameter and 24 ft. long, 1 cu. ft. of steel weighing 490 lb.?

3. If the pressure of steam in a boiler is 120 lb. to the square inch, find the pressure on a valve $2\frac{5}{8}$ in. in diameter.

4. A boiler contains 420 flues, each with an internal diameter of $2\frac{1}{2}$ in. Find the total cross-section area for draft through them.

5. If a pressure of 120 lb. to the square inch is applied to the circular plunger of a cylinder 1 ft. 9 in. in diameter, what is the total pressure?

6. The rim of a flywheel is 8 in. wide and contains 1.56 cu. ft. of cast iron. If the rim were made 1 ft. wide, how much would it weigh, 1 cu. ft. of cast iron weighing 450 lb.?

7. From a rectangular sheet of steel which is $14' 3''$ long, $3' 4\frac{1}{2}''$ wide, and $\frac{1}{2}''$ thick a circular plate 2 ft. in diameter is cut. If 1 cu. ft. of steel weighs 490 lb., what is the weight of the part of the sheet that is left?

8. If the drivewheel of an engine makes 140 revolutions a minute, and if the cylinder is 9 in. in diameter and 14 in. long and is filled with steam twice during each revolution, how many cubic feet of steam are used in a minute?

9. A hollow steel shaft has an external diameter of $9''$ and an internal diameter of $5''$. If 1 cu. ft. of steel weighs 490 lb., find the weight of 10 ft. of this shaft.

10. How high is a smokestack that casts a shadow 80 ft. long at the same time that the shadow of a boy 4 ft. 6 in. tall is 5 ft. long?

MATERIAL FOR FREQUENT DRILL

EXERCISE 1

Taking (a) *7.1*, (b) *8.3*, (c) *15.4*, (d) *21.7*, (e) *24.9*, (f) *26.8*, (g) *35.7*, *or* (h) *28.2*, *as the teacher directs:*

1. Add it to $48.3 + 27.9 + 3.68 + 0.09 + 27$.
2. Subtract it from $23.486 + 76.514$.
3. Multiply it by itself.
4. Divide it by 0.65, carrying the quotient to thousandths.
5. Find $33\frac{1}{3}\%$ of it, using the shortest method.

Except as otherwise directed in these exercises, as in No. 4 above, quotients are to be carried only to the nearest hundredth.

EXERCISE 2

Taking (a) *26.4*, (b) *31.4*, (c) *28.9*, (d) *34.8*, (e) *37.8*, (f) *39.1*, (g) *36.8*, *or* (h) *37.4*, *as the teacher directs:*

1. Add it to $27.9 + 36.7 + 4.28 + 29.09 + 0.08$.
2. Subtract it from $48.378 + 51.622$.
3. Multiply it by twice itself.
4. Divide it by 0.071.
5. Find 25% of it; 125% of it; $133\frac{1}{3}\%$ of it.

This Material for Frequent Drill is so arranged as to give the needed practice in the fundamental operations. By first going through all the exercises, using in each exercise the number denoted by (a), and then the number denoted by (b), and so on, 85 different exercises are found for each letter, or 680 distinct operations in all. Two exercises a week are thus supplied for about 2 yr., or more frequent exercises for 1 yr.

EXERCISE 3

Taking (a) *34.1*, (b) *35.7*, (c) *32.8*, (d) *38.7*, (e) *36.7*, (f) *39.8*, (g) *38.4*, *or* (h) *37.6*, *as the teacher directs:*

1. Add it to the sum of itself and 9 times itself.
2. Subtract it from 429.36 + 570.63 + 0.01.
3. Multiply it by 3.5.
4. Divide it by half itself; by twice itself; by 0.71.
5. Add it to the product of $2\frac{1}{4}$ and 96.

EXERCISE 4

Taking (a) *41.2*, (b) *42.3*, (c) *43.4*, (d) *44.5*, (e) *45.6*, (f) *46.7*, (g) *45.4*, *or* (h) *47.3*, *as the teacher directs:*

1. Add it to 99 times itself.
2. Subtract it from 1001 times itself.
3. Multiply it by the quotient of 625 divided by 5.
4. Divide it by the quotient of itself divided by 47.6.
5. Find 200% of it; 50% of it; 0.1% of it.

EXERCISE 5

Taking (a) *51.8*, (b) *52.7*, (c) *53.6*, (d) *54.5*, (e) *55.4*, (f) *56.3*, (g) *53.2*, *or* (h) *54.9*, *as the teacher directs:*

1. Add it to the difference between 94.073 and itself.
2. Subtract it from 26 times itself.
3. Multiply it by the product of $\frac{2}{3}$ and $\frac{1}{8}$.
4. Divide it by the quotient of $\frac{3}{4} \div \frac{7}{8}$.
5. Divide it by 125%; find 125% of it.

EXERCISE 6

Taking (a) *62.7*, (b) *63.6*, (c) *64.5*, (d) *65.4*, (e) *66.3*, (f) *67.2*, (g) *64.7*, *or* (h) *65.9*, *as the teacher directs:*

1. Add it to $4.096 + 39.8 + 0.987 + 53$.
2. Subtract it from $8.27 + 19.6 + 82 + 0.128$.
3. Multiply it by $2\frac{1}{2} + 3\frac{2}{5}$.
4. Divide it by $3\frac{7}{8} + 4\frac{3}{8}$.
5. Find $12\frac{1}{2}\%$ of it; $1\frac{1}{4}\%$ of it; 125% of it.

EXERCISE 7

Taking (a) *71.9*, (b) *72.8*, (c) *73.7*, (d) *74.6*, (e) *75.5*, (f) *76.4*, (g) *73.9*, *or* (h) *74.8*, *as the teacher directs:*

1. Add it to $48.74 + 6.9 + 39.4 + 95 + 0.082$.
2. Subtract it from $87.4 + 0.96 + 3.4 + 9.857$.
3. Multiply it by 0.625; by 62.5.
4. Divide it by $62\frac{1}{2}\%$; by 6.25; by $6\frac{1}{4}\%$.
5. Find 20% of it; 40% of it; 60% of it.

EXERCISE 8

Taking (a) *82.7*, (b) *83.6*, (c) *84.5*, (d) *85.4*, (e) *86.3*, (f) *87.2*, (g) *86.9*, *or* (h) *87.8*, *as the teacher directs:*

1. Add it to 999 times itself.
2. Subtract it from $6\frac{1}{2}$ times itself.
3. Multiply it by $2\frac{1}{2}\%$; by 25%; by 250%.
4. Divide it by $2\frac{1}{2}\%$; by 25%; by 250%.
5. Add it to the quotient of $42\frac{3}{4} \div \frac{3}{8}$.

EXERCISE 9

Taking (a) *2 ft. 8 in.*, (b) *3 ft. 6 in.*, (c) *5 ft. 4 in.*, (d) *6 ft. 2 in.*, (e) *7 ft. 10 in.*, (f) *8 ft. 4 in.*, (g) *8 ft. 7 in.*, *or* (h) *9 ft. 6 in.*, *as the teacher directs:*

1. Add it to 19 times itself.
2. Subtract it from 10 ft. 1 in.
3. Multiply it by 2; by $2\frac{1}{8}$; by $21\frac{1}{4}$.
4. Divide it by 2; by 4; by 6.
5. Divide it by 3 in.; by 5 in.; by 2 ft.

EXERCISE 10

Taking (a) *92.7*, (b) *97.2*, (c) *93.4*, (d) *94.3*, (e) *98.6*, (f) *96.8*, (g) *97.7*, *or* (h) *98.4*, *as the teacher directs:*

1. Add it to the difference between $137\frac{7}{8}$ and itself.
2. Subtract it from 126 times itself.
3. Multiply it by the quotient of 196.4 divided by 7.856.
4. Divide it by 7.264; by $\frac{1}{2}$ of 7.264.
5. Multiply 34 ft. 2 in. by it.

EXERCISE 11

Taking (a) *1 yd. 15 in.*, (b) *58 in.*, (c) *2 yd. 8 in.*, (d) $2\frac{1}{2}$ *yd.*, (e) $3\frac{1}{2}$ *yd.*, (f) *43 in.*, (g) *62 in.*, *or* (h) $1\frac{1}{2}$ *ft.*, *as the teacher directs:*

1. Add it to 5 ft. 8 in. + 3 ft. 8 in.
2. Subtract it from 6 yd. 1 ft. + 2 yd. 11 in.
3. Multiply it by $2\frac{1}{2}$; by 25; by 25%.
4. Divide it by 3; by 5; by 7; by 12.
5. Divide it by 1 in.; by 0.1 in.; by $\frac{1}{4}$ in.

EXERCISE 12

Taking (a) *2 lb. 3 oz.*, (b) *2 lb. 5 oz.*, (c) *2 lb. 12 oz.*, (d) *42 oz.*, (e) *56 oz.*, (f) *5 lb. 4 oz.*, (g) *7 lb. 7$\frac{1}{2}$ oz.*, *or* (h) *2 lb. 8 oz.*, *as the teacher directs:*

1. Add it to 3 lb. 12 oz. + 4 lb. 2 oz. + 6 lb. 14 oz.
2. Multiply it by $3\frac{1}{3}$; by $\frac{1}{3}$; by $3\frac{1}{3}\%$.
3. Divide it by 2; by 9; by 0.02.
4. Find 25% of it; 75% of it; 225% of it.
5. Divide it by 0.2; by 0.002; by 20.

EXERCISE 13

Taking (a) *147.72*, (b) *129.8*, (c) *136.83*, (d) *141.09*, (e) *150.07*, (f) *158.08*, (g) *156.07*, *or* (h) *215.37*, *as the teacher directs:*

1. Add it to 74.8 + 100.96 + 7.583 + 0.09 + 3.67 + 29.
2. Subtract it from 76 times itself.
3. Find 50% of it; 5% of it; 500% of it.
4. Divide it by 25; by 2.5; by 0.025.
5. Multiply it by the product of $3\frac{1}{2}$ and 9.

EXERCISE 14

Taking (a) *200.02*, (b) *200.16*, (c) *201.3*, (d) *204.04*, (e) *210.9*, (f) *212.12*, (g) *211.36*, *or* (h) *208.28*, *as the teacher directs:*

1. Add it to 49 times itself.
2. Subtract it from $1\frac{1}{2}$ times itself.
3. Multiply it by the quotient of $483\frac{7}{8}$ divided by $2\frac{1}{8}$.
4. Divide it by $6\frac{1}{4}\%$; by $62\frac{1}{2}\%$; by $\frac{5}{8}\%$.
5. Find $16\frac{2}{3}\%$ of it; $66\frac{2}{3}\%$ of it; $37\frac{1}{2}\%$ of it.

EXERCISE 15

Taking (a) $4\frac{1}{2}$ *in.*, (b) $5\frac{1}{8}$ *in.*, (c) $5\frac{3}{16}$ *in.*, (d) $6\frac{7}{8}$ *in.*, (e) $7\frac{11}{16}$ *in.*, (f) $8\frac{1}{32}$ *in.*, (g) $5\frac{1}{64}$ *in.*, *or* (h) $7\frac{3}{64}$ *in.*, *as the teacher directs:*

1. Add it to 2 in. + $3\frac{1}{2}$ in. + $5\frac{7}{8}$ in.
2. Subtract it from $8\frac{3}{8}$ in.
3. Multiply it by 3; by 5; by 32.
4. Divide it by $\frac{1}{2}$; by 0.05; by $\frac{1}{2}\%$.
5. Find what per cent it is of a foot; of a yard.

EXERCISE 16

Taking (a) *3 pt.*, (b) $2\frac{1}{2}$ *qt.*, (c) *4 gal. 2 qt.*, (d) *3 qt. 1 pt.*, (e) $5\frac{1}{2}$ *gal.*, (f) $\frac{1}{2}$ *gal.*, (g) $3\frac{1}{4}$ *pt.*, *or* (h) $2\frac{1}{8}$ *gal.*, *as the teacher directs:*

1. Add it to $7\frac{1}{2}$ qt. + 5 pt.; subtract it from 10 gal.
2. Multiply it by 7; by 12; by $4\frac{1}{2}$.
3. Divide it by 2; by 25%.
4. Find what per cent it is of a cubic foot ($7\frac{1}{2}$ gal.).
5. Add to it 50% of itself; 150% of itself.

EXERCISE 17

Taking (a) *1287.4*, (b) *296.8*, (c) *4296.07*, (d) *374.29*, (e) *1298.72*, (f) *297.68*, (g) *268.42*, *or* (h) *333.33*, *as the teacher directs:*

1. Add it to 4876.39 + 29.786 + 430 + 1.236 + 38.
2. Subtract it from 5000.
3. Multiply it by itself.
4. Divide it by half itself.
5. Find the square root of it to the nearest tenth.

SUPPLEMENT

Nature of this Supplement. As frequently stated in this series, there is a tendency at present to limit arithmetic to the absolute essentials. This has been the policy of the authors in preparing these books. There are certain topics which are essential at times and which should be placed where the teacher can use them if necessary. Such are the topics treated in this supplement.

Land Measure. In surveying land the measures used are:

LENGTH

7.92 inches (in.) = 1 link (li.)
100 links, or 4 rods (rd.) = 1 chain (ch.)
80 chains, or 5280 feet (ft.) = 1 mile (mi.)

City property is usually measured by feet and decimals of a foot.

AREA

16 square rods (sq. rd.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
36 square miles = 1 township (T.)

For example, how many acres are there in a field 32 ch. long and 6 ch. 30 li. wide?

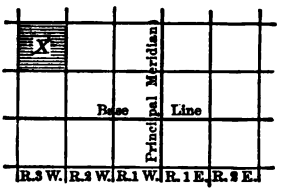
6 ch. 30 li. = 6.30 ch.

32×6.30 sq. ch. $= 32 \times 6.30 \times 0.1$ A. $= 20.16$ A.

The pupils should be given a clear conception of an acre by being shown a field containing one acre or two acres or a half acre.

Laying out Public Lands. In the more recently settled parts of the country, land is laid out as described below.

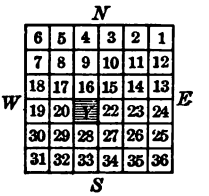
Principal Meridian and Base Line. Through a given tract a meridian is chosen as the *principal meridian.* An east-and-west line is chosen as the *base line.* The principal meridian and base line are here shown.



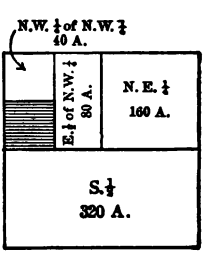
Township. At intervals of 6 mi. lines are run parallel to the principal meridian and other lines parallel to the base line. This divides the land into *townships.*

Range. A north-and-south row of townships is called a *range.* On the first map X is numbered T. 2 N., R. 3 W.; that is, the second township north of the base line, in the third range west of the meridian.

Section. Each township is divided into *sections* 1 mi. square. The second map shows the method of numbering these sections.



Each section is then divided as shown in the third map. If this map represents the shaded part Y of the second map, and the second map represents the shaded part X of the first map, the shaded part here shown would be described thus: S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 21, T. 2 N., R. 3 W. In full form this means the southwest quarter of the northwest quarter of section 21, second township north, third range west.



In parts of the country where land is not laid out in this way little attention should be given to the subject of laying out public lands. In rural schools, however, some attention should be given to the actual measuring of land, particularly by dividing fields of irregular shape into triangles.

LAND MEASURE

Write the description, plot, and find the area:

1. S.E. $\frac{1}{4}$, Sec. 5, T. 3 S., R. 3 W.

2. N.E. $\frac{1}{4}$, Sec. 8, T. 2 N., R. 2 W.

3. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 2, T. 2 N., R. 3 E.

4. S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, Sec. 20, T. 2 S., R. 3 W.

5. N. $\frac{1}{2}$ of S.W. $\frac{1}{4}$, Sec. 10, T. 3 S., R. 2 E.

6. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 30, T. 2 S., R. 3 E.

7. N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$, Sec. 5, T. 1 N., R. 1 W.

8. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 32, T. 1 S., R. 3 E.

9. At \$65 an acre how much is this farm worth: W. $\frac{1}{2}$ of S. $\frac{1}{2}$, Sec. 3, T. 2 N., R. 2 W.?

10. At \$75 an acre how much is this farm worth: N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 5, T. 2 S., R. 2 E.?

11. Find the area of this farm: S.W. $\frac{1}{4}$, Sec. 10, T. 2 S., R. 2 E. Draw a plan of the township and locate the farm.

12. Find the area of this farm: N. $\frac{1}{2}$, Sec. 6, T. 1 N., R. 1 E. Draw a plan of the township and locate the farm.

13. Mr. Read owns the S. $\frac{1}{2}$ of N.E. $\frac{1}{4}$, Sec. 3, T. 2 N., R. 3 E. How many rods of fence are needed to inclose his land?

14. How far is it from Mr. Lang's farm, N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 16, T. 1 N., R. 3 W., to Mr. Taylor's farm, S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 28, T. 1 N., R. 3 W.? Draw the map.

15. A straight road $\frac{1}{2}$ mi. long and 2 rd. wide runs through a farm. How many acres are there in the road? If 2 T. of hay to the acre, worth \$18 per ton, can be cut from along the road, the width of the space on each side being $\frac{1}{6}$ the whole width of the road, how much is gained by cutting this crop?

Longitude and Time. In geography you learned about the longitude of places, and you know that longitude is reckoned in degrees, minutes, and seconds from the prime meridian, which passes through Greenwich, England.

Correspondence of Longitude to Time. Since the earth makes a complete revolution about its axis once in every 24 hr. the place in which we live will pass through 360° between now and this time tomorrow. That is, to the time 24 hr. there corresponds the longitude 360°. Therefore

1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.
1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

Earlier West, Later East. Because the earth revolves 360° in 24 hr., the sun appears to pass through 15° an hour. Therefore, when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hr. earlier, or 6 A.M., 90° west of here.

Since this work belongs partly to geography, no illustrations are here introduced. The teacher should use the globe as needed.

LONGITUDE RELATED TO TIME

All work oral

1. When it is 9 A.M. here, what time is it 15° east of here?

2. When it is noon here, what time is it 30° east of here? 90° west of here? 150° east of here? directly north of here?

3. What difference of time corresponds to a difference of longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

4. Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

Illustrative Problem. Two ships at sea are 65° 7′ 30″ of longitude apart. What is their difference in local time?

Since 1° corresponds to $\frac{1}{15}$ hr., 65° corresponds to $65 \times \frac{1}{15}$ hr., or $4\frac{1}{3}$ hr., or 4 hr. 20 min.

Similarly, 7′ corresponds to $7 \times \frac{1}{15}$ min., or $\frac{7}{15}$ min., or 28 sec., and 30″ corresponds to $30 \times \frac{1}{15}$ sec., or 2 sec.

Therefore the difference in local time between the ships is 4 hr. 20 min. 30 sec.

$$15)\overline{65\ \ 7\ 30}$$
$$\underline{\ \ 4\ 20\ 30}$$

It is apparent that the mere figures of the answer can be obtained by this division, although the reason is not so clear.

DIFFERENCE IN TIME

1. A difference of 27° 4′ 15″ in longitude corresponds to what difference in local time?

2. Two ships are 75° 30′ 30″ of longitude apart. What is their difference in local time?

3. A steamship in longitude 62° 3′ 40″ W. receives at 11 A.M., local time, a wireless telegram from another steamship in longitude 60° 1′ 10″ W. At what hour, local time, was it sent?

4. For time purposes the longitude of Rome is 15° E. and that of Chicago is 90° W. What is the difference in longitude? When it is 2 P.M. in Chicago, what time is it in Rome?

5. For time purposes the longitude of San Francisco is taken as 120° W. When it is noon in London (on the prime meridian), what time is it in San Francisco?

6. At 8 hr. 30 min. A.M., local time, a steamer 68° 10′ 30″ W. sends a wireless telegram to the Nantucket lightship, 70° W. At what hour, local time, is it received? At what hour, local time, is it received in New York, 75° W.?

Correspondence of Time to Longitude. Since, as we have seen, 24 hr. of time corresponds to 360° of longitude, we have:

1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°, of longitude.
1 min. corresponds to $\frac{1}{60}$ of 15°, or $\frac{1}{4}$°, or 15′, of longitude.
1 sec. corresponds to $\frac{1}{60}$ of 15′, or $\frac{1}{4}$′, or 15″, of longitude.

For example, what is the difference in longitude between two ships if the difference in local time is 3 hr. 7 min. 3 sec.?

As above, 3 hr. corresponds to $3 \times 15°$, or 45°; 7 min. corresponds to $7 \times 15'$, or 1° 45′; 3 sec. corresponds to $3 \times 15''$, or 45″.

Therefore the difference in longitude is 46° 45′ 45″.

3°	7′	3″
		15
46°	45′	45″

Evidently the mere figures of the answer can be obtained by thinking of 3 hr. 7 min. 3 sec. as 3° 7′ 3″ and multiplying by 15, although the reason for the work is not so clear.

DIFFERENCE IN LONGITUDE

1. The difference of local time between two steamships is 2 hr. 3 min. 10 sec. What is the difference in longitude?

2. When it is noon at Denver it is 7 P.M. at Greenwich (0°). What longitude does Denver use for its time purposes?

3. When it is noon at Greenwich it is 6 A.M. in Iowa. What longitude does Iowa use for its time purposes?

4. At 10 hr. 4 min. A.M., local time, a steamer in longitude 26° 30′ W. sends a wireless message to another steamer. It is received at 10 hr. 19 min. A.M., local time. What is the longitude of the second steamer?

Questions of the International Date Line and of the Daylight-Saving laws involve no numerical work and should be taken up in geography.

Standard Time. For practical purposes most of the civilized world is now divided into sections, and all places in the same section use the *standard time* of a certain meridian.

The following is the standard-time map of the United States:

When it is noon by Eastern time (75° W.) it is 11 A.M. by Central time (90° W.), 10 A.M. by Mountain time (105° W.), 9 A.M. by Pacific time (120° W.), and 5 P.M. by Greenwich time (0°).

STANDARD TIME

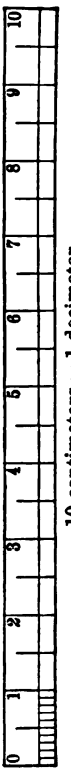
1. When it is 10 A.M. in Illinois, what time is it in Maine?
2. What time is it now in your school? in Oregon?
3. When it is 10 A.M. in New Orleans, what time is it in Dallas? in Memphis? in Seattle? in New York?
4. When it is 9 A.M. in Denver, what time is it in Savannah?
5. When it is noon in Maryland, what time is it in Wyoming?
6. When it is 3 P.M. in Milwaukee, what time is it in Oregon?
7. When it is 9 A.M. in New York, what time is it in Montana?

Metric System. There is a system of measures used in many countries which is known as the *Metric System.*

The unit of length is the *meter.* The meter is about 39.37 in.

The unit of capacity is the *liter* (lētĕr), a cube 0.1 of a meter on an edge. Thè liter is 1 qt., nearly.

The unit of weight is the *gram.* It is practically the weight of a cube of water 0.01 of a meter on an edge. The gram is 15.4 grains, nearly.



Prefixes. Learn the following prefixes:

Just as 1 mill = 0.001 of a dollar,
so 1 millimeter = 0.001 of a meter.
Just as 1 cent = 0.01 of a dollar,
so 1 centimeter = 0.01 of a meter.
Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

Prefix	means	as in	which means
myria-	10,000	myriameter	10,000 meters
kilo-	1000	kilogram	1000 grams
hekto-	100	hektoliter	100 liters
deka-	10	dekameter	10 meters
deci-	0.1	decimeter	0.1 of a meter
centi-	0.01	centigram	0.01 of a gram
milli-	0.001	millimeter	0.001 of a meter

The prefix *myria-* is rarely used.

This system is as much easier than our common one as the system of United States money is easier than the British system. Ease of calculation is the reason why it is so extensively used in other countries and in scientific work. During the World War our army in France used the metric system.

Table of Length. The table of length is as follows:

A **kilometer (km.)** = 1000 meters
A hektometer = 100 meters
A dekameter = 10 meters
Meter (m.)
A decimeter (dm.) = 0.1 of a meter
A **centimeter (cm.)** = 0.01 of a meter
A **millimeter (mm.)** = 0.001 of a meter

The meter is about 39.37 in., $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.6 of a mile.

The measures printed in heavy type should be learned.

The abbreviations in this book are the ones commonly used, but Km., Hm., and Dm. are also used for kilometer, hektometer, and dekameter.

MEASURES OF LENGTH

Express as inches, taking 39.37 in. as 1 m.:

1. 47 m.
2. 34.5 m.
3. 52 m.
4. 67.5 m.
5. 324 cm.
6. 2.83 cm.
7. 4680 mm.
8. 3000 mm.

9. Express in inches the diameter of a 75-millimeter gun.

10. A hill is 203 m. high. Express this height in feet.

11. A tower is 37.5 m. high. Express this height in feet.

12. A wheel is 1.3 m. in diameter. Express this in inches.

13. The distance between two points on a battle front was 78 km. Express this distance in miles.

14. The distance from Paris to a point on the battle front was stated to be 124 km. Express this distance in miles.

15. If it takes a European train 7 hr. to make a trip of 326 km., what is the average rate in miles per hour?

Table of Square Measures. This table is as follows:

A **square kilometer** (km^2.) = 1,000,000 square meters
A square hektometer = 10,000 square meters
A square dekameter = 100 square meters
Square meter (m^2.)
A **square decimeter** (dm^2.) = 0.01 of a square meter
A **square centimeter** (cm^2.) = 0.0001 of a square meter
A square millimeter (mm^2.) = 0.000001 of a square meter

The abbreviation sq. m. is often used for m^2., sq. cm. for cm^2., and so on.

The square dekameter is also called an *are* (är), and the square hektometer is similarly called a *hektare* (ha.). The hektare is 2.47 acres, or nearly $2\frac{1}{2}$ acres. Such facts need not be memorized.

Table of Cubic Measures. This table is as follows:

A cubic kilometer = 1,000,000,000 cubic meters
A cubic hektometer = 1,000,000 cubic meters
A cubic dekameter = 1000 cubic meters
Cubic meter (m^3.)
A **cubic decimeter** (dm^3.) = 0.001 of a cubic meter
A **cubic centimeter** (cm^3.) = 0.000001 of a cubic meter
A cubic millimeter (mm^3.) = 0.000000001 of a cubic meter

The abbreviation cu. m. is often used for m^3., cu. cm. for cm^3., and so on.

In measuring wood a cubic meter is called a *stere* (st.), but this fact need not be memorized.

SQUARE AND CUBIC MEASURES

Express the following as square centimeters:

1. 750 km^2.
2. 37 m^2.
3. 4296 mm^2.
4. 6.25 m^2.

Express the following as cubic meters:

5. 19,750,000 cm^3.
6. 826,427,653.84 mm^3.
7. 7000 cm^3.
8. 3500 dm^3.
9. 937 cm^3.
10. 486 dm^3.

Table of Weight. The table of weight is as follows:

A **metric ton (t.)** = 1,000,000 grams
A **kilogram (kg.)** = 1000 grams
A hektogram = 100 grams
A dekagram = 10 grams
Gram (g.)
A decigram = 0.1 of a gram
A centigram (cg.) = 0.01 of a gram
A milligram (mg.) = 0.001 of a gram

A *quintal* (q.) is 100 kg. and a *myriagram* is 10 kg. A kilogram, commonly called a *kilo*, is about 2⅕ lb. A metric ton is nearly 2204.6 lb.

Table of Capacity. The table of capacity is as follows:

A **hektoliter (hl.)** = 100 liters
A dekaliter = 10 liters
Liter (l.)
A deciliter = 0.1 of a liter
A **centiliter (cl.)** = 0.01 of a liter
A milliliter (ml.) = 0.001 of a liter

A liter is the same as 1 dm^3. A liter is practically the same as a quart.

WEIGHT AND CAPACITY

Express the following as kilograms:

1. 374 lb.	4. 352 oz.	7. 3270 g.	10. 7380 g.
2. 428 lb.	5. 48.4 lb.	8. 7290 g.	11. 4900 g.
3. 204 lb.	6. 300 T.	9. 173.8 lb.	12. 6350 g.

Express the following as liters:

13. 7.5 hl.	15. 2.25 hl.	17. 2500 cl.	19. 15 hl.
14. 8 pt.	16. 12½ gal.	18. 500 gal.	20. 6.83 hl.

Use of Letters for Numbers. There are some problems in which it is convenient to use letters to stand for numbers, as is done in algebra. To take a simple case, find the number which when multiplied by 1.2 is equal to 600. We could say that

$$1.2 \times \text{some number} = 600,$$

and since we have a product (600) and one factor (1.2), the other factor can be found as follows:

$$600 \div 1.2 = 500.$$

But it is shorter to put a letter, as x, in place of the words "some number," and to write $1.2\,x$ for $1.2 \times x$. We then have

$$1.2\,x = 600;$$

whence $$x = 600 \div 1.2 = 500.$$

Illustrative Problems. 1. On an automobile costing \$640 an agent's profit was \$128. What per cent of the cost did he gain?

It is evident from the statement of the problem that

$$x\% \text{ of } 640 = 128.$$

We may divide each of these equals by 640, and we have

$$x\% = 128 \div 640 = 0.20 = 20\%.$$

Therefore the agent gained 20% of the cost.

2. An agent sold an automobile for \$768 and made a profit of 20% of the cost. How much did he pay for the car?

Since the rate of profit on the cost is given, we see that

$$\text{the cost} + 20\% \text{ of the cost} = 1.20 \text{ times the cost.}$$

Then, letting c stand for the number of dollars of cost, we have

$$1.20\,c = 768;$$

whence $$c = 768 \div 1.20 = 640.$$

Therefore the agent paid \$640 for the automobile.

3. Six years ago a boy was half as old as now. How old is he now?

Let $x =$ the number of years in his age now.

Then $x - 6 =$ the number of years 6 yr. ago,

and also $\frac{1}{2}x =$ the number of years 6 yr. ago.

Therefore $x - 6 = \frac{1}{2}x$.

Adding 6 to each of these equals, we have

$$x = \frac{1}{2}x + 6.$$

Subtracting $\frac{1}{2}x$ from each of these equals, we have

$$\frac{1}{2}x = 6;$$

whence $x = 12$.

Therefore the boy is 12 yr. old now.

USES OF THE EQUATION

1. An agent sold some property on a commission of 2%. If his commission was \$46, what was the selling price?

2. An agent sold some property for \$3500. His commission was \$52.50. What was the rate of commission?

3. An airplane traveling at the rate of d miles a minute goes 36.8 mi. in 14 min. Find the value of d.

4. A train traveling at the rate of m miles an hour runs $112\frac{1}{2}$ mi. in $2\frac{1}{2}$ hr. Find the value of m.

5. A baseball team won 65% of the games played, and won 13 games. How many games did it play?

6. A baseball team lost $33\frac{1}{3}$% of the games played, and lost 16 games. How many games did it play?

The use of the algebraic equation is not essential to the study of arithmetic. It simplifies the study of formulas, but only a few of the practical problems of daily life are rendered more simple by its use.

Recreations. Most of the following problems have a long and interesting history. They are part of the inheritance of the school, and they show some of the amusing features of arithmetic. They claim to have no practical bearing, and they should be omitted unless taken in the spirit intended, as mere recreations showing the lighter side of the subject.

RECREATIONS AND CURIOSITIES

1. Multiply 142,857 by 7 and notice the curious result.

2. Multiply 37 by 3; by 6; by 9; by 12; by 15; by 18; by 21. Notice the curious results.

3. How can you write one hundred with Arabic figures without using zeros?

4. How can you write an even number, using only the odd digits?

5. If a goose weighs 10 lb. and half of its own weight, what is the weight of the goose?

6. How many days would it take to cut a piece of cloth 50 yd. long into 1-yard lengths, one yard being cut off daily?

7. If a post in a pond stands $\frac{1}{3}$ in the ground, $\frac{1}{4}$ in the water, and 10 feet above the water, how long is it?

8. A bottle and its stopper cost \$1.10, and the bottle cost \$1 more than the stopper. How much did the stopper cost?

9. A new steel coal car and its contents weigh 139,700 lb., and the contents weigh 60,300 lb. more than the car. How much does the car weigh?

10. Two men purpose to cross the Rocky Mountains. X can walk 13 mi. a day and Y only 10 mi. a day. Therefore Y starts 9 da. ahead. How long will it take X to overtake Y?

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
About 4 weeks = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Money

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

Angles and Arcs

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gr.)
12 gross, or 1728 units = 1 great gross

Paper

24 sheets = 1 quire
500 sheets = 1 ream

Formerly it was the custom to consider a ream as 20 quires, or 480 sheets. Now the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

Weight per Bushel

Wheat . . . 60 lb.	**Clover seed . 60 lb.**	**Barley . . . 48 lb.**
Peas . . . . 60 lb.	**Shelled corn 56 lb.**	**Buckwheat 48 lb.**
Beans . . . 60 lb.	**Rye . . . . . 56 lb.**	**Oats . . . . 32 lb.**
Potatoes . . 60 lb.	**Flaxseed . . 56 lb.**	**Beets . . . . 60 lb.**

These are the legal weights per bushel in many states, but they vary in different parts of the country. In several states, for example, 50 lb. of beets constitute a legal bushel. Onions vary from 48 lb. to 57 lb. in different states, and sweet potatoes from 46 lb. to 60 lb. When possible, pupils in rural schools should verify the above weights for their own state.

Practical Equivalents

1 bu. contains $1\frac{1}{4}$ cu. ft.	**1 cu. ft. water weighs $62\frac{1}{2}$ lb.**
1 bbl. contains $4\frac{1}{5}$ cu. ft.	**1 gal. water weighs $8\frac{1}{3}$ lb.**
1 gal. contains 231 cu. in.	**1 bbl. contains $31\frac{1}{2}$ gal.**
1 cu. ft. contains $7\frac{1}{2}$ gal.	**1 T. hay occupies 500 cu. ft.**

These are generally only approximations, but they serve for ordinary computations.

Convenient Rules for Farmers

Bushels in a Bin. *Take $\frac{4}{5}$ of the product of the three dimensions.*

This gives the number of bushels for a rectangular bin, the dimensions being expressed in feet. In the case of a bin 16 ft. long, 10 ft. wide, and 6 ft. deep the number of bushels is $\frac{4}{5} \times 16 \times 10 \times 6$, or 768.

Bushels in a Circular Bin. *Multiply $\frac{5}{8}$ of the square of the diameter by the depth.*

This gives the number of bushels, the dimensions being given in feet.

Bushels of Corn in a Crib. *Multiply the product of the three dimensions by 0.45 for old, dry corn; by 0.4 for new, fresh corn.*

This gives the number of bushels of shelled corn for a rectangular crib, the dimensions being expressed in feet.

Tons of Hay in a Stack. 1. *Find the approximate height in feet by subtracting the width from the overthrow and dividing by 2. Then divide the product of the three dimensions by 500.*

The overthrow is measured from one side at the ground, over the top, and to the other side at the ground. The dimensions are to be expressed in feet. For wild hay divide by 412; for timothy by 450; for clover and alfalfa by 512. The rule gives only an approximate result.

2. *Divide the product of the three dimensions in yards by 30.*

The divisor varies from 30 to 36 with the kind of hay. This does not give such close results as the first rule and presupposes that there is considerable slope to the stack.

Tons in a Mow. *Measuring in yards, divide the product of the three dimensions by 15 for well-packed hay.*

If the hay is less well packed, the divisor may vary from 15 to 18.

Gallons in a Tank or Cistern. *Measuring in feet, multiply the product of the three dimensions by $\frac{2}{15}$ for rectangular tanks; multiply the product of the depth and the square of the diameter by 5.9 for round (cylindric) cisterns.*

A round, or cylindric, cistern 12 ft. in diameter and 8 ft. deep contains $5.9 \times 8 \times 12^2$ gal., or 6796.8 gal.

Approximate Tonnage of a Silo. The following table is useful:

Height in Feet	Inside Diameter in Feet					
	10	12	14	16	18	20
30	47	67	91			
35	59	84	114	137		
40	70	101	138	180	229	
45				215	269	330
50					310	382

SUGGESTIONS TO TEACHERS

Teachers should observe that the work of Grades VII and VIII may be treated from either of two standpoints. In the junior high schools the pupil is initiated into a general field of mathematics, including arithmetic, intuitive geometry, and algebra, and if he proceeds to the third year of the junior high school, the ninth school year, he may meet with demonstrative geometry, trigonometry, and further work in algebra.

In the elementary school of eight grades, however, the work of Grades VII and VIII is devoted largely to arithmetic, including mensuration, and it is for this type of school that this book is intended.

It should also be observed that the arithmetic of these two years is concerned chiefly with the applications of the processes that the pupil has already learned. Unless square root is taken, the pupil meets with no essentially new operations of a numerical nature. The work is almost entirely in the field of business arithmetic, with such review and drill on the operations as may be necessary.

This being the case, the teacher should put forth every effort to make the subject seem real to the pupil, correlating the work with daily life just as far as possible. This correlation can be made in various ways, first by using the material in the book and then by introducing local problems.

It takes thought to use any book properly. In the first place, it is a great mistake to attempt to give all the problems that appear in any book, however good. Textbooks have to include problems of many types, but those which relate primarily to city life may properly be reduced in number in rural schools, while those relating to farming may properly have less attention in the cities. For example,

problems on mining have a general informational value, but in an agricultural district they need have relatively little attention. Make the textbook your servant, not your master.

As to the outside problems, the best ones are usually brought in by the pupils. It is the fashion to call the more extended of these problems by the more pretentious name of "projects"; but whatever the name, the idea of having the pupils work out the economics of some phase of farming is a valuable one. Let the class figure out the cost and profits of growing an acre of corn, of mining a ton of coal, of a small apple orchard, of building a barn, of running a tractor instead of keeping a team, of putting down a concrete road, or of making some object of local manufacture, and a new life will come into the work in arithmetic. It is this spirit of everyday work with numbers that this series of books seeks to foster, while at the same time insisting upon the accuracy in computing which is essential.

Teachers of arithmetic are urged to bear in mind that the opportunity and privilege are theirs of presenting to their classes much more than the mere art of computation, important as this art is in the life of every citizen. The teachers have the opportunity of seeing to it that the pupils obtain from arithmetic a great deal of valuable information as to the resources of our country and as to the expenses of production. They can stimulate curiosity respecting local industries, household economics, and the demands of everyday life; stimulate originality and independence in the solution of problems; and correlate arithmetic with other subjects of study and with the life of the community. They have an unusual opportunity to teach thrift, to lead the pupils to understand how people can safely invest even small amounts, and to show the dangers of reckless speculation or extravagance. All this can be accomplished without lessening in any degree the scholarly work in arithmetic or neglecting that attention to accuracy, neatness, speed, and intelligence which are essential features in the work which arithmetic offers.

INDEX

	PAGE
Above par	227, 231
Acceptance	178
Accounts	149, 153, 158
Addition	4
Ad valorem duty	200
Alertness test	82, 144, 214, 284
Algebra	332
Aliquot part	14
Altitude	117, 273
Amount	67, 164
Angle measure	336
Annuity	223
Arc measure	336
Areas	86, 115, 121, 274, 276, 321
Bank	149
account	149
commercial	158
discount	172
draft	182
statement	162
Banking information	149, 188
Bar pictogram	247
Base	35, 117
line	322
Bearer	160, 165
Belgian money	186
Below par	231
Beneficiary	222
Bill	53, 54, 59
of exchange	186
Bin, bushels in	337
Board measure	93
Bond	233
Borrowing money	167, 207
British money	186
Broker	231
Brokerage	231
Budget	242
Builder	288
Building a barn	97
a house	94
Bushel, weight per	337
Bushels in a bin or crib	337
Business relations	219
Capacity	331, 337
Capital	227
Cartogram	255
Cash discount	51
Cashier's check	177, 182
Casting out nines	12
Certificate of stock	229
Certified check	188
Chain discounts	57
Check	158, 160, 177, 181, 188
in addition	4
in division	17
in multiplication	12
in subtraction	9
Circle	113
Circular pictogram	250
Circumference	113
Cistern, capacity of	338
Clearing house	184
Collateral	167

PAGE

Collecting by draft 177, 180
Commercial bank 158
discount 51
draft 177
paper 174, 178
Commission 231
Common fraction 23
measures 22
stock 229
Compound interest 152
Concrete 130
Cone 274
Corn crib, capacity of 337
Corporation 227
Counting, table of 336
Coupon 233
bond 233, 235
Credit 53
letter of 188
Creditor 53
Crib, bushels in 337
Cross multiplication 11
Cube 269
root 269
Cubic measure 330, 335
Curiosities 334
Curve-line graph 257
Customhouse 200
Customs revenue 200
Cylinder 119

Daily Life, Problems of 123
See also Problem Material
Debit 53
Debtor 53
Demand curve 257
note 164, 169
Deposit slip 158
Deposits 153, 158, 161

PAGE

Diameter 113
Discount 51, 57, 172, 184
Discounted bill 54, 59
Discounting a note 174
Dividend 223, 227, 229, 233
Division 17
Draft 177, 178, 182
Drawee 177
Drawer 177
Drawing plans 98, 102
to scale 98, 102
Drill work in
addition 4
discount 61
division 18, 20
fractions 23
interest 74, 170
multiplication 13, 16
per cents 34, 41
subtraction 9
Dry measure 336
Duties on imports 200
Duty to the government . . . 191

Efficiency tests 1
Electric meter 90
Equation 332
Equivalents, practical 337
Evolution 269
Examinations 305
Exchange 181, 184, 186
Extremes 106

Face of a note 164
of a policy 219
Farm problems . . 83, 84, 97, 128, 131, 136, 145, 146, 147, 148, 215, 217, 218, 285, 286, 287, 290, 291, 292, 294

PAGE

Farmers' rules 337
Federal Reserve Bank 174
Fertilizers 215
Fire insurance 220
Foreign money 186
Fractional parts 14
Fractions 23
Free list 200
French money 186

Gallons in a tank 338
Garden project 84
Gas meter 88
General Review 293
German money 186
Good roads 197
Government, duty to 191
expenses 199
Graph 247

Handwork 99, 100, 101
Hauling 147, 289, 290
Hay, measuring 338
Height 117, 273
Heights and distances 110
House building 94
Household problems . 125, 126, 127, 132, 135
Hypotenuse 266

Important per cents 33
Income tax 202
Indorsement . . 160, 165, 168, 182
Indorser 168
Industrial Problems . . 83, 84, 145, 215, 285
See Problem Material
Insurance 219
Interest 67, 152, 170

PAGE

Internal revenue 202
Investments 237, 245
Invoice 55
Involution 269
Irrigation 285
Italian money 186

Japanese money 186

Land measure 91, 321
Lateral area 274
Left-to-right computing 6
Length 321, 329, 335
Letter of credit 188
Liberty bonds 208
Life insurance 222
Liquid measure 335
List price 51
Little Examinations . 78, 140, 210, 280
Long division 17
Longitude and time 324
Lumber 93

Maker 165
Manual training 99, 100, 101
Marked price 51
Market price 231
problems 123
Material for Frequent Drill . . 315
Meaning of common terms . . . 3
Means 106
Measurements 85, 110
Measures 22, 335
Mensuration 261
Meridian 322
Meter 328
electric 90
gas 88

PAGE

Metric system 328
Mexican money 186
Minimum Essentials . 79, 141, 211, 281
Money 186, 336
  foreign 186
  order 181
Motor trucks 289, 290
Mortgage 233
Mow, tons of hay in 338
Multiple pictogram 252
Multiplication 11

Net amount 51
  price 51
Nines, casting out 12
Newspaper quotations 234
Note 164, 168, 169, 174

One-name paper 168
Overhead 62

Paper 336
Papering 133
Par 184, 231
  value 227, 233
Parallelogram 86
Parcel post 205
Partial payments 76
Pass book 162
Pay roll 138
Payee 160, 165, 177
Paying bills 181
Per cent 25
Percentage 25
Perfect square 261
Periods 263
Pictogram 247
Plastering 133, 134

PAGE

Policy 219, 223
Post office 156, 181, 204
Postal Savings System 156
Poultry 287
Power 269
Practical equivalents 337
  measurements 85
Preferred stock 229
Premium 182, 184, 219, 220
Price list 60, 296
Principal 67, 164
  meridian 322
Prism 117
Problems of Daily Life 123
  See also Problem Material
Problem Material
  Building 94, 97, 288
  Concrete 130
  Contractor 304
  Corn 136
  Dairy 293
  Farms . . 83, 84, 97, 128, 131, 136, 145, 146, 147, 148, 215, 217, 218, 285, 286, 287, 290, 291, 292, 294
  Fertilizers 215
  Garden project 84
  Hauling 147, 289, 290
  Heating the house 132
  Horses 217
  Household , 125, 126, 127, 132, 135, 297
  Industry at home 126
    at school 124
  Irrigation 285
  Iron industry 302
  Machine shop 300
  Machinist 301
  Manufacturing 298

PAGE

Problem Material
Market 123
Merchant 299
Milk 145
Motor trucks 289, 290
Papering 133
Paving 137
Pay roll 138
Plastering 133
Pottery 303
Poultry 287
Sewing 135
Silo 128
Spraying 294
Tractors 146
Transportation 295
Walls and floors 134
Wastefulness 83, 145
Problems for Completion . 81, 143, 213, 283
without Numbers 80, 142, 212, 282
Proceeds 172, 177
Profit and loss 62
on cost 62
on selling price 64
Projects 84, 87, 94, 97, 340
Promissory note 164
Proportion 106
Proportional numbers 109
sides 109
Public lands 321
Pyramid 273

Radius 113
Range 322
Rate . . 35, 67, 165, 184, 192, 202, 220
of exchange 184
of interest 67, 165
of premium 220

PAGE

Ratio 103
of circumference to diameter 113
Reading numbers 2
per cents 31
Real estate 240
Receipt 54
Recreations 334
Rectangle 86
Rectangular prism 117
solid 86
Review and Drill . 79, 141, 211, 281
Review and Efficiency Tests . . 1
Review Examinations 305
Roman numerals 3

Saving 150, 156, 237, 245
Savings bank 150
Scale, drawing to 98
Section 322
Self-checking additions 5
Selling price 51
Several discounts 57
Shadow reckoning 112
Shares of stock 227, 229
Short division 17
methods 15, 21
Silos 128, 338
Similar figures 109
Six-Per-Cent Method 75, 170
Sixty-Day Method 75, 170
Slant height 274
Solid pictogram 252
Specific duty 200
Speed tests in addition 4, 7
division 18, 20
fractions 24
multiplication 13
subtraction 9
Sphere 276

PAGE

Square 115, 261
measure 321, 330, 335
root 261
Stack, hay in 338
Standard time 327
Statistical graphs 247
Stock 227, 229
certificate 229
quotations 231, 234
Stockholder 229
Subtraction 8
Suggestions to Teachers 339
Supplement 321
Surfaces 121, 276
Swiss money 186

Tables . . . 186, 195, 271, 321, 328, 329, 330, 331, 335, 336, 337, 338
Tank, capacity of 338
Tariff 200
Tax rate 192
table 195
Taxes 191, 202
Teachers, Suggestions to . . . 339
Terms used in arithmetic . . . 3
percentage 35
Thrift 151, 237
Time 324, 336
discount 51
draft 178
note 164, 168

PAGE

Tons in a mow or stack 338
Township 322
Tractors 146
Trade acceptance 178
discount 51
Trapezoid 86
Triangle 86
Two-name paper 168

Using What You Have Learned 47
See also Problem Material
banks 166
house building 96
interest 71, 176
investments 245
measurements 122
percentage . . . 47, 48, 49, 50
ratio and proportion . . . 107
square root 267, 268
tariff 201
volumes and surfaces . 278, 279

Vertex of a pyramid 273
Victory bonds 208
Volumes . . . 86, 92, 117, 119, 269, 273, 275, 276

Wastefulness 83, 145
Weight 331, 335
Weight per bushel 337
Workmen's Compensation Laws 226
Writing numbers 2

ANSWERS

Page 2. **5.** Two million, seven hundred eighteen thousand, three hundred fourteen dollars. **6.** Fourteen million, eight hundred thousand, six hundred twenty-one dollars. **7.** Three million, three. **8.** Fifteen million, fifteen thousand, fifteen. **9.** Five billion, two hundred seven million, eight hundred thirty-six thousand, two hundred ninety-eight dollars and thirty-seven cents. **10.** Eight billion, eight million, eight thousand, eight dollars and eight cents. **11.** 4,000,004. **12.** 7,000,000,021. **13.** $63,000,200.08.

Page 5. **1.** 304. **2.** $50.58.

Page 7. **1.** $21. **2.** $23.19. **3.** $21.05. **4.** $34.23. **5.** $44.94. **6.** $45.83. **7.** $24,993.02. **8.** $22,099.98. **9.** 3661.5. **10.** 24,446.8. **11.** 31.121.

Page 9. **9.** 1214. **10.** 1062. **11.** 2427. **12.** 1739. **13.** 2071. **14.** 3364. **15.** 34,055. **16.** 8187. **17.** 42,894. **18.** 58,874. **19.** 251,494. **20.** 325,479.

Page 10. **6.** $1276. **7.** $3809. **8.** $3355. **9.** $4266. **10.** $4462. **11.** $1710.01. **12.** $3549.25. **13.** $1333.84. **14.** $1233.85. **15.** $1564.24. **16.** $3303.88. **17.** $4244.57. **18.** $1216.19. **19.** $2616.31. **20.** $4729.92. **21.** $12,083.70. **22.** $6282.29. **23.** $12,599.23. **24.** $35,472.39. **25.** $74,031.61. **26.** $678.99.

Page 13. **1.** 413,664. **2.** 1,444,768. **3.** 555,333. **4.** 1,411,592. **5.** 3,255,896. **6.** 4,327,056. **7.** 7,576,569. **8.** 2,518,560. **9.** 4,364,672. **10.** 2,193,734. **11.** 1533. **12.** 5002. **13.** 2496. **14.** 1536. **15.** 2660. **16.** 4872. **17.** 1,469,264. **18.** 1,389,789. **19.** 3,071,880. **20.** 4,015,920. **21.** 5,485,172. **22.** 3,403,944. **23.** 6,195,441. **24.** 39,930,966. **25.** $13,654.75. **26.** $10,174.50. **27.** $13,639.92. **28.** $32,181.32. **29.** 178.7. **30.** 134. **31.** 1.34. **32.** 0.134. **33.** 4400. **34.** 5452.4. **35.** 97. **36.** 74.2. **37.** 742. **38.** 74,200. **39.** 742. **40.** 248.2.

Page 15. **1.** 18,500. **2.** 24,100. **3.** 32,100. **4.** 430,625. **5.** 350,175. **6.** 81,000. **7.** 68,000. **8.** 186,000. **9.** 1,903,750. **10.** 3,454,250. **11.** 4240. **12.** 900. **13.** 1000. **14.** 3200. **15.** 1300. **16.** 1200. **17.** 2000. **18.** 600. **19.** 4500. **20.** 7700. **21.** 6400. **22.** 6500. **23.** $1. **24.** $2. **25.** $6. **26.** $42. **27.** $3. **28.** $6. **29.** $20. **30.** $32. **31.** 1.87\frac{1}{2}$. **32.** 3.33\frac{1}{3}$. **33.** 9.37\frac{1}{2}$. **34.** 23.33\frac{1}{3}$.

Page 16. **1.** $2.50. **2.** $82,510. **3.** $62.50. **4.** $55. **5.** $37. **6.** $287.50. **7.** $250. **8.** $6375. **9.** $3,211,200. **10.** $37,610. **11.** $43,077.50. **12.** $3600.

13. $16,000. 14. $8125. 15. $16,875. 16. $637.50. 17. $1881.25. 18. $80,000. 19. $61,000. 20. $80,000. 21. $800,000. 22. $350,000. 23. 1200. 24. 1600. 25. 4800. 26. $10,410. 27. 210,500. 28. $10,605. 29. 238,824. 30. $1,053,250. 31. $10,560,000. 32. $105,600. 33. 80,000,000.

Page 18. 1. 842; 345; 657; 752; 557. 2. 2593; 2775; 494; 933; 938. 3. 2308; 1131; 692; 741; 881. 4. $1712\frac{1}{2}$; $1392\frac{1}{3}$; $1570\frac{3}{4}$; $1458\frac{1}{5}$; $556\frac{1}{6}$. 5. $920\frac{1}{7}$; $1206\frac{1}{8}$; $385\frac{5}{9}$; $1215\frac{7}{9}$; $1928\frac{3}{5}$. 6. $740\frac{5}{8}$; $915\frac{7}{8}$; $617\frac{3}{8}$; $143\frac{3}{7}$; $429\frac{3}{7}$. 7. 84. 8. 21. 9. 30; 381 *remainder*. 10. 121. 11. 270. 12. 248. 13. 363. 14. 475. 15. 4380. 16. $3.04; 8¢ *remainder*. 17. $3.29; 6¢ *remainder*. 18. $3.85; 6¢ *remainder*. 19. $2.03. 20. $3.21. 21. $3761.69; 25¢ *remainder*. 22. $4606.83; 26¢ *remainder*. 23. $8426.92; 40¢ *remainder*. 24. $7104.91; 38¢ *remainder*.

Page 20. 1. 1936.616. 2. 67.835. 3. 3071.833. 4. 6.701. 5. 2794. 6. 2.01. 7. 1.7. 8. 0.17. 9. 17. 10. 1.7. 11. 2.3. 12. 23. 13. 2.3. 14. 2.1. 15. 21. 16. 2.1. 17. 1000. 18. 10,000. 19. 10,000. 20. 2090. 21. 20.9. 22. 17. 23. 8.2. 24. 8.2. 25. 9100. 26. 51. 27. 6. 28. 980. 29. 78. 30. 7.1. 31. 81. 32. 65. 33. 91. 34. 0.7. 35. 21. 36. 1.2. 37. 513. 38. 0.513. 39. 0.510. 40. 5.102. 41. 1.1. 42. 11. 43. 1.85. 44. 1076. 45. 10.76. 46. 2000. 47. 2.03. 48. 2.03. 49. 24,500. 50. 800. 51. 174.

Page 21. 1. 1589. 2. 155. 3. 634. 4. 290.28. 5. 27. 6. 142.29. 7. 224.34. 8. 250.41. 9. 637.52. 10. 477. 11. 238.68. 12. 107.52. 13. 750. 14. 146.19. 15. 540. 16. 25; 175; $75; $13,459; $115,053. 17. 26; 33; $7; 3950; $19,280. 18. $2001; $3003; $2213; $3621. 19. 3,618,769; 18,093,845.

Page 22. 1. 2. 2. 99. 3. 66. 4. 4. 5. 16. 6. 880; 2640. 7. 278; 1. 8. 4400; 13,200. 9. 4; 960. 10. 2666; 2. 11. 108 qt. 12. 27 qt. 13. 8 qt. 14. 16 qt. 15. 72 qt. 16. 5000 lb. 17. 1500 lb. 18. 3 lb. 19. 4 lb. 20. $\frac{1}{2}$ lb. 21. 18 sq. ft. 22. $272\frac{1}{4}$ sq. ft. 23. 43,560 sq. ft. 24. 3 sq. ft. 25. 1320. 26. 440. 27. 20. 28. $\frac{1}{4}$. 29. 1089 ft.

Page 23. 1. $\frac{7}{8}$. 2. $\frac{7}{16}$. 3. $\frac{9}{16}$. 4. $\frac{3}{16}$. 5. $1\frac{9}{16}$. 6. $\frac{5}{16}$. 7. $\frac{3}{16}$. 8. $1\frac{5}{16}$. 9. $1\frac{9}{16}$. 10. $1\frac{1}{16}$. 11. $\frac{23}{32}$. 12. $\frac{31}{32}$. 13. $1\frac{1}{32}$. 14. $1\frac{23}{32}$. 15. $\frac{17}{32}$. 16. $1\frac{1}{64}$. 17. $1\frac{11}{64}$. 18. $\frac{5}{64}$. 19. $1\frac{7}{64}$. 20. $\frac{21}{64}$. 21. $2\frac{103}{320}$. 22. $3\frac{1}{32}$. 23. $14\frac{1}{32}$. 24. $15\frac{5}{32}$. 25. $9\frac{11}{64}$. 26. $11\frac{17}{64}$. 27. $18\frac{7}{64}$. 28. $16\frac{13}{64}$. 29. $9\frac{1}{8}$. 30. $10\frac{1}{16}$. 31. $92\frac{13}{30}$. 32. $138\frac{3}{16}$. 33. $167\frac{3}{8}$. 34. $142\frac{9}{16}$. 35. $128\frac{3}{8}$. 36. $122\frac{49}{80}$. 37. $113\frac{11}{20}$. 38. $69\frac{49}{80}$. 39. $16\frac{3}{8}$. 40. $16\frac{5}{16}$. 41. $12\frac{29}{30}$. 42. $14\frac{27}{32}$. 43. $12\frac{13}{64}$. 44. 14. 45. $15\frac{21}{64}$. 46. $38\frac{5}{16}$. 47. $\frac{5}{8}$. 48. $\frac{1}{2}$. 49. $\frac{7}{16}$. 50. $\frac{9}{16}$. 51. $\frac{11}{32}$. 52. $\frac{23}{32}$. 53. $\frac{27}{64}$. 54. $27\frac{1}{8}$. 55. $57\frac{1}{4}$. 56. $63\frac{1}{10}$. 57. $44\frac{25}{32}$. 58. $61\frac{13}{16}$. 59. $64\frac{19}{32}$. 60. $70\frac{43}{48}$. 61. $2\frac{1}{16}$. 62. $5\frac{3}{16}$. 63. $2\frac{1}{16}$. 64. $4\frac{1}{16}$. 65. $8\frac{7}{32}$. 66. $3\frac{17}{32}$. 67. $14\frac{15}{16}$.

Page 24. 1. $\frac{3}{4}$. 2. $1\frac{1}{4}$. 3. $\frac{5}{8}$. 4. $\frac{7}{8}$. 5. $1\frac{1}{8}$. 6. $1\frac{3}{8}$. 7. $\frac{3}{8}$. 8. $1\frac{1}{8}$. 9. $1\frac{1}{8}$. 10. $1\frac{3}{8}$. 11. $\frac{1}{4}$. 12. $\frac{1}{4}$. 13. $\frac{3}{8}$. 14. $\frac{1}{8}$. 15. $\frac{3}{8}$. 16. $\frac{1}{8}$. 17. $\frac{1}{8}$. 18. $\frac{5}{8}$. 19. $\frac{3}{8}$. 20. $\frac{1}{8}$. 21. $\frac{1}{8}$. 22. $\frac{3}{8}$. 23. $\frac{1}{16}$. 24. $\frac{3}{16}$. 25. $\frac{5}{16}$. 26. $\frac{7}{16}$. 27. $\frac{1}{32}$. 28. $\frac{21}{32}$. 29. $\frac{9}{32}$. 30. $\frac{15}{32}$. 31. 2. 32. $\frac{1}{2}$. 33. $1\frac{1}{2}$. 34. $\frac{5}{8}$. 35. $\frac{6}{7}$. 36. $1\frac{1}{8}$. 37. $\frac{3}{8}$. 38. $1\frac{1}{8}$. 39. $\frac{16}{21}$. 40. $\frac{9}{10}$. 41. 119. 42. 1116. 43. 2170. 44. 1524. 45. 2280. 46. 1904. 47. 1062. 48. 1740. 49. $1321\frac{7}{8}$. 50. $13\frac{3}{4}$. 51. $27\frac{1}{8}$. 52. $64\frac{3}{16}$. 53. $19\frac{19}{32}$. 54. $37\frac{31}{32}$. 55. $25\frac{7}{8}$. 56. $3\frac{1}{10}$. 57. $2\frac{29}{30}$. 58. $2\frac{3}{4}$. 59. $\frac{3}{8}$.

Page 27. 17. 2; 2; 2.4. 18. 110; 110; 931. 19. 1814; 1814; 4688. 20. 4860; 4860; 2904. 21. 1205; 1205; 1120. 22. $\frac{1}{8}$; 9. 23. $\frac{3}{8}$; 240. 24. 140; 2800. 25. 32; 3224. 26. 1586; 1586; 2834.

Page 28. 10. 0.1; 10%. 11. 0.001; $\frac{1}{1000}$. 12. 8. 13. 0.005. 14. 1.5. 15. 0.00005.

Page 29. 6. 90%. 7. 620%. 8. 59%. 9. 72%. 10. $32\frac{1}{2}$%. 11. 65%. 12. $666\frac{2}{3}$%. 13. 0.375%.

Page 30. 9. $27\frac{7}{9}$%. 10. $8\frac{1}{3}$%; $27\frac{7}{9}$%. 11. $6\frac{1}{4}$%; $1\frac{9}{16}$%. 12. 50%; $62\frac{1}{2}$%; 100%. 13. $6\frac{1}{4}$%; 20%; 100%. 14. $6\frac{2}{33}$%; $18\frac{2}{11}$%; 100%; 200%. 15. $11\frac{1}{9}$%.

Page 31. 4. Four hundred seventy-five per cent. 5. Four and seventy-five hundredths per cent. 6. Three and nine tenths per cent. 7. Thirty-nine per cent. 8. Thirty-nine hundredths per cent. 9. Three hundred ninety per cent. 10. Five and twenty-five hundredths. 11. Seven and thirty-five hundredths. 12. Six tenths per cent. 13. Six hundredths per cent. 14. 2.25. 15. 0.0025. 16. 3. 17. 0.00375. 18. 0.004. 19. $\frac{38}{100}$. 20. $\frac{1}{16}$. 21. $8\frac{1}{4}$. 22. $4\frac{3}{4}$. 23. $\frac{1}{3}$. 24. 60%. 25. $33\frac{1}{3}$%. 26. 75%. 27. 80%. 28. 250%. 29. 375%. 30. 0.28. 31. 0.34. 32. 0.006. 33. $0.66\frac{2}{3}$. 34. 6. 35. 3.55. 36. 2.5. 37. 2.75. 38. 3.25. 39. 50. 40. 0.00006. 41. 45.5.

Page 32. 1. 2; 2; 2. 2. 8; 8; 50; 50%. 3. 4; 4; 4. 4. $\frac{1}{8}$; $12\frac{1}{2}$; $12\frac{1}{2}$%. 5. 8; 8. 6. 1; 3. 7. 6; 12. 8. 16; 48. 9. 5; 25. 10. 14; 24. 11. 36; 80. 12. $\frac{3}{20}$; $\frac{3}{20}$; $\frac{3}{20}$. 13. 0.0075; $\frac{3}{400}$. 14. $6.66\frac{2}{3}$; $\frac{20}{3}$.

Page 34. 16. $1\frac{1}{4}$. 17. $1\frac{3}{5}$. 18. $\frac{5}{12}$. 19. 10%. 20. 50%. 21. 20%. 22. 4. 23. 25%. 24. $1\frac{2}{3}$. 25. \$636.90. 26. \$931.16. 27. \$641.09. 28. \$1461.01. 29. \$6663.12. 30. \$25,721.23. 31. \$53,380.08. 32. \$59,256.75. 33. 98,172.54. 34. 132,451.44.

Page 37. 1. \$6.75; \$38.25. 2. 3; 47. 3. \$15.40. 4. 1.31. 5. \$555. 6. \$3,808,800. 7. \$49.68. 8. 178; 1246.

Page 38. 7. 16.58 lb. 8. 15.96 lb.; \$4.15. 9. Second; 52¢. 10. \$1458.

Page 41. 1. 5%; $3\frac{1}{3}$%. 2. 3%; 20%. 3. 4%; 5%. 4. $6\frac{1}{4}$%; $16\frac{2}{3}$%. 5. 25%; 1%. 6. 1650%; 550%. 7. $266\frac{2}{3}$%; 150%. 8. $33\frac{1}{3}$%; 20%; 10%.

9. 50%; 33⅓%; 25%. 10. 50%; 25%; 16⅔%. 11. 14⅔%. 12. 166⅔%. 13. 12½%. 14. 9%. 15. 5.8%. 16. 20%.

Page 42. 1. 7½%. 2. 25%. 3. 23%. 4. 75%. 5. 39.7%. 6. 48%. 7. 12%. 8. 75%; 58⅓%. 9. 66⅔%; 50%. 10. 6¼%; 12½%. 11. 45%; 55%; 100%.

Page 43. 1. 91,375. 2. 21,800. 3. 87. 4. 288. 5. $16.64. 6. 260; 143. 7. Yes.

Page 44. 1. 400. 2. 1500. 3. 400. 4. 1400. 5. 1100. 6. 296. 7. 1470. 8. 159. 9. 496. 10. 520. 11. 194.7. 12. 340. 13. 20.2. 14. 62. 15. 11,320. 16. 7700. 17. $31; $23.25. 18. $49; $78.40. 19. 113 ft. 4 in.; 249 ft. 4 in. 20. 362 lb. 8 oz.; 6343 lb. 12 oz. 21. $200. 22. 440. 23. 120%; $300. 24. $4220. 25. 120; 96.

Page 46. 1. 10%. 2. 100%. 3. 33⅓%. 4. 20%. 5. 20%. 6. 16⅔%; 12½%. 7. 7%. 8. 7%. 9. 11⅓%. 10. 200%. 11. 66⅔%. 12. 25%. 13. 20%. 14. 7%. 15. 7%. 16. 14%. 17. 5%. 18. 13%. 19. 23%. 20. 9%. 21. 11%.

Page 47. 1. 12½%. 2. 25%; 20%; 25%. 3. 8%; 92%. 4. 132. 5. 371. 6. 19.18 mi. per hr. 7. $2520. 8. 45.

Page 48. 1. 0.8%. 2. $180. 3. 25%. 4. 33⅓%. 5. 6¼%. 6. 300%. 7. 78.8%. 8. 60.3%. 9. 23.3%; 26.7%. 10. 3%. 11. $207.

Page 49. 1. 2%. 2. 50%. 3. 14⅔%. 4. 20%. 5. 15%; 85%. 6. 292; 219. 7. 104. 8. 25%. 9. 0.1%. 10. 24%. 11. 33⅓%.

Page 50. 1. 500. 2. 500. 3. $525. 4. $540. 5. 55 lb. 6. $27; $24. 7. 55¢; $5.

Page 52. 16. $5.88. 17. $23. 18. $33.75. 19. $21.20. 20. $30.20. 21. $33.30. 22. $34.95. 23. $27.45. 24. $50.20. 25. $27.80. 26. $41.25. 27. $77.05. 28. $71.40. 29. $18.85. 30. $49.50. 31. $59.80. 32. $76.20.

Page 56. 1. $213.38. 2. $576.01. 3. $120.15. 4. $529.53. 5. $310.43. 6. $798.06. 7. $1126.54. 8. $10,537.49.

Page 58. 5. $256. 6. $427.50. 7. $472.50. 8. $249.90. 9. $6.25. 10. No; no difference; no difference. 11. 17.2%. 12. 19%. 13. 20.8%. 14. 20.2%. 15. 19.04%. 16. 26.4%. 17. 28%. 18. $1043.10. 19. $38.25. 20. First; $325.50.

Page 59. 1. $167.33. 2. $3720.26. 3. $696.92. 4. $415.97. 5. $2009.93. 6. $2485.65.

Page 60. 1. $30.24. 2. $17.26; $5.08. 3. $13.80; $18.40. 4. $1.79; $4.50. 5. $18.91. 6. $12.96; $38.89.

Page 61. 1. $108.75. 2. $75.60. 3. $67.20. 4. $153.60. 5. $658. 6. $691. 7. $1147. 8. $920. 9. $58.37. 10. $18.75. 11. $26.95. 12. $47.75. 13. $364.80. 14. $755.25. 15. $645.78. 16. $916.05. 17. $677.95. 18. $1824. 19. $2922. 20. $5852. 21. $7030. 22. $8449. 23. $329.09. 24. $511.95. 25. $694.30.

26. $824.76. **27.** $30.59. **28.** $196.35. **29.** $247.70. **30.** $289.86. **31.** $597.21. **32.** $186.09. **33.** $427. **34.** $524.43. **35.** $893.69. **36.** $1080.70. **37.** $734.16. **39.** $580.

Page 63. 1. 30%. 2. 5%. 3. 30%. 4. 7%. 5. 40%. 6. 40%. 7. $6440. 8. $4760. 9. $9800; $11,760. 10. $4000; $28,000. 11. $60, profit.

Page 65. 1. 25%. 2. $5. 3. $2.50. 4. 20%. 5. $33\frac{1}{3}$%. 6. 15%. 7. $16. 8. $30. 9. $5. 10. $3000. 11. $2340.

Page 66. 1. 16.4%. 2. 14.1%. 3. 19.7%. 4. $14.28. 5. $16\frac{2}{3}$%. 6. $153.23. 7. 25%. 8. 15.7%. 9. $3000; $3600.

Page 68. 14. $22.50. 15. $35. 16. $23.75. 17. $25.50. 18. $3.38. 19. $2.98. 20. $4.28. 21. $1.63. 22. $18.78. 23. $22.01. 24. $50.12. 25. $55.51. 26. $3. 27. $5.50. 28. $7. 29. $22.50. 30. $7.13. 31. $3.13. 32. $10.50. 33. $12.38. 34. $10.43. 35. $14.47. 36. $17.27. 37. $23.77.

Page 69. 1. $52.32; $26.16; $17.44; $34.88; $43.60. 2. $36; $18; $30; $3; $27. 3. $43.20; $3.60; $1.80; 60¢; 12¢. 4. $9.02; $4.51; $3.01; 75¢; 25¢.

Page 70. 1. $10.50. 2. $14.67. 3. $62.50. 4. $101.50. 5. $76.50. 6. $1200. 7. $115.50. 8. $14.25. 9. $11.25. 10. $5.40. 11. $98.44. 12. $118.13. 13. $59.06. 14. $78.75. 15. $108.28.

Page 71. 1. $1100.42. 2. $19,228. 3. $86.28. 4. $3695.63. 5. $2278.13. 6. $11.60. 7. $2756.25. 8. $3402. 9. $625.

Page 74. 1. $35.50. 2. $44.38. 3. $53.25. 4. $39.94. 5. $62.13. 6. $26.35. 7. $33.73. 8. $39.36. 9. $47.44. 10. $66.76. 11. $185.68. 12. $210.43. 13. $345.91. 14. $279.14. 15. $3611.84. 16. $66.22. 17. $31; $38.75. 18. $106.40. 19. $78.31. 20. $36.43; $38.35. 21. $623.19; $509.88. 22. $3036.56. 23. $4373.75. 24. $6864.67.

Page 75. 6. $11.25; $9.38. 7. $31.25; $26.04. 8. $18.40; $15.33. 9. $4.35; $3.63. 10. $41.25; $34.38. 11. $90; $75.

Page 77. 1. $393.75. 2. $337.28. 3. $104.58. 4. $651.97. 5. $140.77. 6. $178.85.

Page 78. i. 1. 75%. 2. $33\frac{1}{3}$%. 3. $133; $1330; $13,300. 4. $6125; $60. ii. 1. 0.002. 2. $87\frac{1}{2}$%. 3. $2\frac{7}{9}$%. 4. $\frac{5}{8}$; $\frac{7}{8}$; $\frac{5}{6}$. iii. 1. $487\frac{1}{2}$. 2. $65\frac{5}{8}$. 3. $3.51. 4. $16.62. 5. $86.87. 6. 1.848 ft. 7. 68.6. 8. 4695. 9. 46.95. 10. 251. iv. 1. $8\frac{1}{3}$%; $0.8\frac{1}{3}$%; $0.08\frac{1}{3}$%. 2. 90%; $7\frac{1}{2}$%; $2\frac{1}{2}$%. 3. 50%; $12\frac{1}{2}$%; $2_{\frac{1}{2}}$%. 4. 1200 ft.; 2400 ft. 5. 96 ft. 8 in.; 120 ft. 10 in. v. 1. $13. 2. $240. 3. $13.09. 4. $518.70. 5. $667.92.

Page 79. 1. Four hundred and seventy-five thousandths; four hundred seventy-five thousandths. 2. $57.98. 3. 31 ft. 1 in. 4. $301.63. 5. $476.33. 6. $136.71. 7. $574.35. 8. $285,995.50. 9. $175,854.06. 10. 29.14. 11. 31.91.

12. 346.144. 13. 45.02. 14. $168.75. 15. $17.88. 16. $44.33. 17. $23.93; $29.91; $26.92. 18. $79.72; $67.76; $87.69. 19. $62.91; $47.18; $52.42. 20. $36.74; $33.06; $31.22. 21. $60.28; $42.19; $39.18. 22. $136.14; $102.11; $90.76. 23. $41.98; $46.65; $51.31. 24. $37.35; $44.82; $48.56. 25. $61.25; $68.06; $95.28.

Page 83. 1. $7.50. 2. $90. 3. $67.20.

Page 84. 1. July 4. 2. July 23. 3. July 5. 4. $6.30. 5. $4.50. 6. $9.30.

Page 87. 1. $13\frac{1}{2}$ sq. ft.; $1\frac{1}{2}$ sq. yd. 3. $183.75. 4. 27. 5. 24. 6. 27.

Page 88. 1. $1.80. 2. 55,400; $2.97. 3. $4.29. 4. $7.02. 5. $3; $2.70. 6. 23¢. 7. $5.28. 8. $6.59. 9. 800; $0.88; $0.04; $0.84. 10. 2800; $2.52; $0.13; $2.39. 11. 6100; $7.32; $0.37; $6.95. 12. 4600; $4.60; $0.23; $4.37. 13. 3300; $4.62; $0.23; $4.39. 14. 3900; $3.32; $0.17; $3.15. 15. 3600; $4.86; $0.24; $4.62. 16. 4400; $4.62; $0.23; $4.39. 17. 3700; $3.52; $0.18; $3.34. 18. 7800; $8.97; $0.45; $8.52.

Page 90. 1. 1526. 2. $8.14. 3. $2.32. 4. $2.14. 5. $5.47.

Page 91. 1. 4225. 2. 75 ft.; 6093.75 sq. ft. 3. 9959 sq. ft. 4. 1082.5 sq. ft. 5. 1732 sq. ft. 6. 2706.25 sq. ft. 7. 4330 sq. ft. 8. 2706.25 sq. ft. 9. 2165 sq. ft. 10. 3.11. 11. 600 sq. rd.; $3\frac{3}{4}$ A.

Page 92. 1. 9072. 2. 60 cu. ft.; 103,680 cu. in.; 65 cu. ft.; 112,320 cu. in. 3. $54\frac{1}{8}$. 4. 800. 5. 13,858. 6. 219.14. 7. 200 lb. and 25 lb. 8. 14,400 cu. ft.; $200.

Page 93. 1. 1024. 2. 512. 3. 576. 4. 1680. 5. 640. 6. $36.12.

Page 94. 1. 737. 2. 2632. 3. 3602. 5. $762. 6. $451.01. 7. $2392.50. 8. $4705.51. 9. $143.36.

Page 96. 1. 3200; 4800. 2. 0.46 A.; 0.07 A.; 0.11 A.; 0.28 A. 3. 935 sq. ft. 4. 5310 cu. ft. 5. 6726 cu. ft. 6. 780; 47.27. 7. 288 sq. ft., asparagus; 342 sq. ft., parsnips; 684 sq. ft., beets; 810 sq. ft., tomatoes; 540 sq. ft., cauliflower. 8. 8865.

Page 97. 1. 14,148; $1160.14; $200.20. 2. $80.10. 3. $138.75. 4. $360.75. 5. 22.96; $127.50.

Page 98. 4. $365\frac{5}{8}$ sq. in. 5. $251\frac{7}{8}$ sq. in.

Page 99. 1. $\frac{1}{8}$. 2. 4 ft. 1 in. 3. $\frac{5}{7}$. 4. $99\frac{3}{4}$ sq. in. 5. 8 in.; 9 in.; $4\frac{3}{4}$ in.

Page 100. 1. 9 ft. 11 in. 2. 2 ft. $7\frac{1}{2}$ in. 3. $5\frac{5}{6}$. 4. 3 ft. $3\frac{1}{3}$ in. 5. $\frac{4}{5}\frac{3}{3}$; 0.896. 6. $1; $1.16; 84¢.

Page 101. 1. 8. 2. 3 ft. $\frac{1}{2}$ in.; 6. 3. 9. 4. 5. 5. 2.

Page 105. 7. 80 ft., 20 ft.; 70 ft., 30 ft.; 10 ft., 90 ft. 8. 33 : 40. 9. 18 clear days; 12 cloudy days. 10. $9; $6; $7; $10.50.

Page 107. 1. 46. 2. 5. 3. 214.2 in. 4. 4 hr. 5. 5. 6. 21.9 sec. 7. 12 min. 40 sec. 8. 75 mi. 9. $40; $50.

Page 108. 1. 36. 2. 18. 3. 82. 4. 175. 5. 4. 6. 56. 7. 160. 8. 22.69. 9. 4.135. 10. 15.78. 11. 0.24. 12. 2.00. 13. 1.28. 14. 2.37. 15. 5. 16. 10 min. $18\frac{3}{4}$ sec. 17. 29 min. $59\frac{1}{3}$ sec. 18. \$26.67; \$33.33. 19. 5.76. 20. \$14.70. 21. \$6. 22. 1034.8.

Page 110. 1. 29 ft. 2. 43 ft. 2 in. 3. 55 ft. 4. 51 ft. 5. 57 ft. 8 in.

Page 111. 1. 3. 2. $12\frac{1}{2}$ ft. 4. 30 lb. 5. 80 lb.

Page 112. 1. 38 ft. 6 in. 2. 35 ft. 9 in. 3. 80 ft. 4. 37 ft. 5. 50 ft. 6. 27 ft.

Page 114. 1. 16 ft. 6 in. 2. $14\frac{1}{2}$ in. 3. 1.34 in. 4. 214.34 in. 5. $1329\frac{3}{7}$ in. 6. 16.06 ft. 7. 20.02 in. 8. 14.74 ft. 9. 18.92 in. 10. $42\frac{3}{7}$ ft. 11. 202.4 in. 12. 42 ft. 13. 56 in. 14. 21.7 in. 15. 63 in. 16. 53.41 in. 17. 19.73 in. 18. 10.74 in. 19. 19.37 ft. 20. 17 in. 21. 31 in. 22. 58 in. 23. 308 in. 24. 572 in. 25. 61.6 in. 26. 96.8 ft.

Page 116. 1. 154 sq. in. 2. 15,400 sq. in. 3. 13.86 sq. ft. 4. $38\frac{1}{2}$ sq. in. 5. 6.16 sq. in. 6. $\frac{1}{4}$. 7. 4. 8. 1386 sq. ft. 9. $962\frac{1}{2}$. 10. 7.7 in.; 3.85 in.; 46.59 sq. in. 11. 78.54 sq. in. 12. 11 in. 13. $128\frac{6}{7}$. 14. $94\frac{2}{7}$ in. 15. 37.

Page 118. 1. 72,300. 2. 10,875 cu. in. 3. 325.92 cu. in. 4. 178.38 cu. ft. 5. 0.34 cu. ft. 6. 310.98 cu. in. 7. 199.08 cu. in. 8. 8.38 in. 9. 8 in. 10. 9 in. 11. 9.26 in. 12. 999.6 cu. in. 13. 70.5 cu. in. 14. 113.5 cu. ft. 15. 43.2 cu. ft. 16. 100.3 cu. ft. 17. 2301.3 cu. in. 18. 5088.45 cu. in. 19. 3906.25 cu. in. 20. 10 in.; 10 in.; 11 in.

Page 119. 1. 2100 cu. ft. 2. $50{,}285\frac{5}{7}$ cu. ft. 3. 46.18. 4. 169,273.47. 5. $144\frac{3}{8}$. 6. 2079. 7. 36 cu. in. 8. 72 cu. ft. 9. $19\frac{1}{2}$ cu. in. 10. $17\frac{1}{2}$ cu. in. 11. $57\frac{3}{4}$ cu. in. 12. 42 cu. ft.

Page 120. 1. $10\frac{1}{2}$ ft.; $86\frac{5}{8}$ sq. ft.; $1559\frac{1}{4}$ cu. ft. 2. 351 cu. in. 3. 1320 cu. in. 4. $2085\frac{1}{2}$ cu. in. 5. 3 ft. 6. 6 ft. 6 in. 7. 18 sq. ft. 8. 15.3 sq. ft. 9. $1357\frac{5}{7}$ cu. in. 10. $6436\frac{4}{7}$ cu. in. 11. $377\frac{1}{7}$ cu. in.

Page 121. 13. 2400. 14. 5 sq. ft. 15. 21. 16. $7\frac{1}{2}$.

Page 122. 2. 525. 3. $301\frac{5}{7}$ sq. in. 4. $62\frac{6}{7}$. 5. 5092. 6. $188\frac{4}{7}$. 7. $244\frac{3}{4}$. 8. 110 sq. ft. 9. 27,456 sq. ft. 10. 1100.

Page 123. 1. Yes; \$1.84. 2. \$2.40; 29.6%; 22.9%. 3. \$3335.10.

Page 124. 1. 24. 2. 23 in. 3. $36\frac{3}{8}$ in. 4. \$200. 5. $1\frac{1}{3}$. 6. \$2.24; \$2.80. 7. $\frac{1}{4}$; 25%. 8. $\frac{47}{52}$; 90.4%. 9. $\frac{5}{24}$; $\frac{26}{5}$; 520%.

Page 125. 1. 50¢. 2. \$2.34. 3. \$585,000. 4. \$2.63. 5. \$13.50. 6. 2,281,250. 7. \$196,187.50.

Page 127. 1. 52¢. 2. \$1.14. 3. 25¢. 4. \$1.60. 5. 55¢. 6. 36¢. 7. 10¢. 8. \$1.94. 9. 500; 20¢.

Page 128. 1. $1964\frac{4}{7}$. 2. No; 308 cu. ft. 3. \$220.10; \$418.19. 4. $2828\frac{4}{7}$. 5. 558. 6. 1848. 7. \$73.15. 8. $4812\frac{1}{2}$. 9. 175.22. 10. 120.32. 11. 233.28. 12. 280. 13. 60.

Page 130. 1. $9\frac{3}{4}$ bbl., cement; $3\frac{1}{4}$ cu. yd., sand; $4\frac{7}{8}$ cu. yd., crushed rock. 2. $67\frac{1}{2}$ bbl., cement; 27 cu. yd., sand; $31\frac{1}{2}$ cu. yd., crushed rock. 3. 36. 4. 10. 5. 9 bbl., cement; 3 cu. yd., sand; $4\frac{1}{2}$ cu. yd., crushed rock. 6. $39.38.

Page 131. 1. 10. 2. 3280. 3. 54 T. 4. 77.58. 5. 12 hr. 48 min. 6. $24. 7. 12.5. 8. 11.2 T.

Page 132. 1. $172.50. 2. 10%; $267.50. 3. $1.26; 12.6¢. 4. 4500; 25,500. 5. No; $10.50.

Page 133. 1. 93. 2. $87.78. 3. 18; $8.10. 4. 21; $11.55. 5. $10.20.

Page 134. 1. 132. 2. 23. 3. 31. 4. 8. 5. $94.50; $68.25. 6. No; yes. 7. $41\frac{5}{8}$; 1 yd. 8. Yes; $2\frac{1}{8}$ yd.

Page 135. 1. 20; 34 in. 2. $1\frac{1}{4}$ yd.; 45 in. 3. 24. 4. 4. 5. $15\frac{1}{2}$. 6. $16\frac{2}{3}$%. 7. 52. 8. 20-yard piece; 44¢.

Page 136. 1. $33\frac{1}{3}$%. 2. 100%. 3. 125%. 4. 100%.

Page 137. 1. 3520; $2288. 2. $3696. 3. $4752. 4. $4752. 5. $202.40.

Page 138. 1. $80.80. 2. Total time: $43\frac{1}{2}$; $41\frac{3}{4}$; $39\frac{1}{2}$; $42\frac{1}{4}$; $32\frac{3}{4}$; $242\frac{1}{4}$. Total wages: $30.45; $25.05; $25.68; $23.24; $12.28; $146.45. Hours per day: 47; $43\frac{3}{4}$; 45; $37\frac{3}{4}$; $46\frac{1}{2}$; $22\frac{1}{4}$. 3. $31.54; $27.68; $25.34; $33.35; $34.03; $151.94. 4. $31.78; $28.35; $28.62; $27.64; $22.48; $138.87. 5. $31.90; $29.93; $29.41; $30.06; $32.27; $153.57. 6. *$10*, 12; *$5*, 3; *$2*, 4; *$1*, 1; *50¢*, 2; *25¢*, 3; *10¢*, 5; *5¢*, 2; *1¢*, 10. 7. Ex. 3: *$10*, 13; *$5*, 2; *$2*, 4; *$1*, 2; *50¢*, 2; *25¢*, 2; *10¢*, 2; *5¢*, 2; *1¢*, 14. Ex. 4: *$10*, 11; *$5*, 3; *$2*, 4; *$1*, 3; *50¢*, 3; *25¢*, 3; *10¢*, 5; *1¢*, 12. Ex. 5: *$10*, 13; *$5*, 2; *$2*, 5; *$1*, 1; *50¢*, 2; *25¢*, 4; *10¢*, 3; *5¢*, 4; *1¢*, 7. 8. $146.45. 9. $151.94; $138.87; $153.57.

Page 140. i. 1. $42; $90. 2. $291.60. 3. $207.36. 4. $2079. 5. 125. 6. 2500. ii. 1. 228 sq. ft. 2. 37.5 ft. 3. 126. 4. 42. 5. $49\frac{1}{2}$. 6. $32.

Page 141. 1. $625.88; $228.76. 2. $1619.57; $244.45. 3. $1462.91; $277.55. 4. $948.76; $851.26. 5. $27,545.70. 6. $42,461.94. 7. $77,467.46. 8. 355 ft. 9. 54; 27; 1080. 10. 27; 0.27. 11. 7 ft. 4 in.; 14 ft. 8 in. 12. 7 lb. 2 oz.; 5 lb. $5\frac{1}{2}$ oz. 13. 0.5; 50%. 14. 0.75; 75%. 15. 0.625; $62\frac{1}{2}$%. 16. $0.66\frac{2}{3}$; $66\frac{2}{3}$%. 17. $0.83\frac{1}{3}$; $83\frac{1}{3}$%. 18. 75.46 sq. in. 19. 124.74 sq. ft. 20. 4.95 sq. ft. 21. 2138.4 cu. in. 22. 6.52 sq. ft. 23. $8\frac{1}{4}$ in. 24. $3\frac{1}{2}$ in. 25. $43.32. 26. $122.80. 27. $181.70.

Page 145. 1. 718,750; 262,343,750. 2. 114,063. 3. 14,258. 4. $15,373,343.75.

Page 146. 1. $36.75. 2. $1020.80; $438.75. 3. $582.05. 4. $189; 52¢; $1.92; $1.40.

Page 147. 1. Yes. 2. $\frac{1}{5}$; 1000 lb. 3. 71.

Page 148. 1. $2474; $1379.61. 2. 79.9%; 108.5%; 84.7%; 79.2%; 77.1%. 3. $841.85; $732.09; $73.02; $76.75; $5.47; $27.12; $21.03; $2.67; $109.76; $1632.15; $647.52.

Page 151. 1. $375. 2. $231. 3. $107.18. 4. $5557.50. 5. $595.20. 6. $399.

Page 154. 1. $3000; $3038.76. 2. $4720; $4764.06. 3. $3920; $3987.02. 4. $2520; $2532.40. 5. $1933.75; $1940.25. 6. $713.07; $717.77. 7. $12,400; $12,653.19. 8. $502.09; $507.42. 9. $225.23. 10. $731.59. 11. $765.41. 12. $541.21. 13. $1093.08. 14. $2143.72. 15. $10; $8.75; $8.13; $9.38. 16. $9.50. 17. $110; $1.50. 18. 4½% simple interest; $9.48. 19. $164.84. 20. $488.84. 21. $487.22. 22. $531.62. 23. $1380.85.

Page 157. 16. $50, $20, $10, $5, $2. 17. $31. 18. 40¢. 19. $120; Feb. 1, 1923; 20¢. 20. $2.60. 21. $6.25.

Page 159. 1. $509. 2. $1176.35. 3. $523.73. 4. $2461.90. 5. $883.50. 6. $189.59. 7. $715. 8. $1668.75. 9. $391.17. 10. $348.

Page 161. 1. $560.22. 2. $167.57. 3. $180.92. 4. $1199.10. 5. $1026.45. 6. $1712.20. 7. $2399.67.

Page 163. 1. $16,133.26. 3. $18,854.11. 4. $2721.43.

Page 166. 1. $68.75. 2. $2082.93. 3. $120. 4. $756.67. 5. $1392.06.

Page 170. 1. $1.80. 2. $4.05. 3. $2.63. 4. $38.63. 5. $54.38. 6. $21.25. 7. $38.50. 8. $40. 9. $112.50. 10. $80. 11. $5.25. 12. $9.06. 13. $11.33. 14. $4.13. 15. $10.63. 16. $41.67. 17. $131.25. 18. $146.67. 19. $81.25. 20. $63.75.

Page 171. 1. $3.25. 2. $7. 3. 72¢. 4. 33¢. 5. $15. 6. $19.50. 7. 80¢. 8. $4.20. 9. 46¢. 10. 24¢. 11. $18. 12. $24.36. 13. $3.30. 14. $8.80. 15. 56¢. 16. 45¢. 17. $77. 18. $55.66. 19. 60¢. 20. $6.90. 21. 41¢. 22. 23¢. 23. $67.50. 24. $97.29.

Page 172. 1. 38¢; $74.62. 2. 45¢; $89.55. 3. 50¢; $49.50. 4. $3.75; $246.25. 5. $4.13; $270.87. 6. $8.50; $416.50. 7. $15; $1485. 8. $18.90; $1781.10. 9. $69.75; $4430.25. 10. 17¢. 11. 58¢. 12. $1. 13. $2.19. 14. $4.17. 15. $18.75. 16. $5.21. 17. $18.96. 18. $59.38.

Page 173. 2. $671.62. 3. $1231.25. 4. $1500. 5. $22.50; $1477.50. 6. $750; $6.55 more. 7. $4.38; $870.62. 8. $12.19; $962.81. 9. $2.13; $422.87. 10. $32.06; $2532.94. 11. $44.69; $3205.31. 12. $15.94; $4234.06.

Page 175. 1. $6.25; $743.75. 2. $1.88; $223.12. 3. $5; $495. 4. $3.75; $446.25. 5. $2.81; $222.19. 6. $83.63; $5491.37. 7. $31.25; $7468.75. 8. $23.75; $4726.25. 9. 21¢; $42.29. 10. $1.26; $124.24. 11. $2.75; $272.50. 12. $5.63; $369.87. 13. $1.91; $455.59. 14. $1.24; $295.26. 15. $3.60; $284. 16. $4.69; $370.71. 17. $30.94; $2719.06. 18. $36.84; $3238.16. 19. $1250; $6500. 20. $1747.07. 21. 75¢; $300.25. 22. $6.72; $799.95. 23. $2.59; $552.91. 24. $2.40; $325.31. 25. $5.03; $999.97. 26. $6.68; $1258.95.

Page 176. 1. \$17.50. 2. \$10.13; \$7.13. 3. \$49. 4. \$153. 5. Same. 6. More; 50¢. 7. \$12,528.91. 8. \$27.75.

Page 179. 1. \$497.50. 2. \$971.10. 3. \$348.42. 4. \$718.35. 5. \$1214.72. 6. \$1661.54. 7. \$615.13.

Page 180. 1. \$149.70. 2. 0.3%. 3. 0.1%. 4. \$249.75. 5. \$1480.92. 6. \$2055.73.

Page 183. 1. \$300.60. 2. \$3208.20; \$2501.25. 3. \$37.65; \$63; \$14.40; \$86.80; \$75.70. 4. \$4001.60. 5. 0.3%. 6. \$2750; \$2.75. 7. \$3750. 8. \$850.85. 9. Draft; 10¢. 10. Registered letter; 2¢.

Page 185. 1. \$3756.70. 2. \$751.50. 3. \$2452.45. 4. \$3496.50. 5. \$6753.38. 6. \$17,465. 7. \$5161. 8. \$3408.50. 9. Discount; $\frac{1}{8}$%. 10. Discount; $\frac{1}{8}$%. 11. \$249.75. 12. \$2447.50. 13. \$379.75. 14. \$3240.

Page 187. 1. \$866.40. 2. \$1554.50. 3. £250. 4. 66,000 fr. 5. \$1678.81. 6. \$10,528.85. 7. \$3.88. 8. \$4.44. 9. \$8.92. 10. \$3.08. 11. \$87. 12. \$122.25. 13. \$311.50. 14. \$627.50. 15. \$15.10. 16. \$3.84. 17. \$14.55. 18. \$6974. 19. \$175. 20. \$3750. 21. \$18,640. 22. \$26,362.50.

Page 189. 1. \$609.49. 2. \$813.68. 3. \$1143.77; \$1470.19. 4. \$2343.20. 5. \$1231.25. 6. Proceeds of \$750 note; \$13.25. 7. \$762.74. 8. \$217.50. 9. \$75.19. 12. \$1244.79. 13. \$2479.17.

Page 193. 1. \$40. 2. \$174.40. 3. \$63.75. 4. \$240.90. 5. \$24.60.

Page 194. 1. \$65.28. 2. \$501.60. 3. \$153.75. 4. \$35.75. 5. \$48.13. 6. \$26.81. 7. \$42.21. 8. \$66. 9. \$90.75. 10. \$134.20. 11. \$129.25. 12. \$171.05. 13. \$127.88. 14. \$103.81. 15. \$248.88. 16. \$96.30. 17. \$139.10. 18. \$171.20. 19. \$98.44. 20. \$149.80. 21. \$97.37. 22. \$187.25. 23. \$208.65. 24. \$144.45. 25. \$181.90. 26. \$256.80. 27. \$535. 28. \$695.50. 29. \$823.90. 30. \$968.35. 31. \$1621.05. 32. \$71.20. 33. \$70. 34. \$15,000; \$18,000.

Page 196. 1. \$138.84. 2. \$405.84. 3. \$222.50. 4. \$169.10. 5. \$124.60. 6. \$133.50. 7. \$142.40. 8. \$160.20. 9. \$169.10. 10. \$267. 11. \$400.50. 12. \$445. 13. \$578.50. 14. \$756.50. 15. \$979. 16. \$1335. 17. \$1343.90. 18. \$1424. 19. \$1557.50. 20. \$2225. 21. \$12,015. 22. \$4895. 23. \$6230. 24. \$15,575. 25. \$52.87. 26. \$2139.40.

Page 197. 1. \$36.80; \$10.50. 2. Gained; \$539.50. 3. \$400, macadam; \$18.18, concrete. 4. 2100.2%. 5. 12$\frac{1}{2}$¢. 6. 2%. 7. 25%; 22$\frac{1}{2}$%. 8. 4.44. 9. \$25.27. 10. 10.5¢. 11. \$21.88.

Page 201. 1. \$870; ad valorem. 2. \$525; specific. 3. \$440. 4. \$756.50. 5. \$585. 6. \$11.25. 7. \$250. 8. \$68.75. 9. \$97.50. 10. \$237.50. 11. \$76.70. 12. \$78.89. 13. \$1500. 14. \$1443.75.

Page 203. 1. \$149.51. 2. 30.3%. 3. 1011.1%. 4. \$27,485.70. 5. \$2410. 6. \$2750. 7. \$15,600. 8. \$358,605.

Page 206. 16. 147.

Page 207. 1. $462.50. 2. $70,000,000. 3. $170,000,000. 4. $255,000,000.

Page 208. 1. A little more than $1,000,000,000. 2. $152,350,646. 3. $177,501,966.13. 4. $297,034,497.50. 5. $191,250,000. 6. $118.75. 7. $31.25. 8. $180.63. 9. $437.50.

Page 209. 1. $237.50. 2. $23. 3. $292.25. 4. $36,375. 5. $427.20. 6. 2.1 mills. 7. 1.47%. 8. 28.6%. 9. $211.

Page 210. i. 1. $2282.50. 2. $4620; $4679.70; $4690.85. 3. $6062.50; $6189.97; $6208.40. 4. $14,210; $14,330.76; $14,352.84. 5. $28,500; $28,895.51; $28,951.13. ii. 2. $12.75; $1262.25. 3. $113.25; $7436.75. 4. 88¢; $174.62. 5. $5.32; $1270.48. iii. 1. $16.50. 2. $27.50. 3. $41.25. 4. $82.50. 5. $86.80. 6. $245. 7. $113.40. 8. $256.20. 9. $2625. 10. $6437.50. iv. 1. $1251.25. 2. $1280. 3. $1282.56. 4. $13,736.25. 5. $26,696.50. v. 2. $10.13. 3. $4.50. 4. $3.09. 5. $12.40.

Page 211. 1. I. *a.* $275.28; $274.45; $274.72; $275.55; $274.17; $275.83; $275.14; $274.86; $273.90; $276.10. *b.* $450.45; $449.10; $449.55; $450.90; $448.65; $451.35; $450.23; $449.77; $448.20; $451.80. *c.* $1251.25; $1247.50; $1248.75; $1252.50; $1246.25; $1253.75; $1250.63; $1249.37; $1245; $1255. *d.* $1501.50; $1497; $1498.50; $1503; $1495.50; $1504.50; $1500.75; $1499.25; $1494; $1506. *e.* $1876.88; $1871.25; $1873.12; $1878.75; $1869.37; $1880.63; $1875.94; $1874.06; $1867.50; $1882.50.

III. *a.* $217.72; $217.06; $217.28; $217.94; $216.85; $218.15; $217.61; $217.39; $216.63; $218.37. *b.* $317.07; $316.12; $316.43; $317.38; $315.80; $317.70; $316.91; $316.59; $315.48; $318.02. *c.* $425.68; $424.40; $424.82; $426.10; $423.97; $426.53; $425.46; $425.04; $423.55; $426.95. *d.* $676.43; $674.40; $675.07; $677.10; $673.72; $677.78; $676.09; $675.41; $673.05; $678.45. *e.* $843.20; $840.68; $841.52; $844.04; $839.83; $844.89; $842.78; $841.94; $838.99; $845.73.

2. $291.48. 3. $29.45. 4. $255,000,000. 5. $3\frac{3}{2}$; 0.55; 103.4; 3.28; $13\frac{7}{10}$; 25,425. 6. $3\frac{1}{2}$; 0.09; 20.8; 0.73; $1\frac{7}{20}$; 925. 7. $1\frac{3}{16}$; 0.91; 113.1; 6.15; $15\frac{5}{12}$; 31,935. 8. $3\frac{5}{2}$; 0.64; 109.2; 5.26; $14\frac{1}{2}$; 28,272. 9. $\frac{1}{8}$; 0.45; 30.5; 3.60; $2\frac{1}{2}$; 7435. 10. 4; 1.78; 1.64; 3.12; $1\frac{2}{5}$; 1.31. 11. $\frac{1}{2}\frac{1}{8}$; 0.09; 2804.27; 5.08; $54\frac{1}{8}$; 196,269,500. 12. $\frac{5}{5}\frac{1}{2}$; 0.22; 4458.78; 9.78; $69\frac{2}{3}$; 259,349,875. 13. $\frac{3}{16}$; 0.73; 130; 5.99; $16\frac{4}{15}$; 29,197. 14. $\frac{1}{16}$; 0.09; 5.8; 1.98; $1\frac{1}{15}$; 2847. 15. $\frac{9}{3\frac{1}{2}}$; 1.09; 139.7; 8.85; $17\frac{5}{8}$; 35,707. 16. $3\frac{1}{2}$; 0.27; 3.9; 0.89; $\frac{1}{2}$; 3663. 17. $5\frac{1}{2}$; 0.07; 2564.73; 2.56; $47\frac{1}{2}$; 161,393,750. 18. 2; 1.39; 1.50; 1.57; $1\frac{27}{25}$; 1.08. 19. $10\frac{6}{24}$; 0.16; 2965.34; 6.22; $57\frac{7}{24}$; 241,141,250. 20. 5; 2.96; 1.74; 3.82; $1\frac{7}{15}$; 1.61.

Page 215. 1. $7.74; $2.58. 2. No; $1.16, loss. 3. $3.55, gain. 4. 40 lb., nitrogen; 160 lb., phosphoric acid; 40 lb., potash. 5. 12 lb., nitrogen; 24 lb.,

phosphoric acid; 12 lb., potash. 6. Phosphoric acid; 12 lb., nitrogen; 12 lb., potash. 80 lb., nitrogen; 80 lb., potash. 7. 12 lb., nitrogen; 24 lb., phosphoric acid; no potash. No nitrogen; 24 lb., phosphoric acid; 12 lb., potash.

Page 217. 1. 2.13 lb. 2. 2.38 lb. 3. 2.44 lb. 4. 1.12; 1.15.

Page 218. 1. 2.0% nitrogen; 0.7% phosphoric acid; 2.2% potash. 2. 1053. 3. $76.34. 4. $1.12; 12¢. 5. $2.81. 6. $396. 7. 10%; 9.1%.

Page 221. 1. $23.75. 2. $30.80. 3. $43.13. 4. $88. 5. $75. 6. $3500. 7. $2052. 8. $202.13. 9. $1.65; 55¢. 10. $1.20; 40¢. 11. $10,000. 12. $10,500; $16,800. 13. $6050. 14. $31.25; $1500.

Page 224. 1. $410.40; $4104. 2. $2739. 3. $171; $3420. 4. $148.25. 5. $263.60; $11,731.50. 6. $4076; $1108.80. 7. $2739; $4108.50.

Page 225. 1. $1912; $1523. 2. $487.75. 3. $1474.11. 4. $1209.09. 5. $1097. 6. $665. 7. $11.50.

Page 226. 1. $8.40. 2. $145.60. 3. $408.

Page 230. 1. $1,250,000. 2. $500. 3. $270. 4. $2,750,000. 5. 2% quarterly. 6. 5%.

Page 232. 7. $8727.50. 8. $41,010. 9. $21,810. 10. $48,025. 11. $7141.25. 12. $12,961.88. 13. $12,237.50. 14. $1200.25. 15. $317.88. 16. $8832. 17. $25,410. 18. $18,487.50. 19. $57,735. 20. $4986.25. 21. $5214.30. 22. $632.50. 23. $841.50. 24. $192,840. 25. $5177. 26. $265.25. 27. $5371. 28. $11,001.75.

Page 236. 1. $5000; $350. 2. 5.8%. 3. $195, gain. 4. $221.25, loss. 5. $58.50, gain. 6. $11.75, loss. 7. $1230, gain. 8. $3558.75, loss. 9. 3.9%. 10. 4.9%. 11. 5.0%. 12. $65.

Page 237. 1. $1.50. 2. $50. 3. $220. 4. $104.30; savings bank. 5. Yes; $245. 6. $1125; $450; the one at 6%. 7. $97.50. 8. $20. 9. $23.75. 10. $23. 11. $29. 12. $39. 13. $52.50. 14. $50.88. 15. $39.38. 16. $50. 17. $63. 18. $312.50. 19. $375. 20. $525. 21. $493.50. 22. $195. 23. $337.50. 24. $595. 25. $175. 26. Same. 27. Same. 28. $125; $300; the 5% security. 29. $25.38; $50.75; $101.50; $253.75. 30. $1376.55. 31. $5.25. 32. $149.70. 33. 6.4%.

Page 240. 1. 7%. 2. $315. 3. 5.6%. 4. $599.50. 5. $310; $50. 6. 7%. 7. Yes; $622.60. 8. $17\frac{2}{3}$%.

Page 244. 1. House, $300; Food, $375; Clothing, $300; Higher Life, $300; Miscellaneous, $150; Investments, $75. 2. House, $500; Food, $625; Clothing, $375; Higher Life, $625; Miscellaneous, $250; Investments, $125. 3. House, $228; Food, $516; Clothing, $132; Miscellaneous, $264; Investments, $60. 4. House, $750; Food, $750; Clothing, $450; Higher Life, $300; Health, $150; Miscellaneous, $300; Investments, $300.

Page 245. 1. $495. 2. $378.46; 4.2%. 3. 24; 5.6%. 4. 5.1%. 5. $520; 5.8%.

Page 246. 1. \$312; \$188. 2. Decreased; \$8. 3. \$3650; \$219. 4. \$23.06; \$2.94, less. 5. \$278.40, loss. 6. \$441.60, gain.

Page 253. 1. *A*, \$266.67; *B*, \$155.56. 2. *A*, 66.0%; *B*, 34.0%. 3. *A*, 68.9%; *B*, 31.1%. 4. *A*, 65.3%; *B*, 34.7%. 5. *A*, 63.2%; *B*, 36.8%. 10. 71.4%; 142.9%; 2000%. 11. 200%.

Page 256. 1. Massachusetts; New Hampshire. 3. Montana; Wyoming; New Mexico; Idaho; Ohio; Utah; Oregon; California; Michigan; Colorado.

Page 262. 1. 5.29. 2. 22.09. 3. $6\frac{1}{4}$. 4. $11\frac{1}{9}$. 5. 5.76. 6. 12.96. 7. 0.49. 8. 0.6561. 9. 1,440,000. 10. 12,974,404. 11. 11. 12. 13. 13. 25. 14. 18. 15. 27. 16. 24. 17. 36. 18. 33. 19. 3.5. 20. 6.4. 21. 2.2 in. 22. 1.4 ft. 23. 2.1 in. 24. 66 in. 25. 77 yd. 26. 3.3 yd. 27. 324 ft. 28. 440 in. 29. 560 ft. 30. 48.4 in. 31. 512 in. 32. 520 ft. 33. 320 ft.; 960 ft. 34. 540 ft.; 1080 ft.; 2. 35. 80 ft.; 50 ft. 36. 680 ft.; 2040 ft.; 3.

Page 264. 1. 57. 2. 61. 3. 63. 4. 71. 5. 82 ft. 6. 87 ft. 7. 95 ft. 8. 97 ft. 9. 89 ft. 10. 83 yd. 11. $\frac{1}{2}\frac{2}{3}$. 12. $\frac{3}{3}\frac{1}{2}$. 13. $\frac{3}{3}\frac{3}{4}$. 14. $\frac{4}{7}\frac{4}{1}$.

Page 266. 1. 25.63 ft. 2. 63 in. 3. 24.04 in. 4. 1.73 in. 5. 2.45 in. 6. 12.73 in.

Page 267. 1. 85 ft. 2. 2.83 ft. 3. 36.06 in. 4. 95 in. 5. 125 ft. 6. 22 ft. 6 in. 7. 16 ft. 6 in. 8. 42.52 ft. 9. 85 ft. 10. 535 ft. 11. 76.5 in. 12. 153 ft. 13. 91.92 in. 14. 36 ft. 15. 20.62 ft.

Page 268. 1. 45.45 ft. 2. 88.24 ft. 3. 18.03 in. 4. 76.62 ft. 5. 8.49 in. 6. 9.76 in. 7. 9.29 in. 8. 41.05 in. 9. 4.34 in. 10. 21.93 ft. 11. 3.46 in. 12. 50 ft. 13. 7.5 mi. per hr. 14. 8.94 mi. per hr.

Page 269. 1. 9. 2. 11. 3. 8. 4. 12. 5. 14. 6. 25. 7. 16. 8. 18. 9. 2.4 in. 10. 22 in.

Page 272. 1. 5. 2. 7. 3. 8. 4. 9. 5. 2.646. 6. 3.606. 7. 5.385. 8. 6.856. 9. 8.185. 10. 9.539. 11. 2. 12. 3. 13. 4. 14. 2.351. 15. 2.571. 16. 2.668. 17. 3.037. 18. 3.583. 19. 3.708. 20. 4.626. 21. 15. 22. 23. 23. 31. 24. 46. 25. 72. 26. 83. 27. 88. 28. 93. 29. 95. 30. 98. 31. 18. 32. 21. 33. 27. 34. 35. 35. 44. 36. 46. 37. 81. 38. 93. 39. 15. 40. 19. 41. 22. 42. 29. 43. 38. 44. 41. 45. 62. 46. 69. 47. 82. 48. 93. 49. 14. 50. 28. 51. 42. 52. 87. 53. 13 in. 54. 3.39 ft. 55. 688.90 cu. in. 56. 14.69 cu. in.

Page 273. 1. 160.33 cu. in. 2. 600.94 cu. ft. 3. 3933.38 cu. in. 4. 667.04 cu. ft. 5. 21 sq. in. 6. 27 sq. in. 7. 21 sq. in. 8. 18 sq. in. 9. 94,765,056 cu. ft.; 7,960,264.7 T.

Page 275. 1. 192 sq. in. 2. $229\frac{1}{2}$ sq. in. 3. $2\frac{3}{8}$ sq. ft. 4. 37.41 sq. in. 5. 15 sq. ft. 6. $2\frac{3}{16}$ sq. ft. 7. 1490.72 cu. in. 8. 623.7 cu. in. 9. 33,264 cu. in. 10. 946,176 sq. ft. 11. 960. 12. $183\frac{1}{3}$.

Page 315. *Exercise 1.* **1.** 114.07; 115.27; 122.37; 128.67; 131.87; 133.77; 142.67; 135.17. **2.** 92.9; 91.7; 84.6; 78.3; 75.1; 73.2; 64.3; 71.8. **3.** 50.41; 68.89; 237.16; 470.89; 620.01; 718.24; 1274.49; 795.24. **4.** 10.923; 12.769; 23.692; 33.385; 38.308; 41.231; 54.923; 43.385. **5.** 2.37; 2.77; 5.13; 7.23; 8.3; 8.93; 11.9; 9.4.

Exercise 2. **1.** 124.45; 129.45; 126.95; 132.85; 135.85; 137.15; 134.85; 135.45. **2.** 73.6; 68.6; 71.1; 65.2; 62.2; 60.9; 63.2; 62.6. **3.** 1393.92; 1971.92; 1670.42; 2422.08; 2857.68; 3057.62; 2708.48; 2797.52. **4.** 371.83; 442.25; 407.04; 490.14; 532.39; 550.70; 518.31; 526.76. **5.** 6.6; 7.85; 7.23; 8.7; 9.45; 9.78; 9.2; 9.35. 33; 39.25; 36.13; 43.5; 47.25; 48.88; 46; 46.75. 35.2; 41.87; 38.53; 46.4; 50.4; 52.13; 49.07; 49.87.

Page 316. *Exercise 3.* **1.** 375.1; 392.7; 360.8; 425.7; 403.7; 437.8; 422.4; 413.6. **2.** 965.9; 964.3; 967.2; 961.3; 963.3; 960.2; 961.6; 962.4. **3.** 119.35; 124.95; 114.8; 135.45; 128.45; 139.3; 134.4; 131.6. **4.** 2; 2; 2; 2; 2; 2; 2; 2. $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{1}{2}$. 48.03; 50.28; 46.20; 54.51; 51.69; 56.06; 54.08; 52.96. **5.** 250.1; 251.7; 248.8; 254.7; 252.7; 255.8; 254.4; 253.6.

Exercise 4. **1.** 4120; 4230; 4340; 4450; 4560; 4670; 4540; 4730. **2.** 41,200; 42,300; 43,400; 44,500; 45,600; 46,700; 45,400; 47,300. **3.** 5150; 5287.5; 5425; 5562.5; 5700; 5837.5; 5675; 5912.5. **4.** 47.6; 47.6; 47.6; 47.6; 47.6; 47.6; 47.6; 47.6. **5.** 82.4; 84.6; 86.8; 89; 91.2; 93.4; 90.8; 94.6. 20.6; 21.15; 21.7; 22.25; 22.8; 23.35; 22.7; 23.65. 0.0412; 0.0423; 0.0434; 0.0445; 0.0456; 0.0467; 0.0454; 0.0473.

Exercise 5. **1.** 94.073; 94.073; 94.073; 94.073; 94.073; 94.073; 94.073; 94.073. **2.** 1295; 1317.5; 1340; 1362.5; 1385; 1407.5; 1330; 1372.5. **3.** 4.32; 4.39; 4.47; 4.54; 4.62; 4.69; 4.43; 4.58. **4.** 60.43; 61.48; 62.53; 63.58; 64.63; 65.68; 62.07; 64.05. **5.** 41.44; 42.16; 42.88; 43.6; 44.32; 45.04; 42.56; 43.92. 64.75; 65.88; 67; 68.13; 69.25; 70.38; 66.5; 68.63.

Page 317. *Exercise 6.* **1.** 160.583; 161.483; 162.383; 163.283; 164.183; 165.083; 162.583; 163.783. **2.** 47.298; 46.398; 45.498; 44.598; 43.698; 42.798; 45.298; 44.098. **3.** 369.93; 375.24; 380.55; 385.86; 391.17; 396.48; 381.73; 388.81. **4.** 7.6; 7.71; 7.82; 7.93; 8.04; 8.15; 7.84; 7.99. **5.** 7.84; 7.95; 8.06; 8.18; 8.29; 8.4; 8.09; 8.24. 0.78; 0.80; 0.81; 0.82; 0.83; 0.84; 0.81; 0.82. 78.38; 79.5; 80.63; 81.75; 82.88; 84; 80.88; 82.38.

Exercise 7. **1.** 262.022; 262.922; 263.822; 264.722; 265.622; 266.522; 264.022; 264.922. **2.** 29.717; 28.817; 27.917; 27.017; 26.117; 25.217; 27.717; 26.817. **3.** 44.94; 45.5; 46.06; 46.63; 47.19; 47.75; 46.19; 46.75. 4493.75; 4550; 4606.25; 4662.5; 4718.75; 4775; 4618.75; 4675. **4.** 115.04; 116.48; 117.92; 119.36; 120.8; 122.24; 118.24; 119.68. 11.50; 11.65; 11.79; 11.94; 12.08; 12.22; 11.82; 11.97. 1150.4; 1164.8; 1179.2; 1193.6; 1208; 1222.4; 1182.4; 1196.8. **5.** 14.38; 14.56; 14.74; 14.92; 15.1; 15.28; 14.78; 14.96.

28.76; 29.12; 29.48; 29.84; 30.2; 30.56; 29.56; 29.92. 43.14; 43.68; 44.22; 44.76; 45.3; 45.84; 44.34; 44.88.

Exercise 8. 1. 82,700; 83,600; 84,500; 85,400; 86,300; 87,200; 86,900; 87,800. 2. 454.85; 459.8; 464.75; 469.7; 474.65; 479.6; 477.95; 482.9. 3. 2.07; 2.09; 2.11; 2.14; 2.16; 2.18; 2.17; 2.20. 20.68; 20.9; 21.13; 21.35; 21.58; 21.8; 21.73; 21.95. 206.75; 209; 211.25; 213.5; 215.75; 218; 217.25; 219.5. 4. 3308; 3344; 3380; 3416; 3452; 3488; 3476; 3512. 330.8; 334.4; 338; 341.6; 345.2; 348.8; 347.6; 351.2. 33.08; 33.44; 33.8; 34.16; 34.52; 34.88; 34.76; 35.12. 5. 196.7; 197.6; 198.5; 199.4; 200.3; 201.2; 200.9; 201.8.

Page 318. *Exercise 9.* 1. 53 ft. 4 in.; 70 ft.; 106 ft. 8 in.; 123 ft. 4 in.; 156 ft. 8 in.; 166 ft. 8 in.; 171 ft. 8 in.; 190 ft. 2. 7 ft. 5 in.; 6 ft. 7 in.; 4 ft. 9 in.; 3 ft. 11 in.; 2 ft. 3 in.; 1 ft. 9 in.; 1 ft. 6 in.; 7 in. 3. 5 ft. 4 in.; 7 ft.; 10 ft. 8 in.; 12 ft. 4 in.; 15 ft. 8 in.; 16 ft. 8 in.; 17 ft. 2 in.; 19 ft. 5 ft. 8 in.; 7 ft. $5\frac{1}{4}$ in.; 11 ft. 4 in.; 13 ft. $1\frac{1}{4}$ in.; 16 ft. $7\frac{3}{4}$ in.; 17 ft. $8\frac{1}{2}$ in.; 18 ft. $2\frac{7}{8}$ in.; 20 ft. $2\frac{1}{4}$ in. 56 ft. 8 in.; 74 ft. $4\frac{1}{2}$ in.; 113 ft. 4 in.; 131 ft. $\frac{1}{2}$ in.; 166 ft. $5\frac{1}{2}$ in.; 177 ft. 1 in.; 182 ft. $4\frac{3}{4}$ in.; 201 ft. $10\frac{1}{2}$ in. 4. 1 ft. 4 in.; 1 ft. 9 in.; 2 ft. 8 in.; 3 ft. 1 in.; 3 ft. 11 in.; 4 ft. 2 in.; 4 ft. $3\frac{1}{2}$ in.; 4 ft. 9 in. 8 in.; $10\frac{1}{2}$ in.; 1 ft. 4 in.; 1 ft. $6\frac{1}{2}$ in.; 1 ft. $11\frac{1}{2}$ in.; 2 ft. 1 in.; 2 ft. $1\frac{3}{4}$ in.; 2 ft. $4\frac{1}{2}$ in. $5\frac{1}{2}$ in.; 7 in.; $10\frac{3}{8}$ in.; 1 ft. $\frac{1}{2}$ in.; 1 ft. $3\frac{3}{4}$ in.; 1 ft. $4\frac{3}{8}$ in.; 1 ft. $5\frac{1}{8}$ in.; 1 ft. 7 in. 5. $10\frac{3}{4}$; 14; $21\frac{1}{3}$; $24\frac{3}{4}$; $31\frac{1}{3}$; $33\frac{1}{3}$; $34\frac{1}{3}$; 38. $6\frac{3}{8}$; $8\frac{3}{8}$; $12\frac{1}{4}$; $14\frac{3}{8}$; $18\frac{3}{4}$; 20; $20\frac{3}{8}$; $22\frac{1}{4}$. $1\frac{1}{3}$; $1\frac{3}{4}$; $2\frac{3}{8}$; $3\frac{1}{12}$; $3\frac{11}{12}$; $4\frac{1}{8}$; $4\frac{7}{24}$; $4\frac{3}{4}$.

Exercise 10. 1. $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$; $137\frac{7}{8}$. 2. 11,587.5; 12,150; 11,675; 11,787.5; 12,325; 12,100; 12,212.5; 12,300. 3. 2317.5; 2430; 2335; 2357.5; 2465; 2420; 2442.5; 2460. 4. 12.76; 13.38; 12.86; 12.98; 13.57; 13.33; 13.45; 13.55. 25.52; 26.76; 25.72; 25.96; 27.15; 26.65; 26.90; 27.09. 5. 3167 ft. 3 in.; 3321 ft.; 3191 ft. 2 in.; 3221 ft. 11 in.; 3368 ft. 10 in.; 3307 ft. 4 in.; 3338 ft. 1 in.; 3362 ft.

Exercise 11. 1. 4 yd. 19 in.; 14 ft. 2 in.; 5 yd. 12 in.; 5 yd. 22 in.; 6 yd. 22 in.; 12 ft. 11 in.; 14 ft. 6 in.; 10 ft. 10 in. 2. 7 yd. 8 in.; 21 ft. 1 in.; 6 yd. 15 in.; 6 yd. 5 in.; 5 yd. 5 in.; 22 ft. 4 in.; 20 ft. 9 in.; 24 ft. 5 in. 3. 3 yd. $19\frac{1}{2}$ in.; 145 in.; 5 yd. 20 in.; $6\frac{1}{4}$ yd.; $8\frac{3}{4}$ yd.; $107\frac{1}{2}$ in.; 155 in.; $3\frac{3}{4}$ ft. 35 yd. 15 in.; 1450 in.; 55 yd. 20 in.; $62\frac{1}{2}$ yd.; $87\frac{1}{2}$ yd.; 1075 in.; 1550 in.; $37\frac{1}{2}$ ft. $12\frac{3}{4}$ in.; $14\frac{1}{2}$ in.; 20 in.; $\frac{5}{8}$ yd.; $\frac{7}{8}$ yd.; $10\frac{3}{4}$ in.; $15\frac{1}{2}$ in.; $\frac{3}{8}$ ft. 4. 17 in.; $19\frac{1}{3}$ in.; $26\frac{3}{8}$ in.; $\frac{5}{8}$ yd.; $1\frac{1}{8}$ yd.; $14\frac{1}{3}$ in.; $20\frac{3}{8}$ in.; $\frac{1}{2}$ ft. $10\frac{1}{8}$ in.; $11\frac{3}{8}$ in.; 16 in.; $\frac{1}{2}$ yd.; $\frac{7}{10}$ yd.; $8\frac{3}{8}$ in.; $12\frac{3}{8}$ in.; $\frac{3}{10}$ ft. $7\frac{3}{7}$ in.; $8\frac{3}{7}$ in.; $11\frac{3}{7}$ in.; $\frac{5}{14}$ yd.; $\frac{1}{2}$ yd.; $6\frac{1}{7}$ in.; $8\frac{3}{7}$ in.; $\frac{3}{14}$ ft. $4\frac{1}{4}$ in.; $4\frac{5}{8}$ in.; $6\frac{3}{4}$ in.; $\frac{5}{24}$ yd.; $\frac{7}{24}$ yd.; $3\frac{7}{12}$ in.; $5\frac{1}{8}$ in.; $\frac{1}{8}$ ft. 5. 51; 58; 80; 90; 126; 43; 62; 18. 510; 580; 800; 900; 1260; 430; 620; 180. 204; 232; 320; 360; 504; 172; 248; 72.

Page 319. *Exercise 12.* 1. 16 lb. 15 oz.; 17 lb. 1 oz.; 17 lb. 8 oz.; 17 lb. 6 oz.; 18 lb. 4 oz.; 20 lb.; 22 lb. $3\frac{1}{2}$ oz.; 17 lb. 4 oz. 2. 7 lb. $4\frac{3}{8}$ oz.; 7 lb. $11\frac{1}{3}$ oz.;

9 lb. $2\frac{3}{5}$ oz.; 140 oz.; $186\frac{3}{5}$ oz.; 17 lb. 8 oz.; 24 lb. $14\frac{1}{3}$ oz.; 8 lb. $5\frac{1}{3}$ oz. $11\frac{2}{3}$ oz.; $12\frac{1}{3}$ oz.; $14\frac{2}{3}$ oz.; 14 oz.; $18\frac{2}{3}$ oz.; 1 lb. 12 oz.; 2 lb. $7\frac{5}{8}$ oz.; $13\frac{1}{3}$ oz. 1.17 oz.; 1.23 oz.; 1.47 oz.; 1.4 oz.; 1.87 oz.; 2.8 oz.; 3.98 oz.; 1.33 oz. **3.** 1 lb. $1\frac{1}{2}$ oz.; 1 lb. $2\frac{1}{2}$ oz.; 1 lb. 6 oz.; 21 oz.; 28 oz.; 2 lb. 10 oz.; 3 lb. $11\frac{3}{4}$ oz.; 1 lb. 4 oz. $3\frac{5}{8}$ oz.; $4\frac{1}{2}$ oz.; $4\frac{5}{8}$ oz.; $4\frac{3}{5}$ oz.; $6\frac{3}{8}$ oz.; $9\frac{1}{2}$ oz.; $13\frac{5}{16}$ oz.; $4\frac{4}{5}$ oz. 109 lb. 6 oz.; 115 lb. 10 oz.; 137 lb. 8 oz.; 2100 oz.; 2800 oz.; 262 lb. 8 oz.; 373 lb. 7 oz.; 125 lb. **4.** $8\frac{3}{4}$ oz.; $9\frac{1}{4}$ oz.; 11 oz.; $10\frac{1}{2}$ oz.; 14 oz.; 1 lb. 5 oz.; 1 lb. $13\frac{7}{8}$ oz.; 10 oz. 1 lb. $10\frac{1}{4}$ oz.; 1 lb. $11\frac{3}{4}$ oz.; 2 lb. 1 oz.; $31\frac{1}{2}$ oz.; 42 oz.; 3 lb. 15 oz.; 5 lb. $9\frac{5}{8}$ oz.; 1 lb. 14 oz. 4 lb. $14\frac{3}{4}$ oz.; 5 lb. $3\frac{1}{4}$ oz.; 6 lb. 3 oz.; $94\frac{1}{2}$ oz.; 126 oz.; 11 lb. 13 oz.; 16 lb. $12\frac{7}{8}$ oz.; 5 lb. 10 oz. **5.** 10 lb. 15 oz.; 11 lb. 9 oz.; 13 lb. 12 oz.; 210 oz.; 280 oz.; 26 lb. 4 oz.; 37 lb. $5\frac{1}{2}$ oz.; 12 lb. 8 oz. 1093 lb. 12 oz.; 1156 lb. 4 oz.; 1375 lb.; 21,000 oz.; 28,000 oz.; 2625 lb.; 3734 lb. 6 oz.; 1250 lb. 1.75 oz.; 1.85 oz.; 2.2 oz.; 2.1 oz.; 2.8 oz.; 4.2 oz.; 5.98 oz.; 2 oz.

Exercise 13. **1.** 363.823; 345.903; 352.933; 357.193; 366.173; 374.183; 372.173; 431.473. **2.** 11,079; 9735; 10,262.25; 10,581.75; 11,255.25; 11,856; 11,705.25; 16,152.75. **3.** 73.86; 64.9; 68.42; 70.55; 75.04; 79.04; 78.04; 107.69. 7.39; 6.49; 6.84; 7.05; 7.50; 7.90; 7.80; 10.77. 738.6; 649; 684.15; 705.45; 750.35; 790.4; 780.35; 1076.85. **4.** 5.91; 5.19; 5.47; 5.64; 6.00; 6.32; 6.24; 8.61. 59.09; 51.92; 54.73; 56.44; 60.03; 63.23; 62.43; 86.15. 5908.8; 5192; 5473.2; 5643.6; 6002.8; 6323.2; 6242.8; 8614.8. **5.** 4653.18; 4088.7; 4310.15; 4444.34; 4727.21; 4979.52; 4916.21; 6784.16.

Exercise 14. **1.** 10,001; 10,008; 10,065; 10,202; 10,545; 10,606; 10,568; 10,414. **2.** 100.01; 100.08; 100.65; 102.02; 105.45; 106.06; 105.68; 104.14. **3.** 41,479.15; 41,508.18; 41,744.59; 42,312.80; 43,735.39; 43,988.39; 43,830.78; 43,192.07. **4.** 3200.32; 3202.56; 3220.8; 3264.64; 3374.4; 3398.92; 3381.76; 3332.48. 320.03; 320.26; 322.08; 326.46; 337.44; 339.39; 338.18; 333.25. 32,003.2; 32,025.6; 32,208; 32,646.4; 33,744; 33,939.2; 33,817.6; 33,324.8. **5.** 33.34; 33.36; 33.55; 34.01; 35.15; 35.35; 35.23; 34.71. 133.35; 133.44; 134.2; 136.03; 140.6; 141.41; 140.91; 138.85. 75.01; 75.06; 75.49; 76.52; 79.09; 79.55; 79.26; 78.11.

Page 320. *Exercise 15.* **1.** $15\frac{7}{8}$ in.; $16\frac{1}{2}$ in.; $16\frac{9}{16}$ in.; $18\frac{1}{4}$ in.; $19\frac{1}{16}$ in.; $19\frac{2}{3}\frac{3}{2}$ in.; $16\frac{2}{3}\frac{5}{2}$ in.; $18\frac{7}{8}\frac{7}{4}$ in. **2.** $3\frac{7}{8}$ in.; $3\frac{1}{4}$ in.; $3\frac{3}{16}$ in.; $1\frac{1}{2}$ in.; $\frac{1}{16}$ in.; $\frac{1}{32}$ in.; $3\frac{3}{8}\frac{3}{4}$ in.; $1\frac{2}{8}\frac{1}{4}$ in. **3.** $13\frac{1}{2}$ in.; $15\frac{3}{8}$ in.; $15\frac{9}{16}$ in.; $20\frac{5}{8}$ in.; $23\frac{1}{16}$ in.; $24\frac{3}{32}$ in.; $15\frac{3}{8}\frac{3}{4}$ in.; $21\frac{9}{8}\frac{9}{4}$ in. $22\frac{1}{2}$ in.; $25\frac{5}{8}$ in.; $25\frac{5}{16}$ in.; $34\frac{3}{8}$ in.; $38\frac{7}{16}$ in.; $40\frac{5}{3}\frac{5}{2}$ in.; $25\frac{5}{6}\frac{5}{4}$ in. 144 in.; 164 in.; 166 in.; 220 in.; 246 in.; 257 in.; $160\frac{1}{2}$ in.; $225\frac{1}{2}$ in. **4.** 9 in.; $10\frac{1}{4}$ in.; $10\frac{3}{8}$ in.; $13\frac{3}{4}$ in.; $15\frac{3}{8}$ in.; $16\frac{1}{16}$ in.; $10\frac{1}{3}\frac{1}{2}$ in.; $14\frac{3}{3}\frac{3}{2}$ in. 90 in.; $102\frac{1}{2}$ in.; $103\frac{3}{4}$ in.; $137\frac{1}{2}$ in.; $153\frac{3}{4}$ in.; $160\frac{5}{8}$ in.; $100\frac{5}{16}$ in.; $140\frac{1}{16}$ in. 900 in.; 1025 in.; $1037\frac{1}{2}$ in.; 1375 in.; $1537\frac{1}{2}$ in.; $1606\frac{1}{4}$ in.; $1003\frac{1}{8}$ in.; $1409\frac{3}{8}$ in. **5.** $37\frac{1}{2}\%$; 42.7%; 43.2%; 57.3%; 64.1%; 66.9%; 41.8%; 58.7%. $12\frac{1}{2}\%$; 14.2%; 14.4%; 19.1%; 21.4%; 22.3%; 13.9%; 19.6%.

Exercise 16. 1. 11 qt. 1 pt.; 12½ qt.; 7 gal.; 13 qt. 1 pt.; 8 gal.; 3 gal.; 11 qt. 1¼ pt.; 4⅝ gal. 38 qt. 1 pt.; 37½ qt.; 5 gal. 2 qt.; 36 qt. 1 pt.; 4½ gal.; 9½ gal.; 38 qt. ¾ pt.; 7⅞ gal. 2. 21 pt.; 17½ qt.; 31 gal. 2 qt.; 24 qt. 1 pt.; 38½ gal.; 8½ gal.; 22¾ pt.; 14⅞ gal. 36 pt.; 30 qt.; 54 gal.; 42 qt.; 66 gal.; 6 gal.; 39 pt.; 25½ gal. 13½ pt.; 11¼ qt.; 20 gal. 1 qt.; 15 qt. 1½ pt.; 24¾ gal.; 2¼ gal.; 14⅝ pt.; $9\frac{9}{16}$ gal. 3. 1½ pt.; 1¼ qt.; 2 gal. 1 qt.; 1 qt. 1½ pt.; 2¾ gal.; ¼ gal.; 1⅝ pt.; $1\frac{1}{16}$ gal. 12 pt.; 10 qt.; 18 gal.; 14 qt.; 22 gal.; 2 gal.; 13 pt.; 8½ gal. 4. 5%; 8⅓%; 60%; 11⅔%; 73½%; 6⅔%; 5.4%; 28½%. 5. 4½ pt.; 3¾ qt.; 6 gal. 3 qt.; 5 qt. ½ pt.; 8¼ gal.; ¾ gal.; 4⅞ pt.; $3\frac{3}{16}$ gal. 7½ pt.; 6¼ qt.; 11 gal. 1 qt.; 8 qt. 1½ pt.; 13¾ gal.; 1¼ gal.; 8⅛ pt.; $5\frac{5}{16}$ gal.

Exercise 17. 1. 6662.812; 5672.212; 9671.482; 5749.702; 6674.182; 5673.092; 5643.832; 5708.742. 2. 3712.6; 4703.2; 703.93; 4625.71; 3701.28; 4702.32; 4731.58; 4666.67. 3. 1,657,398.76; 88,090.24; 18,456,217.4449; 140,093.0041; 1,686,673.6384; 88,613.3824; 72,049.2964; 111,108.8889. 4. 2; 2; 2; 2; 2; 2; 2; 2. 5. 35.9; 17.2; 65.5; 19.3; 36.0; 17.3; 16.4; 18.3.

Page 323. 1. 160 A. 2. 160 A. 3. 80 A. 4. 80 A. 5. 80 A. 6. 80 A. 7. 40 A. 8. 40 A. 9. $10,400. 10. $3000. 11. 160 A. 12. 320 A. 13. 480. 14. 2 mi. 15. 2; $24.

Page 325. 1. 1 hr. 48 min. 17 sec. 2. 5 hr. 2 min. 2 sec. 3. 11 hr. 8 min. 10 sec. A.M. 4. 105°; 9 P.M. 5. 4 A.M. 6. 8 hr. 22 min. 42 sec. A.M.; 8 hr. 2 min. 42 sec. A.M.

Page 326. 1. 30° 47′ 30″. 2. 105° W. 3. 90° W. 4. 22° 45′ W.

Page 327. 1. 11 A.M. 3. 10 A.M.; 10 A.M.; 8 A.M.; 11 A.M. 4. 11 A.M. 5. 10 A.M. 6. 1 P.M. 7. 7 A.M.

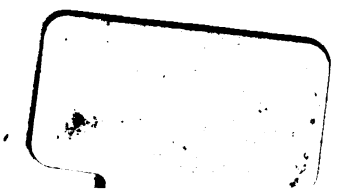
Page 329. 1. 1850.39 in. 2. 1358.27 in. 3. 2047.24 in. 4. 2657.48 in. 5. 127.56 in. 6. 1.11 in. 7. 184.25 in. 8. 118.11 in. 9. 2.95 in. 10. 666.01 ft. 11. 123.03 ft. 12. 51.18 in. 13. 46.8 mi. 14. 74.4 mi. 15. 27.9.

Page 330. 1. 7,500,000,000,000 cm^2. 2. 370,000 cm^2. 3. 42.96 cm^2. 4. 62,500 cm^2. 5. 19.75 m^3. 6. 0.82642765384 m^3. 7. 0.007 m^3. 8. 3.5 m^3. 9. 0.000937 m^3. 10. 0.486 m^3.

Page 331. 1. 170 kg. 2. 194.55 kg. 3. 92.73 kg. 4. 10 kg. 5. 22 kg. 6. 272,727.27 kg. 7. 3.27 kg. 8. 7.29 kg. 9. 79 kg. 10. 7.38 kg. 11. 4.9 kg. 12. 6.35 kg. 13. 750 l. 14. 4 l. 15. 225 l. 16. 50 l. 17. 25 l. 18. 2000 l. 19. 1500 l. 20. 683 l.

Page 333. 1. $2300. 2. 1.5%. 3. 2.63. 4. 45. 5. 20. 6. 48.

Page 334. 1. 999,999. 2. 111; 222; 333; 444; 555; 666; 777. 5. 20 lb. 6. 49. 7. 24 ft. 8. 5¢. 9. 39,700 lb. 10. 30 da.



8 1 6
3 5 7
4 9 2